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A double pendulum with friction under the electromagnetic forcing and kinematic excitation: modeling and numerical simulation

Godiya Yakubu[1]*, Paweł Olejnik[2], Jan Awrejcewicz[3]

[1,2,3]Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, 1/15 Stefanowski Str., 90-924 Lodz, Poland

[1]godiya.yakubu@dokt.p.lodz.pl, [2]pawel.olejnik@p.lodz.pl, [3]jan.awrejcewicz@p.lodz.pl,

[G.Y. ORCID: 0000-0001-8295-7890, P.O. ORCID: 0000-0002-3310-0951, J.A. ORCID: 0000-0003-0387-921X]

* Presenting Author

Plan of the presentation

1. Introduction

- motivation to undertake research on the basis of literature analysis
- research innovativeness
- research aim and hypothesis

2. Materials and Methods: Research Methodology

- Mathematical modeling of an extended swinging Atwood machine with Friction
- Modeling friction in pulley bearings
- Mathematical Model of Electromagnetic Actuator

3. Numerical results of simulation

4. Conclusions

5. References

Introduction

- The variable-length pendulum: the swinging Atwood machine
- The double pendulum: the extended swinging Atwood machine
- Electromagnetic forcing and Kinematic excitation of suspension points

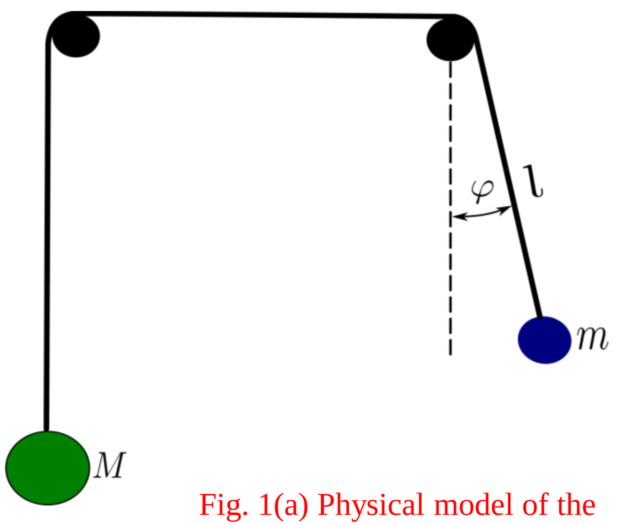


Fig. 1(a) Physical model of the Swinging Atwood Machine

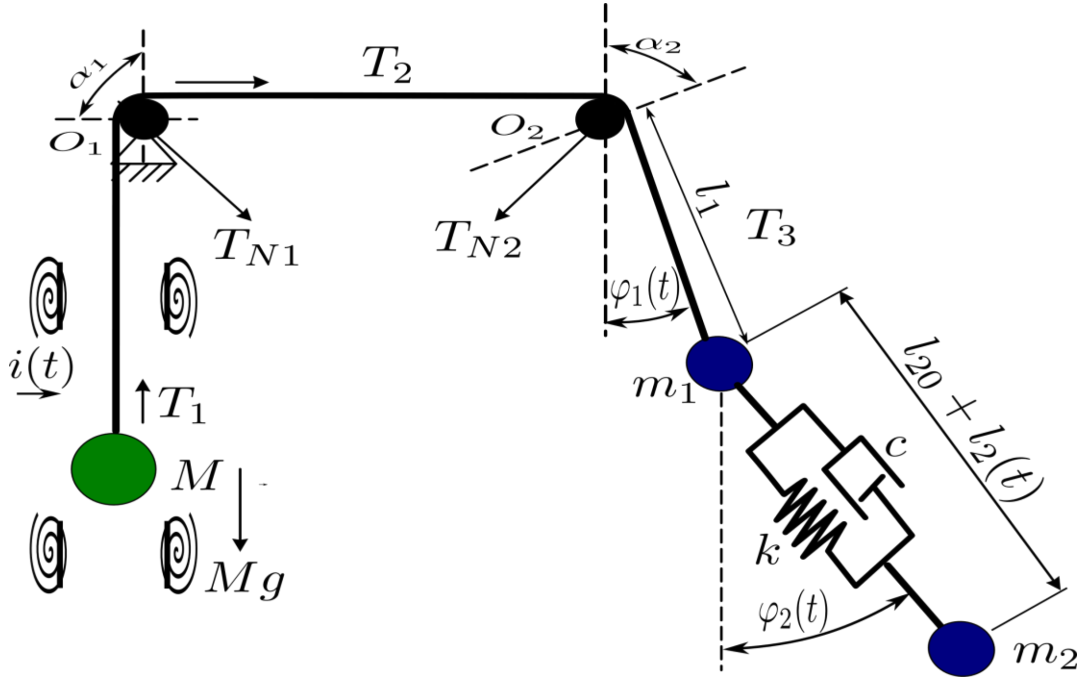


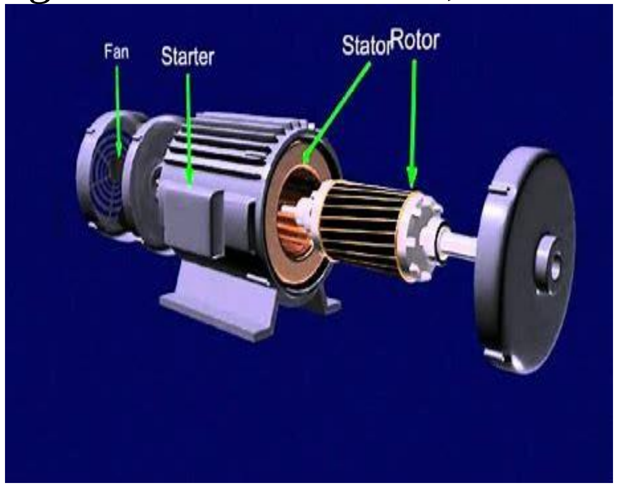
Fig. 1 (b) Original Modified Swinging Atwood Machine

