External and internal resonances in a mass-spring-damper system with 3-dof

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Abstract. The investigation is focused on dynamical response of the nonlinear mechanical system of 3 degrees of freedom. The obtained approximate analytical solution of the governing equations allows to carry out the qualitative/quantitative analysis of the system dynamics. The applied multiple scale method gives possibility, among others, to recognize and test all possible resonances.

Introduction

The pendulum-like systems may serve as a reliable model for many machine and measurement devices, manipulators and the human body parts. Moreover, they can be used to develop, test and validate the new approaches employed to study dynamics of multi-body systems. The pendulums could exhibit very complicated behavior mainly due to strong nonlinearity of a geometrical nature and couplings between their components. The couplings result in the case of the autoparametric excitation are related to the energy exchange between vibration modes [1].
Many pendulum-like systems are analytically studied in reference [2]. The analytical approach is often more useful from theoretical but also practical point of view than direct numerical simulations. The asymptotic approach aimed on analysis of the kinematically excited spring pendulum near parametric resonances is presented in [3-4]. In this work we are focused on a study of the external and internal resonances of the spring physical pendulum by means of the method of multiple scales (MSM) in time domain and we demonstrate feasibility of MSM and we detect almost all system resonances as well as predict novel nonlinear phenomena.

Description of the problem

The studied physical pendulum consists of a rigid body of mass m suspended by a massless elastic-damping link. The scheme of the system is presented in Fig. 1. The distance between the point A and the body mass centre C is denoted by S and called further the eccentricity. The spring is assumed to be nonlinear of a cubic type, where k_1 and k_2 are constant coefficients. Both dampers are assumed to be viscous. In the plane motion, the pendulum has three degrees of freedom. The angles Φ and Ψ and the total spring deformation X, which includes also the static elongation X_r, are chosen as the general coordinates. Two torques $M_1(t)=M_{01}\cos(\Omega_2 t)$ and $M_2(t)=M_{02}\cos(\Omega_3 t)$ as well as the harmonic force $F(t)=F_0\cos(\Omega_1 t)$ play a role of the system excitation. There is also assumed a torque of viscous nature attenuated the swing vibration related to the angle Ψ, and C_3 stands for a viscous coefficient.

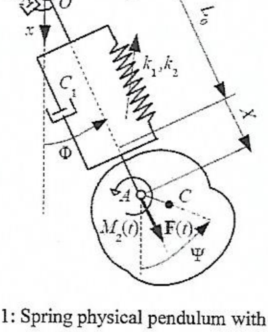

Figure 1: Spring physical pendulum with 3-dof.

Mathematical model

The equations of motion have been derived using the Lagrange formalism. All external loadings and damping effects are considered as the generalized forces. The dimensionless form of the governing equation is as follows

$$\ddot{\xi}+c_1\dot{\xi}+\xi+\alpha\xi^3+3\xi_r\alpha\xi^2+3\xi_r^2\alpha\xi-w_2^2(\cos\varphi-1)-(1+\xi)\dot{\varphi}^2-s\cos(\varphi-\gamma)\dot{\gamma}^2+e\sin(\varphi-\gamma)\ddot{\gamma}=f_1\cos(p_1\tau), \quad (1)$$

$$\ddot{\varphi}(1+2\xi+\xi^2)+w_2^2\sin\varphi(1+\xi)+c_2\dot{\varphi}+2\dot{\xi}\dot{\varphi}+2\xi\dot{\xi}\dot{\varphi}+s\sin(\varphi-\gamma)(1+\xi)\dot{\gamma}^2+s\cos(\varphi-\gamma)(1+\xi)\ddot{\gamma}=f_2\cos(p_2\tau), \quad (2)$$

$$\ddot{\gamma} + w_3^2\sin\gamma + c_3\dot{\gamma} + 2\frac{w_3^2}{w_2^2}\cos(\varphi-\gamma)\dot{x}\dot{\varphi} - \frac{w_3^2}{w_2^2}(1+\xi)\sin(\varphi-\gamma)\dot{\varphi}^2 + \frac{w_3^2}{w_2^2}\sin(\varphi-\gamma)\ddot{\xi} + \frac{w_3^2}{w_2^2}\cos(\varphi-\gamma)\ddot{\varphi} = f_3\cos(p_3\tau), \quad (3)$$

where $\tau = \omega_1 t$ is the dimensionless time, and $\omega_1^2 = k_1/m$.

