

DEVELOPMENT AND FEM VALIDATION OF A HENCKY-TYPE NONLINEAR BEAM MODEL

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ABSTRACT

A Hencky-type discrete model is developed and validated for dynamic analysis of geometrically nonlinear slender beams [1]. The beam is represented by rigid segments connected through rotational springs and dampers, which allows large rotations and large transverse deflections to be described with a relatively small number of generalized coordinates. The governing equations are written in an explicit matrix form for an arbitrary number of elements, which makes the formulation convenient for numerical implementation and parameter studies [1]. This approach is closely related to finite-difference descriptions of discrete elastica based on the Hencky system [2]. Finite-element elastica formulations based on distributed microstructure have also been studied in the literature [3].

A key feature of the model is the unified treatment of right-end boundary conditions. Horizontal, vertical and rotational boundary springs, together with an additional axial stiffness, allow one to represent classical, non-classical and imperfect supports. In particular, the same formulation can describe fully shortened, fully stretched and intermediate partly-shortened or partly-stretched configurations [1]. This is important in nonlinear beam dynamics, since boundary flexibility may significantly modify the frequency response and may lead to transitions between softening- and hardening-type behaviour.

The validated mechanical core may be coupled with additional functional elements, such as passive viscous dampers, shear-thickening-fluid dampers, piezoelectric actuators or vibration energy-harvesting devices [4]. Therefore, the model is intended not only for isolated beam simulations, but also as a compact basis for smart beam systems in which geometric nonlinearity, support flexibility and local functional components interact.

In the validation example shown in Fig. 1, a cantilever beam subjected to a harmonic tip bending moment is analysed. The Hencky solution is compared with an independently implemented co-rotational finite element method (FEM) model using the same number of elements and comparable numerical accuracy. The comparison includes the dimensionless tip-deflection time history, the tip phase portrait and the instantaneous deformed shape.

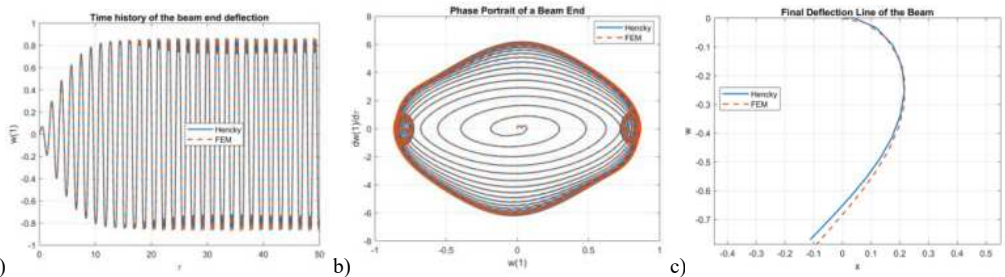


Fig. 1. FEM validation of the Hencky-type nonlinear beam model for a cantilever beam subjected to a harmonic tip bending moment: (a) dimensionless tip-deflection time history; (b) tip phase portrait; (c) instantaneous deflected shape at the final time.

Very good agreement is obtained in both the transient and steady-state parts of the response. The phase portraits practically overlap, and only a small local difference is visible near the beam tip in the final deformed configuration, consistently with the expected discretisation error. At the same time, in the tested implementation, the Hencky model reduced the computational time by about 10 times compared with FEM. This makes it particularly useful for long-time simulations, parameter sweeps, bifurcation analysis and optimisation of nonlinear damping, actuation or energy-harvesting beam systems.

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