Parametric vibrations of the functionally graded sandwich plates with complex form

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Abstract. Buckling behavior and parametric vibrations of the sandwich plates with an arbitrary form made of the isotropic and functionally graded materials (FGM) are studied. The different types of lamination schemes have been considered: sandwich plate with FGM face sheets and isotropic (metal or ceramic) core and plate with the FGM core and ceramics or metal on top and bottom face sheets. It was assumed that FGM layers are made of a mixture of metal and ceramics and effective material properties of layers are varied according to Voigt's rule. To calculate different mechanical characteristics for different types of lamination schemes the analytical expressions are obtained. Formulation of the problem was carried out using the first-order shear deformation theory (FSDT) of the plate. Subcritical state of the plate is taken into account.

Introduction

Nowadays functionally graded materials (FGM) are widely used materials in many industry fields as heat-resistant thin-constructed elements. There are many published works devoted to study and investigation of dynamic and static behaviour of FGM plates and shells as designed objects [1, 2]. Therefore many theories and methods were proposed last time for their mathematic modelling and analysis [3], variational Ritz's method is one of them. It is known that one of the main difficulties that occurs at application of Ritz's method is a choice of basic system of functions satisfying the boundary conditions. In this work the R-functions theory [4] is used for solution of this problem especially for complex geometry of plates and shallow shells with different boundary conditions. Earlier the variational method of Ritz and R-functions theory (RFM) were effectively applied for vibration investigation of layered plates and shells [5]. This time the development of RFM for research of linear and parametric vibrations of FGM plates is proposed.

Sandwich plates with complex plan form made of isotropic and functionally graded materials (Al / Al_2O_3) are considered. It is assumed that plate is subjected to a periodic in-plane load of the form $p_N = p_{st} + p_{din}\cos\theta t$, where p_{st} is a static component, p_{din} is the amplitude of a periodic part, and θ is the frequency of an applied load. The proposed method includes the following steps: a) calculation of the subcritical state; b) solution of the linear vibration problem for compressed plate; c) buckling problem; d) investigation of the parametric vibration; d) determination of the dynamic instability regions by Bolotin's approach; e) research of the geometrical nonlinear vibration. In order to implement these steps variational Ritz's method and the R-functions theory [4] are used. Validation of the proposed method and developed software has been examined on test problems for sandwich FGM plate with rectangular plan form and different boundary conditions.
In order to demonstrate possibilities of the proposed approach, new results for simply supported laminated FGM plate (Figure 1) of the complex plan form (Figure 2) are presented. To construct the system of basis function on each step the tools of the R-functions theory are used.

Table 1. Effect of gradient index p on buckling load $\widehat{N}_{cr}$ for FGM plate (Al/Al_2O_3; $c/2a$=0.3, $d/2a$=0.25, $h/2a = 0.1$)

Type	p=0	p=0.5	p=1	p=5	p=10	p=100
1-1	9.915	7.205	5.966	3.905	3.560	3.245
1-2	8.215	6.537	5.597	3.343	2.705	1.931
2-2	5.864	4/841	4.381	3.754	3.685	3.625

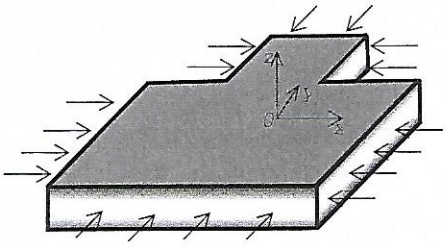

Figure 1. FGM sandwich plate of complex geometry

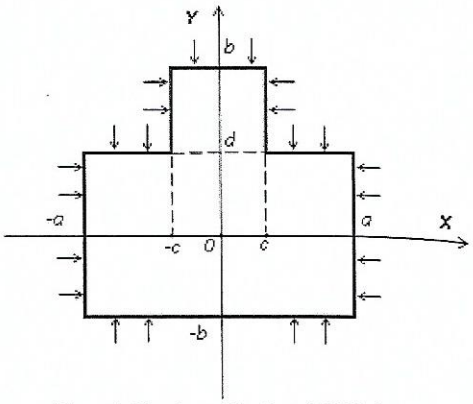

Figure 2. Plan form of laminated FGM plate

The values of the buckling load for uniformly compressed plate (Figure 1) and different Types of the material distribution along thickness of the plate are presented in Table 1. Type 1-1 and Type 1-2 correspond to sandwich plate with FGM face sheets and isotropic metal or ceramic core relatively. The plate of Type 2-2 corresponds to a sandwich plate with the FGM core and ceramics on top and metal on bottom face sheets. The ratio of the layers thickness is the same for all Types: $h^{(1)}:h^{(2)}:h^{(3)}=1:2:1$.
Backbone curves for the plate of the Type 1-1 and ratio of the thickness 1:2:1 and gradient index p=1 are presented in Figure 3.

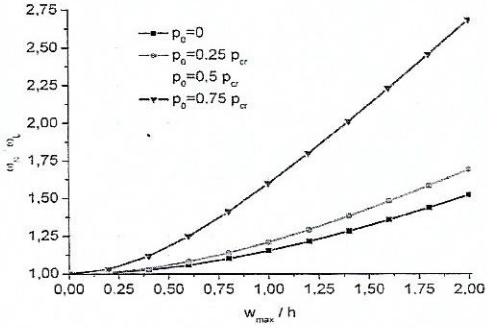

Figure 3. Backbone curves for plate (Type 1-1, ratio of thickness 1:2:1, p=1)

Effects of different distribution of materials, lamination schemes, boundary conditions, and geometrical parameters on natural frequencies and backbone curves are reported and analysed. Zones of the instability are constructed.

Conclusions

Numerically analytical approach for investigation of parametric vibrations of FGM plates is suggested. Proposed method allows to study FGM plates with complex geometry and various boundary conditions. This work demonstrates the possibilities of application of the R-functions theory to analyse the observed elements under different distribution of materials, lamination schemes, boundary conditions, and geometrical parameters.

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