Introduction cont…

- Various applications in mechatronic systems, e.g., robots, electro-mechanical systems like induction motors, purely electrical networks like dc-dc-power converters, lifting devices like cranes, mine elevators, earthquake detections, wave energy converters, etc.

Robots

Induction motor



Mine elevator

Crane

Motivation to undertake research on the basis of literature analysis

- Based on the carried-out literature analysis, many variable-length pendulums are very demanding in the modeling and analysis of parametric dynamical systems.

- Applications in mechanical and mechatronic systems are observed in practice and in theory as well.

- The problem employs modern tools in methods of solution and analysis of parametric dynamical systems.

Motivation to undertake research on the basis of literature analysis

The extended swinging Atwood machine with friction and electromagnetic forcing also aimed at modeling physical systems and dynamics of loads carried by cranes placed on ships sailing at sea

Research innovativeness

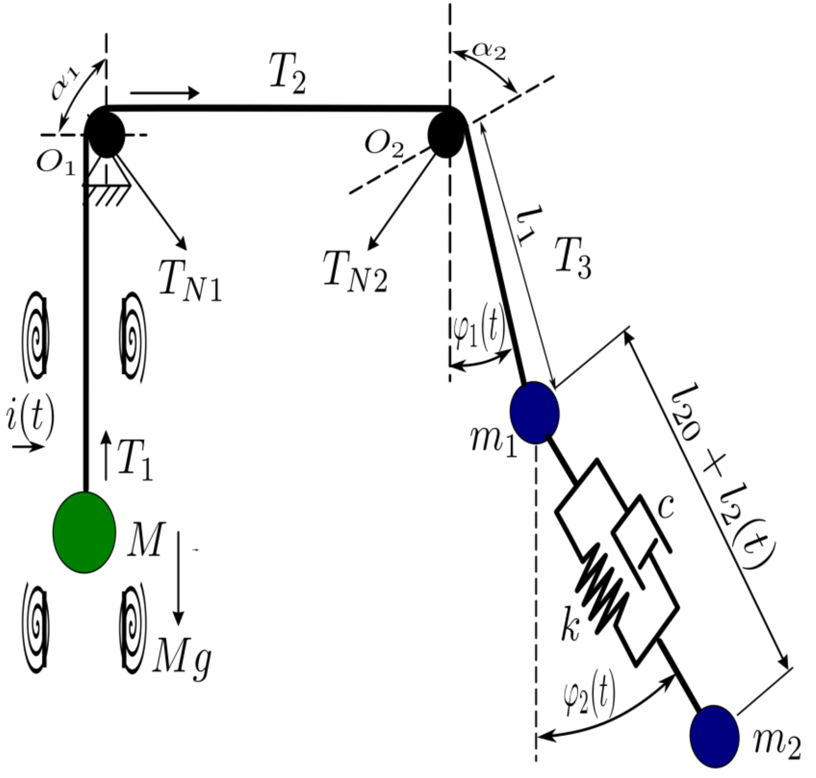
- Modification of a swinging Atwood's mechanism to present influence of damping and length variability on a kinematically and magnetically forced triple pendulum with friction in pulley bearings

Research aim and hypothesis

- An in-depth literature review
- Development of the mathematical model
- Numerical simulations and deep dynamical analysis in Python/Julia/LabVIEW

