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### ***Prospective insight***

*The jubilee of the conference, similarly as the jubilee of every important event in our lives, invites us to reflect on the past and the future.*

*It was April 1960 when prof. Edmund Karaśkiewicz as a chairman of the Poznan Department of the Polish Society of Theoretical and Applied Mechanics (PTMTS) organized and headed the first two-day symposium on linear and nonlinear vibrations. It took place in Poznan. The symposium became an event organized every two years. The chairmen of the conference changed, but all of them set themselves the goal of caring for high scientific level of the symposium. It resulted in obtaining by the conference a high reputation in the Polish scientific world.*

*More than 60 years have passed. At that time, we observed the rapid development of technology, which fundamentally affected the world, the life of societies and every single person. The development of new technologies was possible thanks to science. On the other hand, we see how much we can support the development of science through the use of modern technical solutions. Faced with the task of organizing the 30th edition of the VIBSYS conference, we asked ourselves a number of questions. First of all, which research topics are currently the most relevant and important from the scientific and application point of view. The second issue was to define an attractive way to exchange knowledge, popularize science and encourage young scientists to conduct research.*

*We decided to answer the first of these questions together with the conference participants who represent various modern trends in the broadly understood subject of vibrations in physical systems. The current and subsequent editions of VIBSYS will allow us to decide which of the topics are particularly worth considering during the conference. In terms of organization, we plan to maintain new ideas that turned out to be right during the conference in 2020. These include a hybrid form of participation both stationary face-to-face on the spot and remote via an online platform, a competition for young scientists on the best presentation of the research results, popularization of history and art through trips to interesting places in the Greater Poland region and the emission of short films encouraging to see, e.g., the exhibition of the National Museum in Poznan during breaks in the sessions.*

*The special moment during the 30 edition of VIBSYS will be a session dedicated to the memory of prof. Czesław Cempel. Prof. Cempel worked in the Institute of Applied Mechanics and organized the VIBSYS conference many times. He was a chairman and honorary member of the scientific committee. Employees of our Institute, co-authors and friends will present the profile of the professor and his scientific achievements.*

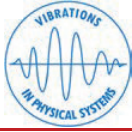
*At the end of this introduction, we wish the participants fruitful discussions and many pleasant moments during the VIBSYS conference at the Poznan University of Technology.*

*Chairs of the Conference*



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## SYNCHRONIZATION AND ENERGY TRANSFER IN 4DOF FRICTION-INDUCED SELF- AND PARAMETRICALLY EXCITED OSCILLATORS

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### ABSTRACT

We consider a practical case in which parametric excitation is provided by a massless rotor of rectangular cross-section with a cylinder-like mass concentrated at the center. The rotor arrangement is placed on a friction-induced self-excited support in the form of a frame placed on a belt moving with constant velocity. This frame is connected to a supplementary mass. A Stribeck friction model is considered for the mass in contact with the belt. The frictional force between the mass and the belt is oscillatory in nature because of the variation of normal force due to parametric excitation from the rotor.

The model shown in Fig. 1 is an expanded form of the one in which zones of instability were studied by Authors in [1]. Studies in [2–3] dealt with synchronization, chaos and other phenomena which are based on continuous system of lesser degrees of freedom. In this work, we show the effect of friction-induced self-excitation and how it interacts with the parametric excitation of the rotating mass. Synchronization has been identified as one of the features, and energy transfer between the masses which leads to this effect have been studied.

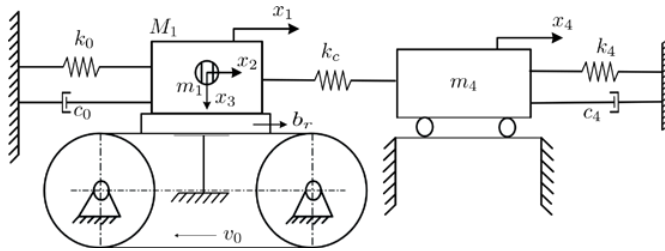


Fig. 1. Model of rotor with parametric excitation placed on friction-induced self-excited support, connected to a supplementary degree of freedom

