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Synchronization and Energy Transfer in 4DoF Friction-Induced Self- and Parametrically Excited Oscillators

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Presentation Plan

1. Introduction
- Innovativeness

2. System Model
- Derived Equations of motion
- Investigation scenarios

3. Discussion of Scientific Results
4. Conclusion
5. References
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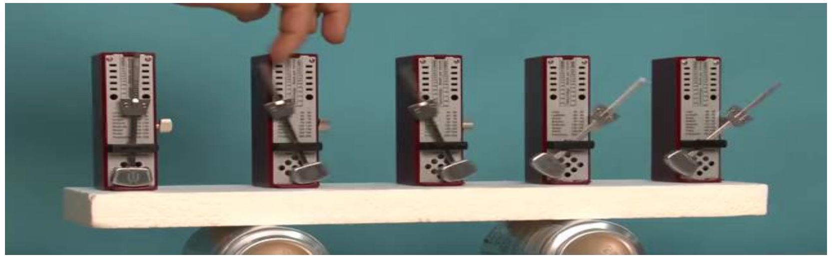
Introduction

A phenomenon where oscillators with different frequencies adjust their rhythms to oscillate in unison. The coupling is responsible for the flow of energy between them.



Synchronization and Energy Transfer in 4DoF Friction-Induced Self- and Parametrically Excited Oscillators

Oscillations are driven by varying some parameter(s) at some frequenc(y/ies), typically different from the natural frequency

Present in mechanical/ electromechanical devices where rubbing surfaces introduce some kinds of energies (negative damping) into the oscillating unit.

Other works VS Innovativeness

- Phenomenon such as vibration quenching, anti-resonance, and synchronization have been studied by Authors such as Warmiński, Litak, Dohnal and Ecker in [1-3], and simultaneously occurring parametric and self excited vibrations by Tondl, Szabelski and Warmiński [5-8]. However, their study is limited to continuous/2DoF systems. We are studying this phenomena in discontinuous system

- The system under study is an expansion of the model in [4]*, it models a caliper mass and stiffness, with parametric excitation provided by unbalanced rotating parts.

- Found in deep drilling, and automobile brake system

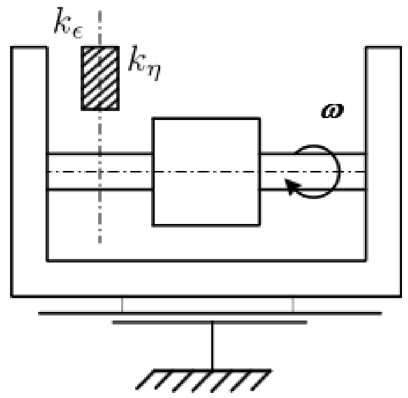


Fig. 1: 3DoF system

[4] Awrejcewicz, et al "Asymptotic Approaches in Nonlinear Dynamics," Springer ,1998

System model:

- A 4DoF electromechanical system (Fig. 2) with frame M_1 placed of moving belt with constant velocity v_0
- The frame encloses a weightless shaft with rectangular cross-section, and a cylinder-like mass m_1 concentrated at the center
- M_1 is coupled to another mass m_4, placed on a frictionless surface, using a linear coupling spring of stiffness k_c

