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POZNAN UNIVERSITY OF TECHNOLOGY



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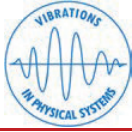
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### ***Prospective insight***

*The jubilee of the conference, similarly as the jubilee of every important event in our lives, invites us to reflect on the past and the future.*

*It was April 1960 when prof. Edmund Karaśkiewicz as a chairman of the Poznan Department of the Polish Society of Theoretical and Applied Mechanics (PTMTS) organized and headed the first two-day symposium on linear and nonlinear vibrations. It took place in Poznan. The symposium became an event organized every two years. The chairmen of the conference changed, but all of them set themselves the goal of caring for high scientific level of the symposium. It resulted in obtaining by the conference a high reputation in the Polish scientific world.*

*More than 60 years have passed. At that time, we observed the rapid development of technology, which fundamentally affected the world, the life of societies and every single person. The development of new technologies was possible thanks to science. On the other hand, we see how much we can support the development of science through the use of modern technical solutions. Faced with the task of organizing the 30th edition of the VIBSYS conference, we asked ourselves a number of questions. First of all, which research topics are currently the most relevant and important from the scientific and application point of view. The second issue was to define an attractive way to exchange knowledge, popularize science and encourage young scientists to conduct research.*

*We decided to answer the first of these questions together with the conference participants who represent various modern trends in the broadly understood subject of vibrations in physical systems. The current and subsequent editions of VIBSYS will allow us to decide which of the topics are particularly worth considering during the conference. In terms of organization, we plan to maintain new ideas that turned out to be right during the conference in 2020. These include a hybrid form of participation both stationary face-to-face on the spot and remote via an online platform, a competition for young scientists on the best presentation of the research results, popularization of history and art through trips to interesting places in the Greater Poland region and the emission of short films encouraging to see, e.g., the exhibition of the National Museum in Poznan during breaks in the sessions.*

*The special moment during the 30 edition of VIBSYS will be a session dedicated to the memory of prof. Czesław Cempel. Prof. Cempel worked in the Institute of Applied Mechanics and organized the VIBSYS conference many times. He was a chairman and honorary member of the scientific committee. Employees of our Institute, co-authors and friends will present the profile of the professor and his scientific achievements.*

*At the end of this introduction, we wish the participants fruitful discussions and many pleasant moments during the VIBSYS conference at the Poznan University of Technology.*

*Chairs of the Conference*



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## A DOUBLE PENDULUM WITH FRICTION UNDER THE ELECTROMAGNETIC FORCING AND KINEMATIC EXCITATION: SIMULATION AND EXPERIMENTATION

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### ABSTRACT

A mathematical model of a double pendulum with friction is derived from the extended swinging Atwood machine. The novel system is modeled under electromagnetic forcing and kinematic excitation. Furthermore, we present some results from the computational analysis and the system's real-time laboratory set-up for experimentation. The system shows promising results as more observations based on system identifications are in progress.

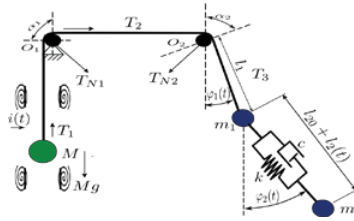


Fig. 1. Schematic diagram of the proposed double pendulum under the influence of electromagnetic forcing

The system is modeled with a frictional force acting on the two pulleys under the action of electromagnetic forcing. The equation of motion is derived from Newton's second law of motion and verified by the Euler-Lagrange method. The friction has to be modeled as well for better apprehension and precise responses to the dynamical system by using extra state variable  $T_R$  Eqs. (1)-(4).

### Equations of Motion

$$\ddot{l}_1(t) = \frac{-(m_1 s_{f2} + M) \ddot{x}_0 + m_1 (l_1(t) \dot{\phi}_1^2 + g c_{f1}) + F_{ck} (c_{f1} c_{f2} - s_{f1} s_{f2}) + T_R - Mg}{m_1 + M} \quad (1)$$

$$\ddot{l}_2(t) = \left( \frac{1}{m_1 m_2 (m_1 + M)} \right) \left( - \left( (s_{f2} (s_{f1}^2 - s_{f1}) + c_{f1} c_{f2} s_{f1} - c_{f1} c_{f2}) M m_1 m_2 \ddot{x}_0 + M m_2 s_{f1}^2 F_{ck} (1 - 2s_{f1}^2) + M m_2 (m_1 l_1(t) \dot{\phi}_1^2(t) + 2c_{f1} c_{f2} F_{ck} + g m_1 c_{f1}) + m_1 m_2 s_{f1} s_{f2} (T_R - Mg) + M m_2 s_{f1}^2 (F_{ck} + g m_1 c_{f2}) - (m_1 m_2 (m_1 + M) (l_2(t) + l_{20s})) \dot{\phi}_2^2(t) - M m_1 m_2 c_{f1} c_{f2} l_1(t) \dot{\phi}_1^2(t) + (m_2 + m_1 + M) m_1 F_{ck} + m_1 m_2 c_{f2} (c_{f1} T_R - M g c_{f1} - M g) \right) \right) \quad (2)$$