Materials and Methods

Mathematical modeling of an extended swinging Atwood machine with Friction



$$\ddot{l}_1=\frac{1}{m_1+M}\Big(T_{t,3}\cos\varphi_{21}-\ddot{X}_0\big(m_1\sin\varphi_1+M\big)+m_1\big(g\cos\varphi_1+l_1\dot{\varphi}_1^2\big)-T_R-Mg\Big),$$

$$\ddot{l}_2=\frac{1}{2m_1m_2(m_1+M)}\Big(Mm_2T_{t,3}\big(\cos2\varphi_{21}-1\big)+m_1m_2\Big(M\ddot{X}_0\big(2\cos\varphi_{21}+\sin(\varphi_2-2\varphi_1)-\sin\varphi_2\big)$$
$$+gM\big(2\cos\varphi_{21}+\cos(\varphi_2-2\varphi_1)+\cos\varphi_2\big)+2\big((Ml_1\dot{\varphi}_1+T_R)\cos\varphi_{21}+(m_1+M)L_2\dot{\varphi}_2^2-T_{t,3}\big)\Big)$$
$$-2m_1T_{t,3}(m_1+M)\Big)\qquad(1)$$

$$\ddot{\varphi}_1=\frac{1}{m_1l_1}\Big(T_{t,3}\sin\varphi_{21}-m_1\big(g\sin\varphi_1+\ddot{X}_0\cos\varphi_1+2\dot{l}_1\dot{\varphi}_1\big)\Big),$$

$$\ddot{\varphi}_2=\frac{1}{2m_1L_2(m_1+M)}\Big(-MT_{t,3}\sin2\varphi_{21}-Mm_1\Big(g\big(2\sin\varphi_{21}+\sin(\varphi_2-2\varphi_1)+\sin\varphi_2\big)$$
$$+\ddot{X}_0\big(2\sin\varphi_{21}-\cos(\varphi_2-2\varphi_1)+\cos\varphi_2\big)+2l_1\dot{\varphi}_1^2\sin\varphi_{21}\Big)-2m_1\big(T_R\sin\varphi_{21}+2(m_1+M)\dot{l}_2\dot{\varphi}_2\big)\Big),$$

Materials and Methods

➢ **Modeling friction in pulley bearings** The friction variable T_R is the sum of the friction at the first pulley T_{R1} and the friction at the second pulley T_{R2}.

$$T_R = T_{R1} + T_{R2}$$

$$T_{R1} = \mu_{p1} T_{N1} + T_{pv1}(\dot{l}_1)$$

$$T_{R2} = \mu_{p2} T_{N2} + T_{pv2}(\dot{l}_1)$$

$$T_{N1} = \sqrt{T_1^2 - 2T_1T_2\cos(\alpha_1) + T_2^2} \qquad (5)$$

$$T_{N2} = \sqrt{T_2^2 - 2T_2T_3\cos(\alpha_2) + T_3^2}$$

$$T_1 = Mg - m_1\ddot{l}_1(t)$$

$$T_1 = \frac{T_2\left(2 + sin(\dot{l}_1)\mu_{p1}\sqrt{2(1-\cos(\alpha_1))}\right) + 2T_{pv1}(\dot{l}_1)}{\left(2 - sin(\dot{l}_1)\mu_{p1}\sqrt{2(1-\cos(\alpha_1))}\right)}$$

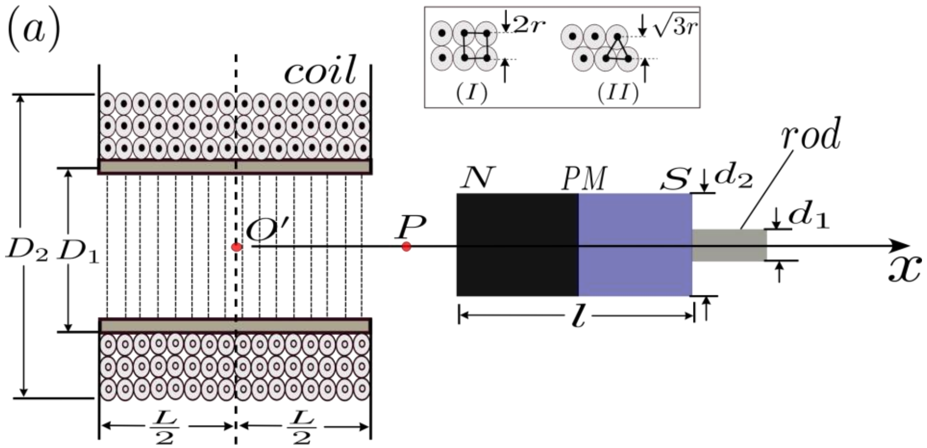
$$T_2 = \frac{T_3\left(2 + sin(\dot{l}_1)\mu_{p2}\sqrt{2(1-\cos(\alpha_2))}\right) + 2T_{pv2}(\dot{l}_1)}{\left(2 - sin(\dot{l}_1)\mu_{p2}\sqrt{2(1-\cos(\alpha_2))}\right)} \qquad (6)$$

The wrapping angle at the first and second pulley is given by
$\alpha_1 = 90^0$ and $\alpha_2 = (90 - \varphi_1)^0$ respectively.