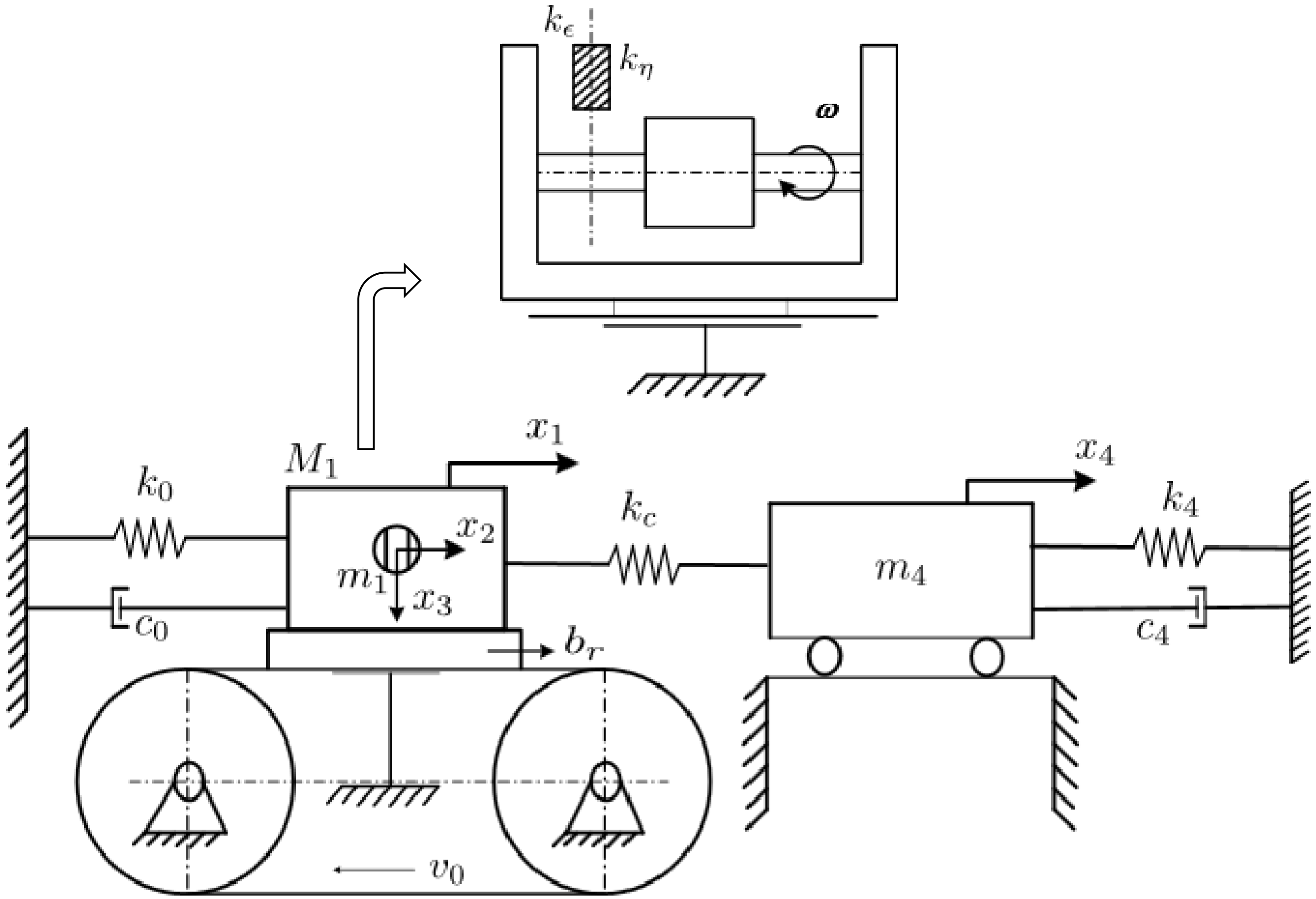


Fig. 2: Model of rotor with parametric excitation placed on friction-induced self-excited support, connected to a supplementary degree of freedom

Derived Equations of Motion

In nondimensional form

$$\ddot{X}_1 = -X_1(A_0 + \eta_2 d + \eta_1 b_1 + \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 + X_2(\eta_1 b_1 + \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 - X_3(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) + X_4 \eta_2 d\gamma^2 - \dot{X}_1 h_1 \gamma + [X_1(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) - X_2(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) + X_3(\eta_1 b_1 - \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 + 1]b_r \quad (1)$$

$$\ddot{X}_2 = X_1(b_1 + b_2 \cos(2\gamma\tau))\gamma^2 - X_2(b_1 + b_2 \cos(2\gamma\tau))\gamma^2 + X_3(b_2 \gamma^2 \sin(2\gamma\tau)) + \kappa\gamma^2 \sin(\gamma\tau + \phi_0) \quad (2)$$

$$\ddot{X}_3 = -X_1(b_2 \gamma^2 \sin(2\gamma\tau)) + X_2(b_2 \gamma^2 \sin(2\gamma\tau)) - X_3(b_1 \gamma^2 - b_2 \gamma^2 \cos(2\gamma\tau)) + \kappa\gamma^2 \cos(\gamma\tau + \phi_0) + 1 \quad (3)$$

$$\ddot{X}_4 = X_1 d\gamma^2 - X_4 - \dot{X}_4 h_4 \gamma \quad (4)$$

State space variables: $(y_1, v_1, y_2, v_2, y_3, v_3, y_4, v_4)$

Power/Energy Balance Equation

$$\dot{X}_1\ddot{X}_1 + \dot{X}_1 X_1 A_0 \gamma^2 + \dot{X}_1 \eta_2 d\gamma^2 (X_1 - X_4) + \dot{X}_1 \dot{X}_1 h_1 \gamma = \dot{X}_2 F_{pe} + \dot{X}_1 F_{br} \quad (5)$$

$$\dot{X}_4\ddot{X}_4 - \dot{X}_4 X_1 d\gamma^2 + \dot{X}_4 X_4 + \dot{X}_4 \dot{X}_4 h_4 \gamma = 0 \quad (6)$$

where

$$F_{pe} = -X_1(\eta_1 b_1 + \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 + X_2(\eta_1 b_1 + \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 - X_3(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) \quad (7)$$

$$F_{br} = (X_1(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) - X_2(\eta_1 b_2 \gamma^2 \sin(2\gamma\tau)) + X_3(\eta_1 b_1 - \eta_1 b_2 \cos(2\gamma\tau))\gamma^2 + 1) b_r \quad (8)$$