$$\dot{\phi}_1(t) = - \frac{m_1 (2l_1(t) \dot{\phi}_1(t) + c_{f1} \ddot{x}_0 + g s_{f1}) + F_{ck} (s_{f1} c_{f2} - s_{f2} c_{f1})}{m_1 l_1(t)} \quad (3)$$

$$\dot{\phi}_2(t) = \left( \frac{1}{(m_1 + M) (m_1 l_2(t) + m_1 l_{20s})} \right) \left( \left( (s_{f1} - 1) s_{f2} c_{f1} - s_{f1} c_{f2} (s_{f1} - 1) \right) M m_1 \ddot{x}_0 - 2M s_{f1} c_{f1} s_{f2}^2 F_{ck} + \left( (2c_{f2} s_{f1}^2 F_{ck} + g m_1 s_{f1}^2 - m_1 c_{f1} l_2(t) \dot{\phi}_1^2(t) - c_{f2} F_{ck}) M + (T_R - Mg) m_1 c_{f1} - M g m_1 \right) s_{f1} + \left( (m_1 c_{f2} l_1(t) \dot{\phi}_1^2(t) + c_{f1} F_{ck}) M + (M g c_{f1} - T_R + M g) m_1 c_{f2} \right) s_{f1} 2m_1 l_2 \dot{\phi}_1^2(m_1 M) \right) \quad (4)$$

The friction variable  $T_R$  is the sum of the friction at the first pulley  $T_{R1}$  and the friction at the second pulley  $T_{R2}$ . That is  $T_R = T_{R1} + T_{R2}$  where  $T_{R1} = \mu_{p1}T_{N1} + T_{pv1}(\dot{l}_1)$ ,  $T_{R2} = \mu_{p2}T_{N2} + T_{pv2}(\dot{l}_1)$ ,  $T_{N1} = \sqrt{T_1^2 - 2T_1T_2 \cos(\alpha_1) + T_2^2}$ ,  $T_{N2} = \sqrt{T_2^2 - 2T_2T_3 \cos(\alpha_2) + T_3^2}$ ,  $T_1 = Mg - m_1\ddot{l}_1(t)$ ,

$T_2 = \frac{T_1(2 + \sin(i_1)\mu_{p1}\sqrt{2(1 - \cos(\alpha_1))}) + 2T_{pv1}(\dot{l}_1)}{(2 - \sin(i_1)\mu_{p1}\sqrt{2(1 - \cos(\alpha_1))})}$ ,  $T_3 = \frac{T_2(2 + \sin(i_2)\mu_{p2}\sqrt{2(1 - \cos(\alpha_2))}) + 2T_{pv2}(\dot{l}_1)}{(2 - \sin(i_2)\mu_{p2}\sqrt{2(1 - \cos(\alpha_2))})}$ . The wrapping angle at the first and second pulley is given by  $\alpha_1 = 90^\circ$  and  $\alpha_2 = (90 - \varphi_1)^\circ$  respectively.

### Simulation Results and Discussion

The mathematical model of the double pendulum with friction has been solved numerically using the Runge-Kutta method with adaptive step size. The results described the double pendulum with friction concepts demonstrating the system behavior with a suspension system between the two pendulums excited by an electromagnet and armature. The excitation frequency has an impact on the system.

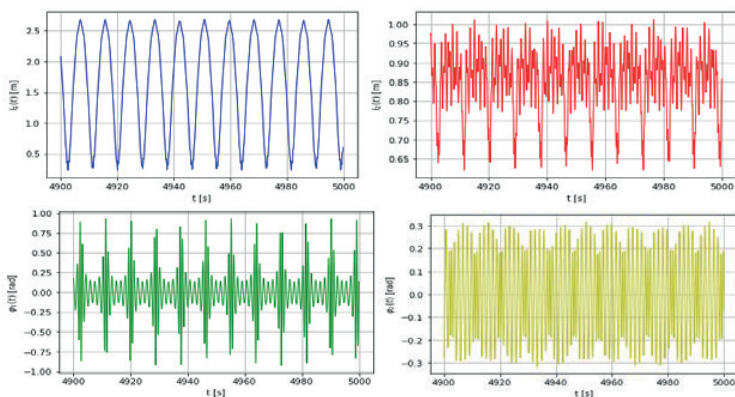


Fig. 2. Numerical results of simulation of the double pendulum: Time Histories for  $\check{l}_1(t)$ ,  $\check{l}_2(t)$ ,  $\check{\varphi}_1(t)$ ,  $\check{\varphi}_2(t)$

### Conclusion

The presented results show the time series of the double pendulum, with no physical contact between the swinging assemble and the fixed points. Most importantly, in some regimes, compact regions of attraction, such as in Fig. 2, appear in the system. Therefore, the nonlinear dynamics of the double pendulum with friction can be vigorously studied, and more possibilities for modification.

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