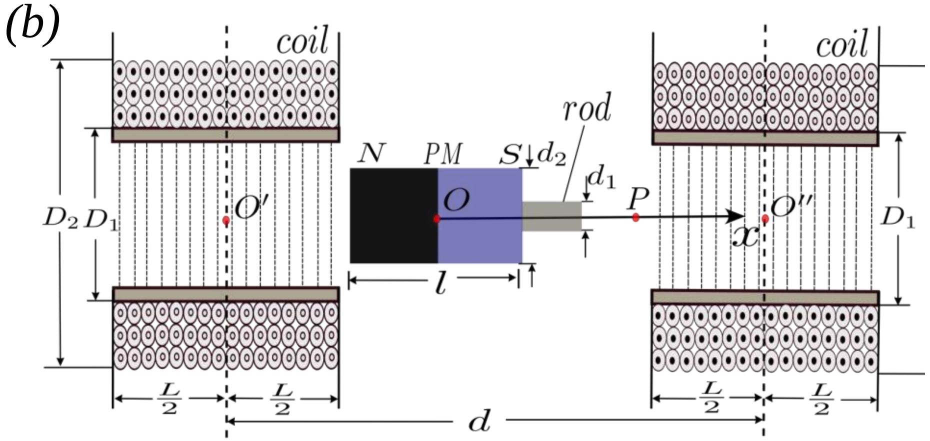
Materials and Methods

➢ Mathematical Model of Electromagnetic Actuator

$\ddot{X}_0 = -\omega^2 f_0 \sin \omega t$ Where f_0 is the excitation force and ω is the excitation frequency



2 (a) The single solenoid actuator



2 (b) The sectional view of the twin-solenoids actuator

$$F_x = \frac{B_r \pi (d_2^2 - d_1^2)}{4\mu_0}\left[B_x\left(x - \frac{l}{2}\right) - B_x\left(x + \frac{l}{2}\right)\right] \quad (6)$$

$$B_x = \frac{\mu_0 N I R^2}{2(D_2 - D_1)L}\left[(L-2x)\ln\frac{D_2 + \sqrt{(2x-L)^2 + D_2^2}}{D_1 + \sqrt{(2x-L)^2 + D_1^2}} + (L+2x)\ln\frac{D_2 + \sqrt{(2x+L)^2 + D_2^2}}{D_1 + \sqrt{(2x+L)^2 + D_1^2}}\right]$$

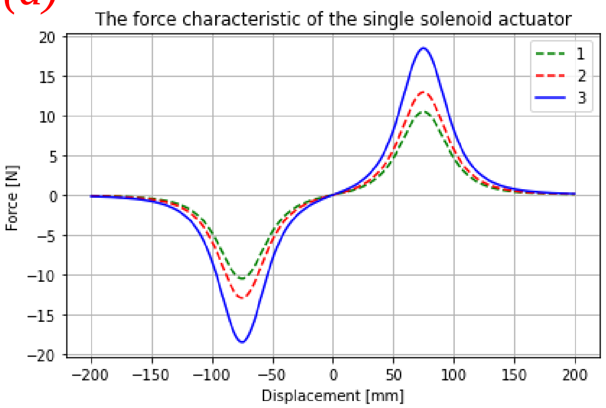
$B_x(\cdot)$ is the function with respect to the variable x

$$f_0 = F_x\left(x - \frac{d}{2}\right) + \left(x - \frac{d}{2}\right) \quad (7)$$

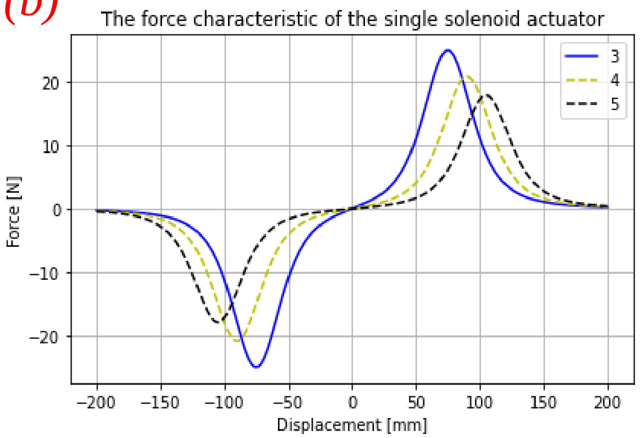
$F_x(\cdot)$ is the function of the variable x