With ref to the work of Kulke and Ostermeyer in [9]

State space variables: $(y_1, v_1, y_2, v_2, y_3, v_3, y_4, v_4)$

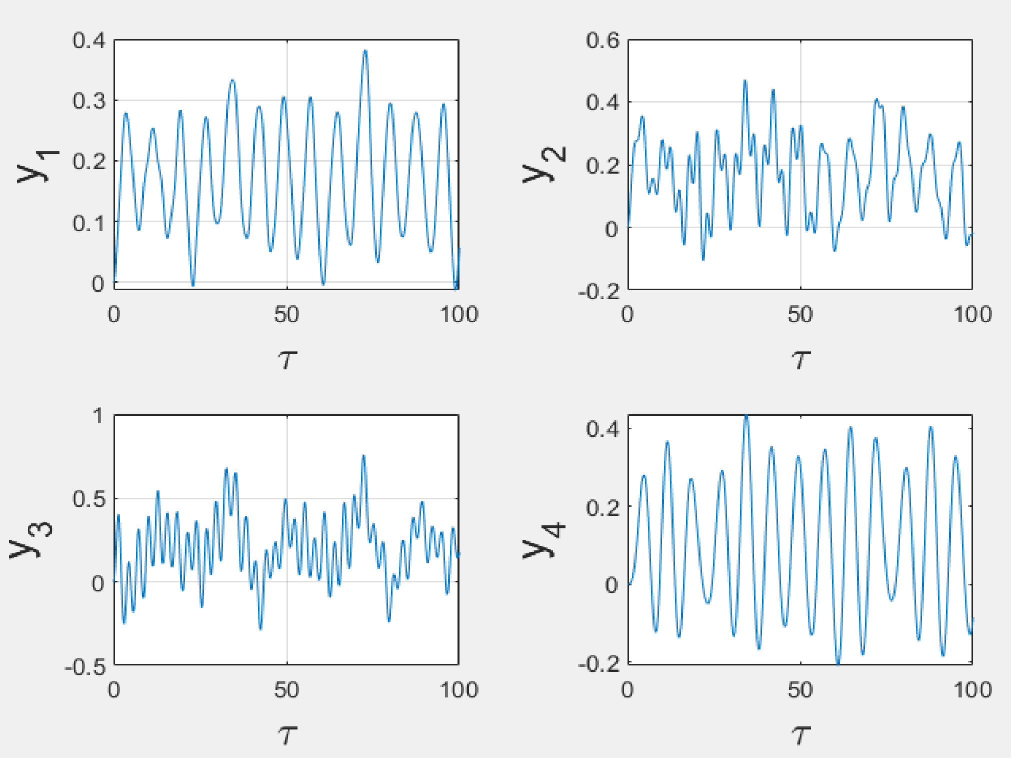
Investigation scenarios and parameters

Table 1: Simulation parameters

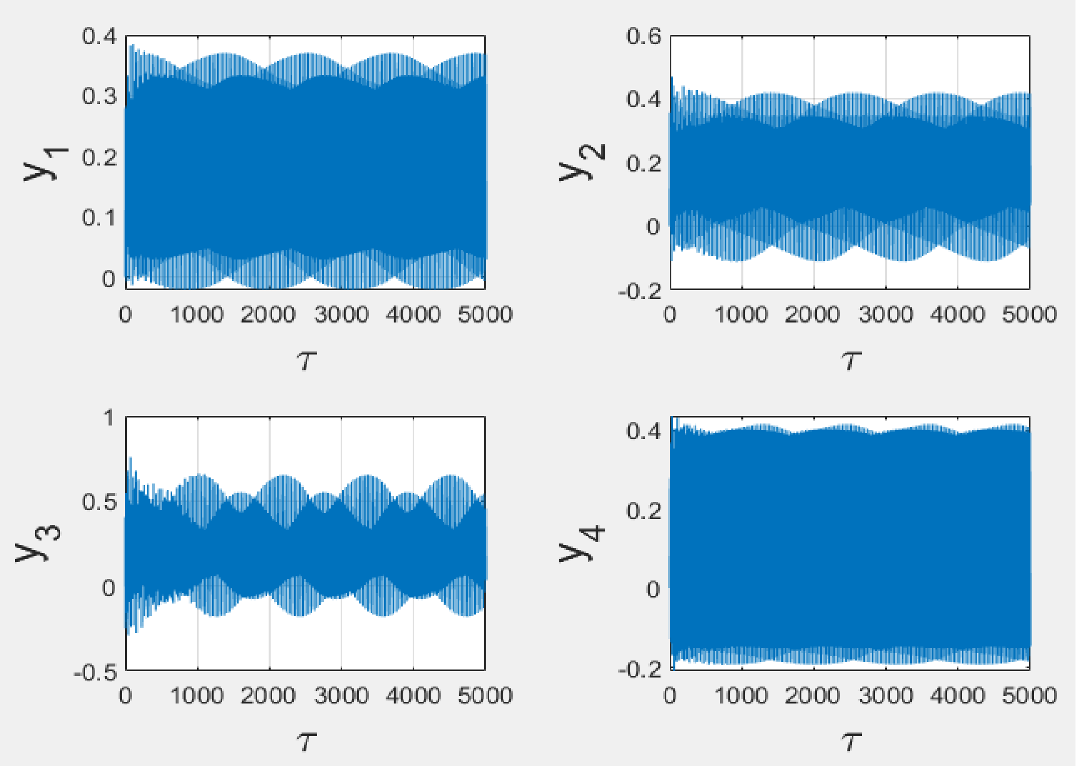
Fixed Parameters		
$\varphi_0 = \frac{4\pi}{9}, \gamma = 0.066, \kappa = 8.632, h1 = 1.9231, h4 = 0.1, \eta_1 = 0.1, \eta_2 = 0.2$		
Data Sets		
Parameter	***Data Set 1 (DS1)***	***Data Set 2 (DS2)***
b_1	177.5148	710.0592
b_2	5.3184	8.1462
d	22.1893	44.3787
A_0	33.2840	532.5444