The dimensionless quantities are defined as follows: $s = S/L$, $\xi = X/L$, $L = L_0 + X_r$. The functions $\xi(\tau), \varphi(\tau), \gamma(\tau)$ correspond to the generalized coordinates $X(t), \Phi(t), \Psi(t)$, respectively. The dimensionless counterpart of X_r is denoted by $\xi_r = X_r/L$ and satisfies the equilibrium equation

$$\alpha\xi_r^3 + \xi_r = w_2^2. \quad (4)$$

Assuming ω_1 as the reference quantity, we define the remaining dimensionless parameters

$$c_1 = \frac{C_1}{m\omega_1},\ c_2 = \frac{C_2}{mL^2\omega_1},\ c_3 = \frac{C_3}{\omega_1 m r_A^2 L^2},\ f_1 = \frac{F_0}{mL\omega_1^2},\ f_2 = \frac{M_{01}}{mL^2\omega_1^2},\ f_3 = \frac{M_{02}}{\omega_1^2 m R_A^2 L^2},\ \alpha = \frac{k_2 L^2}{\omega_1^2 m},\ w_2 = \frac{\omega_2}{\omega_1},$$

$$w_3 = \frac{\omega_3}{\omega_1},\ p_1 = \frac{\Omega_1}{\omega_1},\ p_2 = \frac{\Omega_2}{\omega_1},\ p_3 = \frac{\Omega_3}{\omega_1},\ \omega_2^2 = \frac{g}{L},\ \omega_3^2 = \frac{Sg}{R_A}.$$

Equations (1)–(3) are supplemented by the initial conditions

$$\xi(0) = u_{01},\ \dot{\xi}(0) = u_{02},\ \varphi(0) = u_{03},\ \dot{\varphi}(0) = u_{04},\ \gamma(0) = u_{05},\ \dot{\gamma}(0) = u_{06}, \quad (5)$$

where dimensionless quantities $u_{01}, ..., u_{06}$ are known.

The analytical study of the mathematical model (1) – (5) allows one to identify the following types of resonances: external resonances, when $p_1=1$, $p_2 = w_2$, $p_3 = w_3$, and internal or combined resonances, when $w_2 = 1/2$, $w_2 = w_3$, $w_3 = 1/2$, $p_2=w_3$, $w_3=1$, $w_2=3w_3$, $w_3=3w_2$, $w_2+w_3=1$, $w_2-w_3=1$, $w_2+w_3=2$, $w_2-w_3=2$.

Case study

The example of time courses of the generalized coordinates are presented in Fig. 2 (fixed parameters: $\alpha=0.25$, $f_2=0.05$, $f_3=0.01$, $f_1=1$, $c_1=0.1$, $c_2=0.01$, $c_3=0.001$, $w_2=0.32$, $w_3=0.24$, $p_1=3.13$, $p_2=1.58$, $p_3=1.78$, $e=0.03$, $a_{10}=0.04$, $a_{20}=0.04$, $a_{30}=0.004$, $\psi_{10}=0.$, $\psi_{20}=0.$, $\psi_{30}=0$).

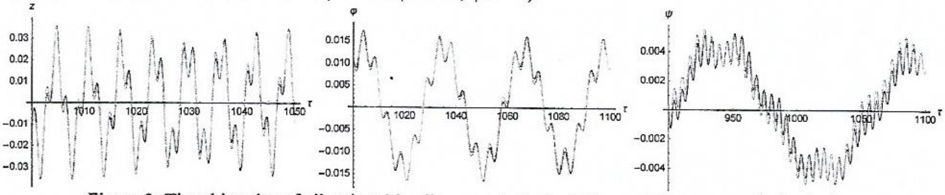

Figure 2: Time histories of vibration; blue line – analytical solution, red line – numerical solution.

The reported time histories are obtained by direct numerical integration of the equations (1) – (5) and based on the MSM analytical solution.

Conclusions

The mathematical model of the nonlinear lumped mass system (3-dof) has been derived, and the asymptotic solution of the equations of motion has been obtained up to the third order of approximation. This approach allows to detect all possible kind of resonances which could appear in the system, and enables to determine various amplitude-frequency relations. High accuracy of the approximate analytical solutions has been verified by numerical calculations. The carried out analysis based on dimensionless variables allows to generalize the obtained results to other physical systems.

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