

NONLINEAR DYNAMICS OF A PARAMETRICALLY EXCITED 2DOF OSCILLATOR WITH MAGNETIC RESTORING FORCE AND DRY FRICTION

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ABSTRACT

This work presents an analytical and numerical investigation of a nonlinear two-degree-of-freedom mechanical oscillator subjected to parametric excitation, magnetic stiffness nonlinearity, and dry friction. The proposed system consists of two coupled oscillating masses connected through a rotating rectangular beam, whose directional stiffness produces time-periodic parametric excitation. Strong nonlinear restoring forces are introduced through repulsive magnetic springs, while dry friction is considered to represent motion resistance in the guiding mechanism.

The complexification-averaging method is employed to derive approximate analytical modulation equations and amplitude–frequency responses of the system. The obtained analytical solutions are validated through direct numerical simulations using the Runge–Kutta method. To examine the system dynamics in detail, bifurcation diagrams, phase portraits, Poincaré maps, and Lyapunov exponents are computed for both equal- and unequal-mass configurations.

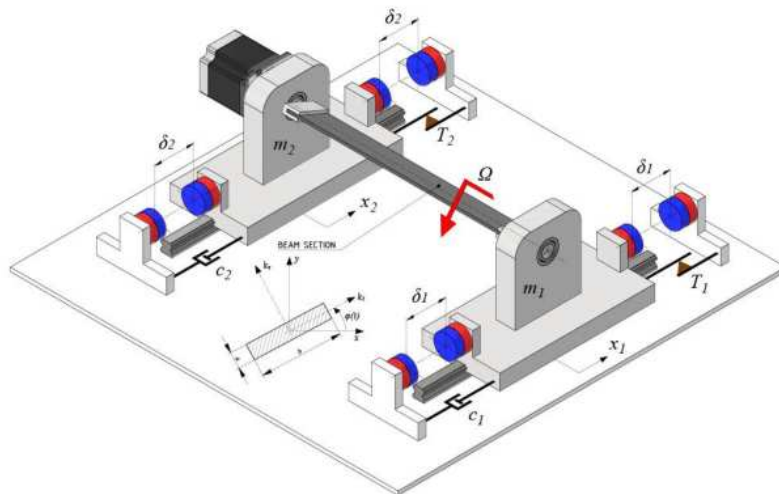


Fig. 1. Physical concept of 2DOF mechanical parametric oscillator.

The results show that the 2DOF coupled parametric oscillator exhibits a rich variety of nonlinear responses, including periodic, quasi-periodic, and chaotic motion. For unequal masses, the response is mainly periodic and closely resembles the behaviour of the corresponding single-degree-of-freedom system [1, 2]. In contrast, the equal-mass configuration produces more complex bifurcation patterns, multistability, and transitions to quasi-periodic and chaotic regimes. The analysis also demonstrates that magnetic nonlinearity, dry friction, and mass distribution significantly influence resonance behaviour, stability, and vibration amplitude [3]. These findings provide useful insight into the dynamics of coupled nonlinear mechanical systems and support the design of advanced vibration mitigation devices, nonlinear energy harvesters, and mechanical systems with tunable stiffness characteristics.

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