*Primary Set: **DS1**
Secondary: DS2,

Discussion of Scientific results



(a) There exist local stability at the origin



(b) There is loss of local stability resulting in bifurcation to quasiperiodic states

Fig. 3: Nonlinear dynamical responses to initial conditions [0,0,0,0,0,0,0,0], on DS1 with (a) Equilibrium states at the origin, (b) Hopf (secondary) bifurcation to quasiperiodic states in finite time τ

Discussion of Scientific Results Cont…

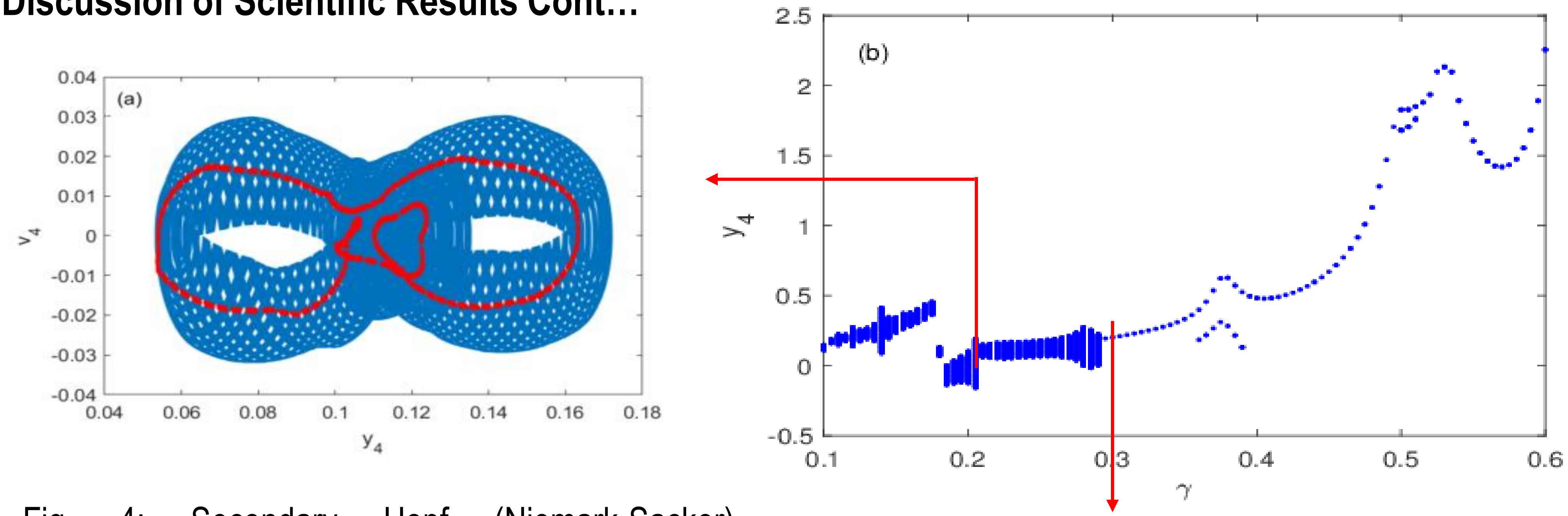
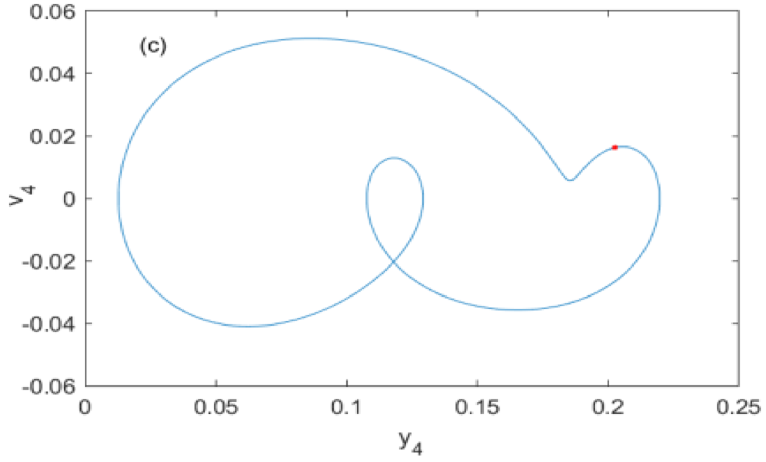