Numerical results of simulation of the Electromagnetic Actuator

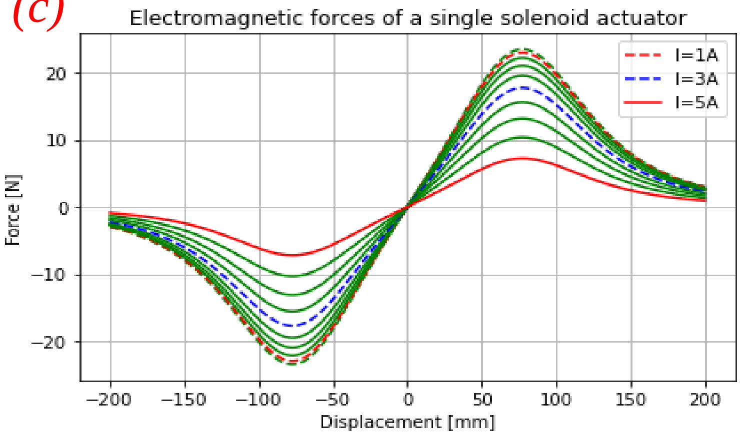
(a)



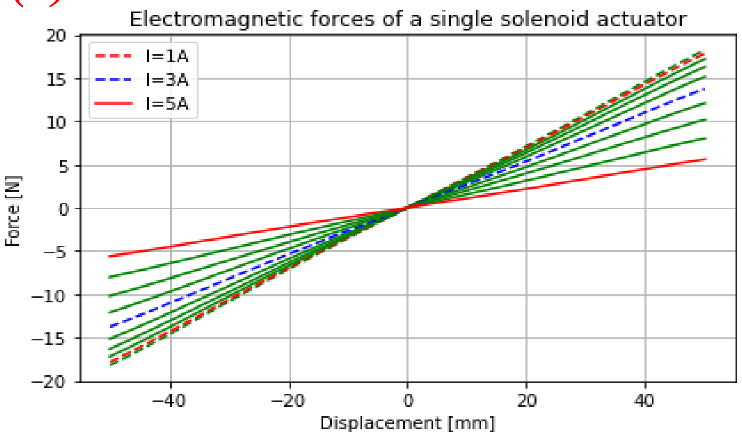
(b)



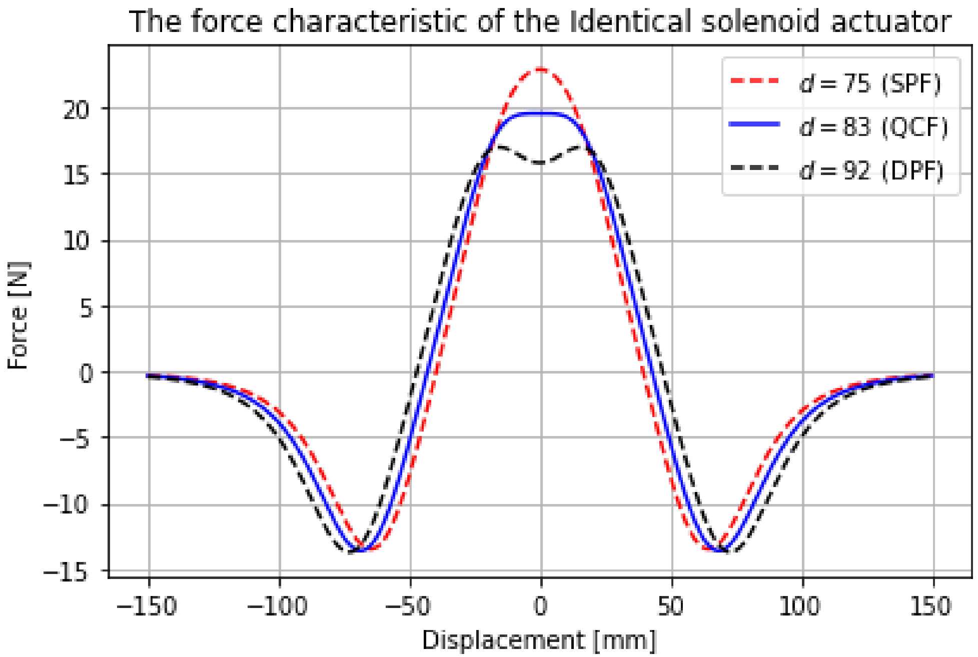
(c)