### Equation of motion

The governing equation of motion for the system has been derived for which, in nondimensional form, it is given in Equation (1)-(4):

$$\begin{aligned} \ddot{X}_1 = & -X_1(A\gamma^2 + A_1\gamma^2 + A_2\gamma^2 \cos(2\gamma\tau)) + X_2(A_1\gamma^2 + A_2\gamma^2 \cos(2\gamma\tau)) - \\ & X_3(A_2\gamma^2 \sin(2\gamma\tau)) + X_4D\gamma^2 - \dot{X}_1h_1\gamma + [X_1(A_2\gamma^2 \sin(2\gamma\tau)) - X_2(A_2\gamma^2 \sin(2\gamma\tau)) + \\ & X_3(A_1\gamma^2 - A_2\gamma^2 \cos(2\gamma\tau)) + 1]b_r \end{aligned} \quad (1)$$

$$\ddot{X}_2 = X_1(b_1\gamma^2 + b_2\gamma^2 \cos(2\gamma\tau)) - X_2(b_1\gamma^2 + b_2\gamma^2 \cos(2\gamma\tau)) + X_3(b_2\gamma^2 \sin(2\gamma\tau)) + \kappa\gamma^2 \sin(\gamma\tau + \phi_0) \quad (2)$$

$$\ddot{X}_3 = -X_1(b_2\gamma^2 \sin(2\gamma\tau)) + X_2(b_2\gamma^2 \sin(2\gamma\tau)) - X_3(b_1\gamma^2 - b_2\gamma^2 \cos(2\gamma\tau)) + \kappa\gamma^2 \cos(\gamma\tau + \phi_0) + 1 \quad (3)$$

$$\ddot{X}_4 = X_1d\gamma^2 - X_4 - \dot{X}_4h_4\gamma \quad (4)$$

where  $A_{1,2}$ ,  $b_{1,2}$  represent the sum and difference of the excitation frequencies,  $b_r$  – dry friction,  $h_{1,4}$  – damping, while  $d$  – stiffness of the coupling spring, and  $0 < \gamma < 1$ .

### Results

Mass  $m_4$  is of practical importance and as such, we consider its displacement with respect to other structures ( $M_1$ ). Synchronization between the two stems from complex pattern due to the present of higher harmonics to simple one at m-periodic windows (Fig. 2(c)). This effect reduces the self-excitation such that the  $M_1$  behaves in a manner that is congruent to the slipping phase. Fig. 2 (a) shows its phase portrait and Poincare section at  $\gamma = 0.22$  while Fig. 2 (b), (d) show the bifurcation diagram and the average power in the system. There are creation and destruction of limit cycles in the interval  $0.1 < \gamma < 0.3$  leading to Neimark-Sacker bifurcation at  $\gamma = 0.295$ . In same vein, the energy plots show that the kinetic energy of  $m_4$  is highest at the point greatest point of absorption.

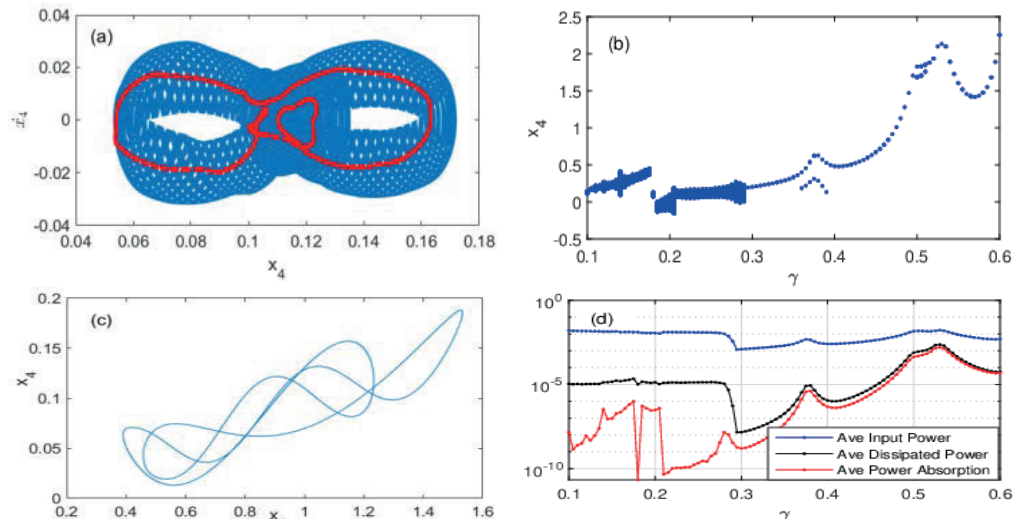


Fig. 2. (a) Phase portrait and Poincare section at  $\gamma = 0.22$ , (b) Bifurcation diagram of mass  $m_4$ , (c) Lissajous curve showing synchronisation of  $M_1$  and  $m_4$ , (d) Power plots from energy transfer

### Conclusion

Synchronization results from the interactions between the self- and parametric excitations, and are vivid in the periodic windows. Energy exchange between the masses is balanced at those points.

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