Fig. 4: Secondary Hopf (Niemark-Sacker) bifurcation scenario with phase portrait and Poincare section showing the existence of limit cycles (a)[$\gamma = 0.22$], in the bifurcation diagram (b), and destruction of the limit cycles to periodic states in (c) [$\gamma = 0.3$], with respect to structure m_4



Discussion of Scientific Results Cont…

Synchronization phenomenon

…from mutual interaction between self- and parametric excitations

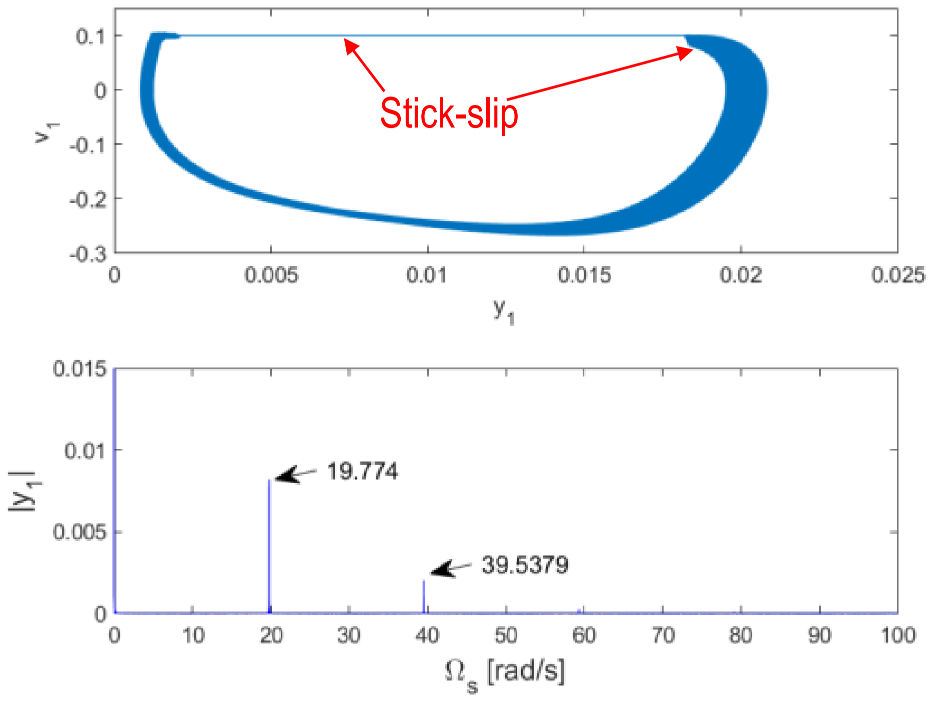