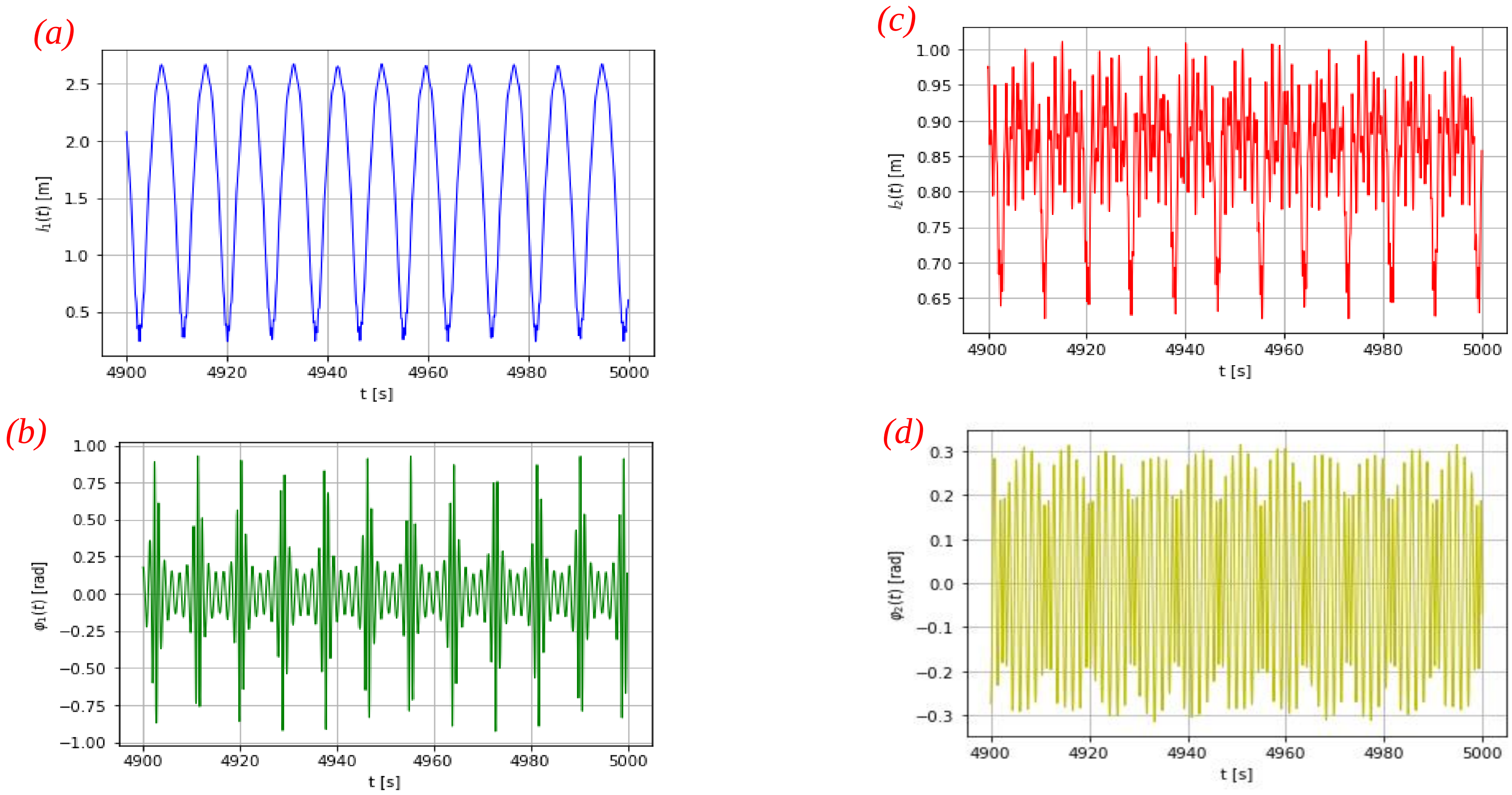
(d)



(e)