Fig. 5: Phase portrait and FFT of y_1, having $\Omega_{s1} = 19.77\ rad/s$

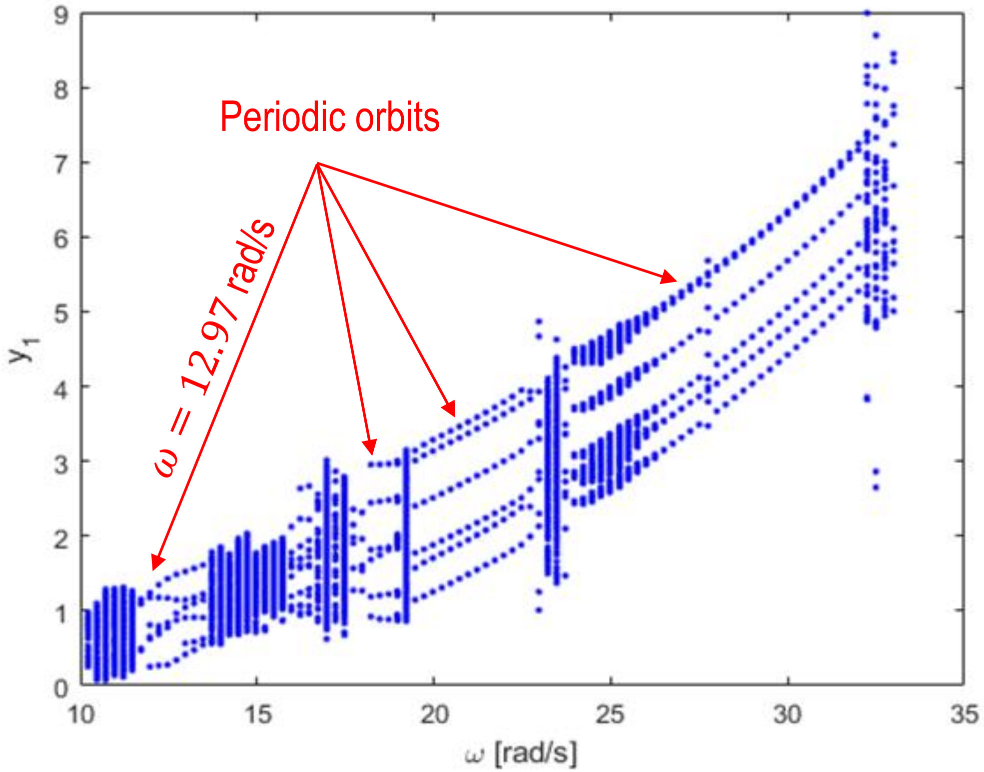


Fig. 6: Bifurcation diagram w.r.t y_1 using parametric frequency ω as the control parameter

Discussion of Scientific Results Cont…

Synchronization phenomenon…

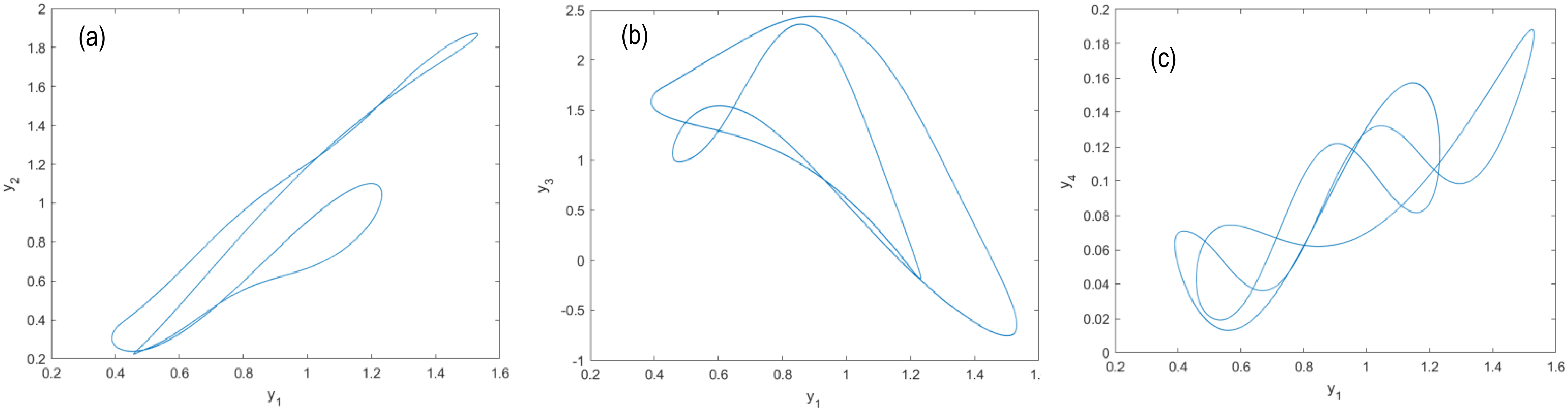


Fig. 7: Lissajous plot of mass M_1 with other oscillators at $\omega = 12.97$ rad/sec. Closed curves in all plots show synchronization in the system for (a) (y_1, y_2), (b) (y_1, y_3), (a) (y_1, y_4)