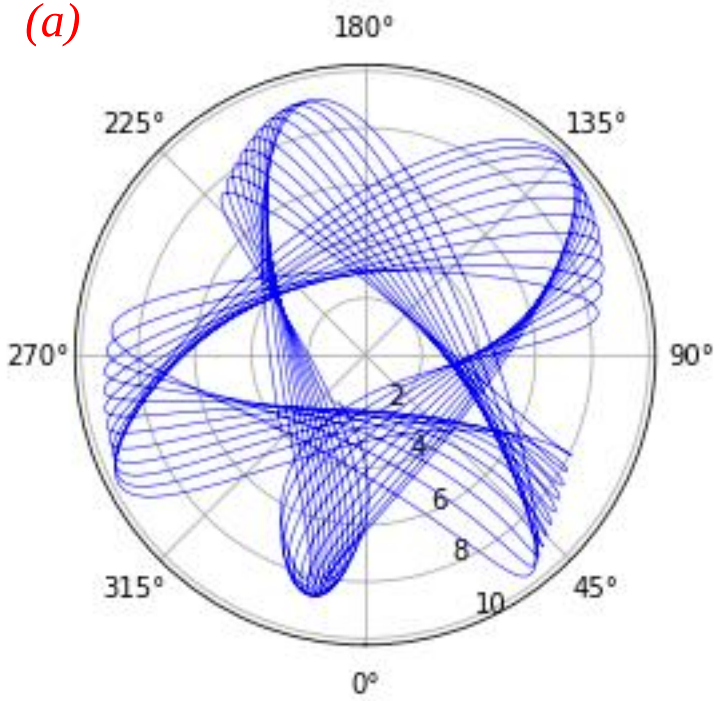


Numerical results of simulation of the modified SAM – Time History



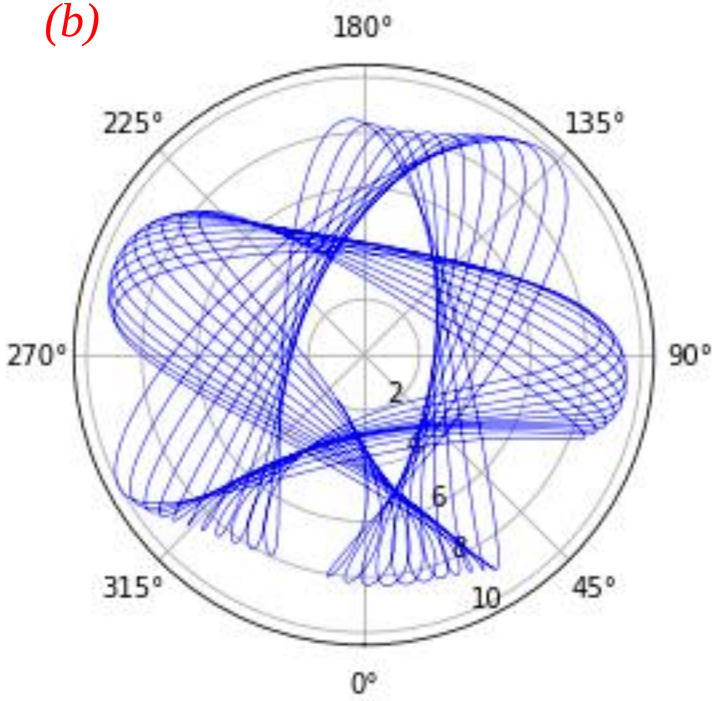
Numerical results of simulation of the modified SAM: An orbit of the extended SAM for $M = 77, m_1 = 25, m_2 = 9, \varphi_1 = \frac{\pi}{6}, \varphi_2 = -\frac{\pi}{6}$

(a)



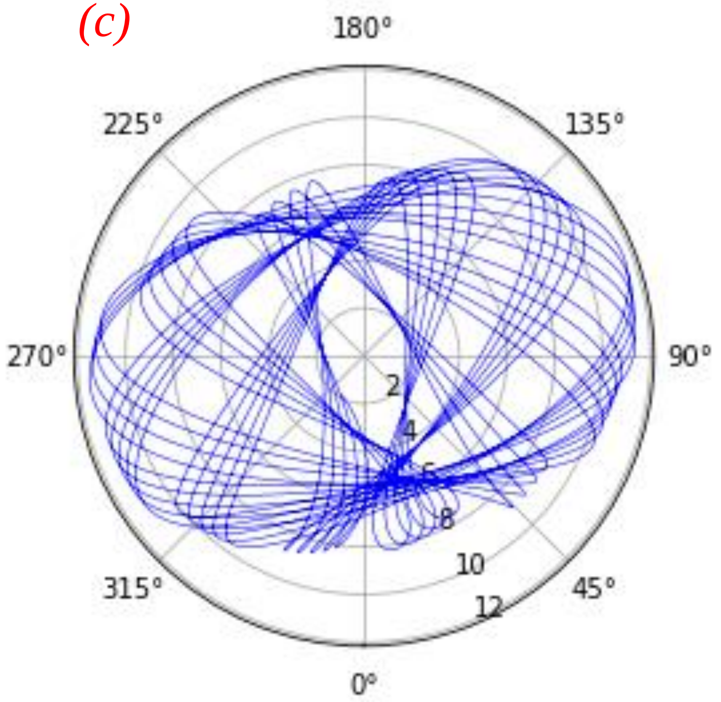
(a) initial velocity equal to zero

(b)



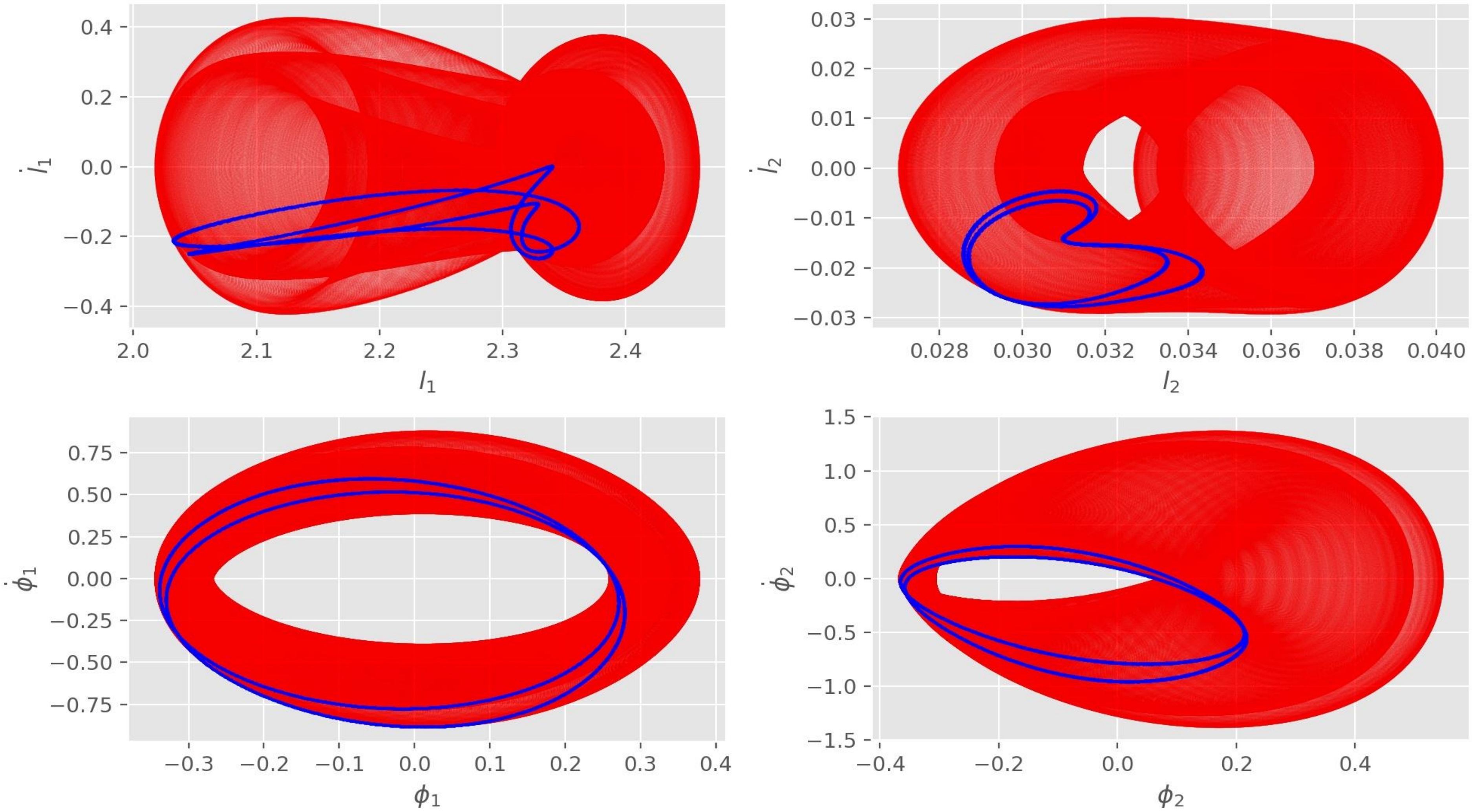
(b) initial velocity equal to 0.01

(c)



(c) initial velocity equal to 0.05

Numerical results of simulation of the modified SAM: Poincare Section (Blue) on Phase Portrait (Red) - Quasi-Periodic



More results

Extended work on the dynamical analysis is ongoing and will be sent for publication in springer soon

Numerical simulation and analysis of Mathiu and Hill's equations

4. Conclusions

- The extended SAM presents a novel SAM concept applicable in the modeling of engineering objects.

- It is bases on a variable-length double pendulum with friction under the electromagnetic forcing and kinematic excitation and a suspension between the two pendulums.

- The numerical simulation results, clearly show that the nonlinear dynamics of the extended SAM presented can be thoroughly studied, and more modifications can be achieved.

- Interestingly, in some regimes, compact regions of attraction appear in the system.

- The new technique can reduce residual vibrations through damping when the desired level of an engineering object, e.g., a crane, is reached. It can also be used in the modeling of nonlinear mechatronic and robotic.

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