Discussion of Scientific Results Cont…

Energy transfer

(A): Change of attractor → low power absorption

(B): New bifurcation point → low dissipation

(C): Near resonance ($p = 0.8$) → equal absorption and dissipation

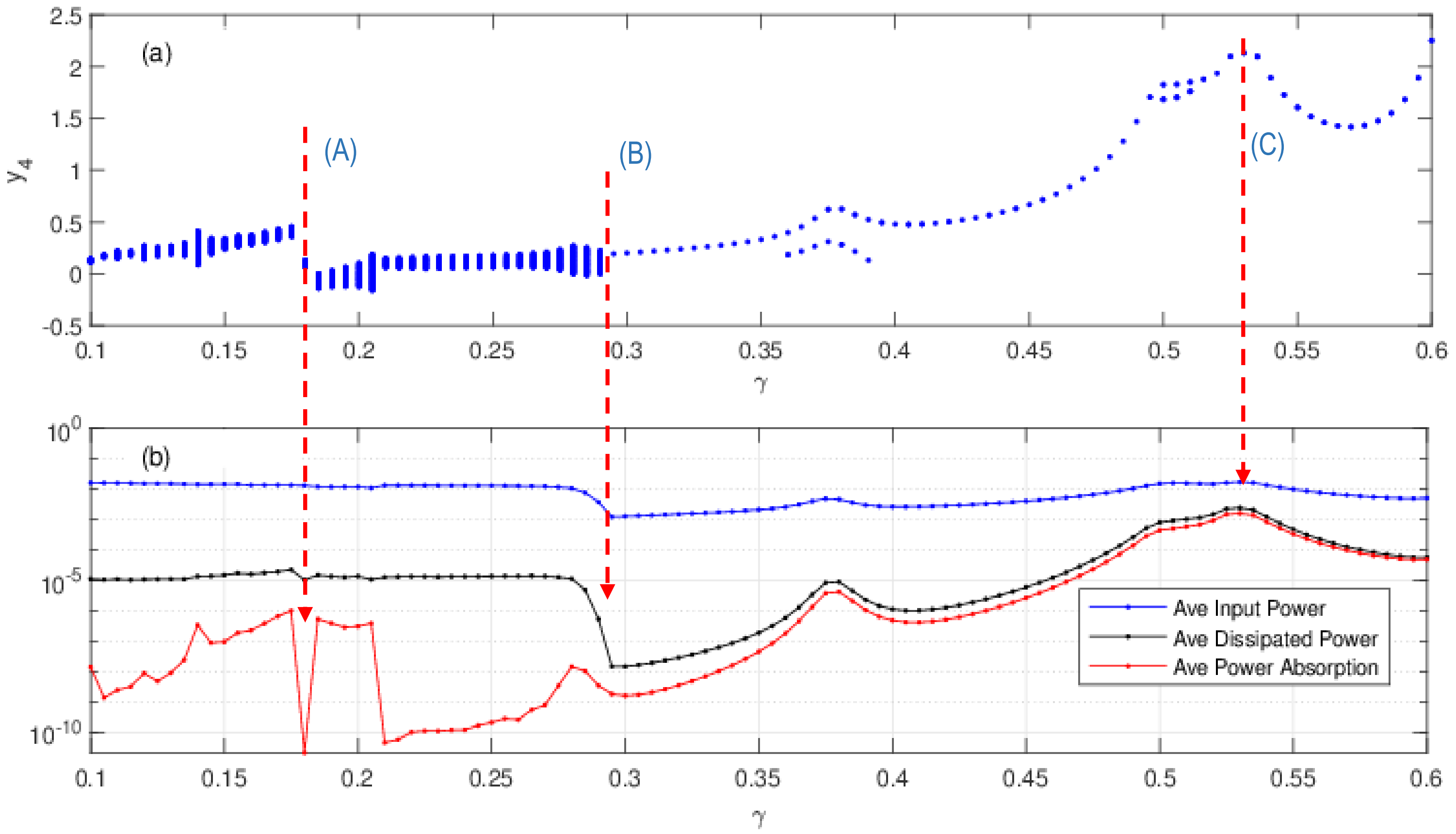


Fig. 8: Energy transfer relations of m_4 w.r.t v on DS1

Discussion of Scientific Results Cont…

Energy transfer…

Small amplitude until critical $v_c = 0.124$

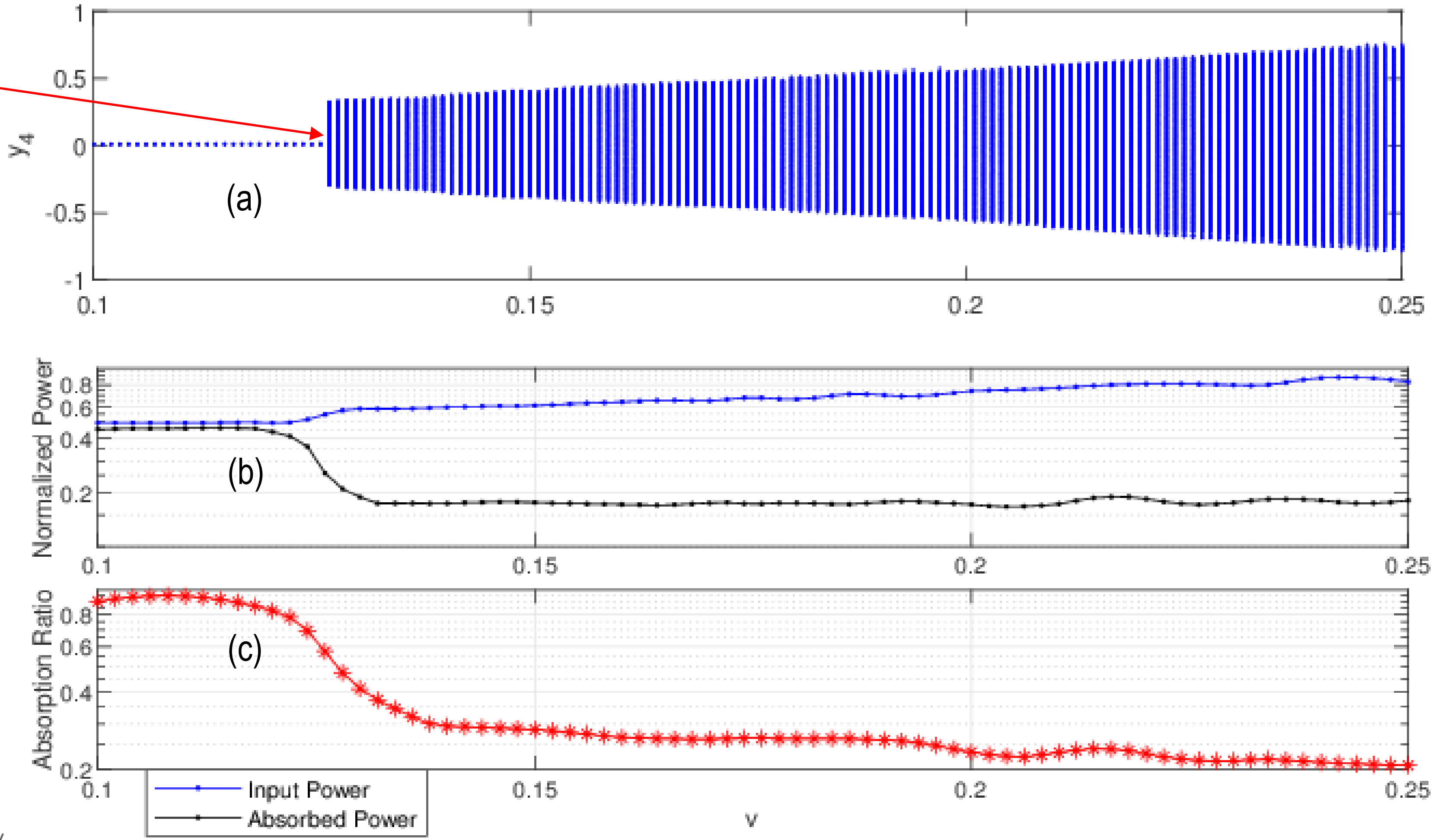


Fig. 9: Bifurcation diagram of structure m_4 in (a), normalized energy relations in (b), and power absorption ration in (c) w.r.t v on DS2

Conclusion

- There is mutual interaction between friction-induced self-excitation and parametric excitation leading to some phenomena such as **synchronization**-to-periodic-orbits, **Neimark-Sacker bifurcation**, small-amplitude vibration, and **energy transfer in the system**
- Lissajous curves show the complex synchronization patterns for all harmonics in the system
- Presence of changing attractors (bifurcation points) are reflected in the energy balance mechanism of the system
- Practical applications involves energy harvesting, automobile braking system, deep drilling, etc.

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Acknowledgement

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Thank you for your kind attention!