Asymptotic Solutions and Resonance Responses for 3-DOF Planar Physical Pendulum

G. Sypniewska-Kamińska[1a], R. Starosta[1b], J. Awrejcewicz[2c]

[1] *Poznan University of Technology*
Piotrowo 3, 60-965 Poznań, Poland

[2] *Technical University of Lodz*
Stefanowskiego 1/15, 90-924 Łódź, Poland

[a] *grazyna.sypniewska-kaminska@put.poznan.pl*
[b] *roman.starosta@put.poznan.pl,* [c] *awrejcew@p.lodz.pl*

The dynamical behaviour of a harmonically excited and linear damped 3-dof system is investigated in the paper. The system consists of a rigid body of mass, m, suspended at point A on a massless and linear spring, whose other end is fixed at point O as is shown in Figure 1. The point C is the mass centre of the body. Let $S = AC$ denote the eccentricity. The dynamical extension of the spring, Z, and the angles, φ and ψ, are used as the generalized coordinates. The external excitations, i.e. the force, $F(t) = F_0 \cos(\Omega_1 t)$, the moment, $M_\varphi(t) = M_2 \cos(\Omega_2 t)$, and the moment, $M_\psi(t) = M_3 \cos(\Omega_3 t)$ are taken into consideration. The pendulum motion is damped by the viscous force, $C_1 \dot{Z}$, and two linear moments, $C_1 \dot{\varphi}$ and $C_3 \dot{\psi}$.

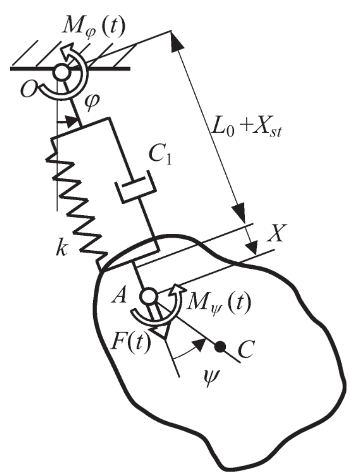

Figure 1: Physical spring pendulum

Applying the Lagrange equations we obtain the motion equations. They have the following dimensionless form:

$$\ddot{z}(\tau)+c_1\dot{z}(\tau)+w_1^2 z(\tau)+1-\cos\big(\varphi(\tau)\big)-\big(1+z(\tau)\big)\big(\dot{\varphi}(\tau)\big)^2+s\sin\big(\varphi(\tau)-\psi(\tau)\big)-\\ s\cos\big(\varphi(\tau)-\psi(\tau)\big)\big(\psi(\tau)\big)^2=f_1\cos(p_1\tau),$$

$$\big(1+z(\tau)\big)^2\ddot{\varphi}(\tau)+c_2\dot{\varphi}(\tau)+\big(1+z(\tau)\big)\Big(\sin\big(\varphi(\tau)\big)+2\dot{z}(\tau)\dot{\varphi}(\tau)\Big)+\\ s\big(1+z(\tau)\big)\Big(\sin\big(\varphi(\tau)-\psi(\tau)\big)\big(\dot{\psi}(\tau)\big)^2+\cos\big(\varphi(\tau)-\psi(\tau)\big)\ddot{\psi}(\tau)\Big)=f_2\cos(p_2\tau),$$

$$\ddot{\psi}(\tau)+c_3\dot{\psi}(\tau)+w_3^2\sin\big(\psi(\tau)\big)+\\ w_3^2\big(1+z(\tau)\big)\Big(\sin\big(\varphi(\tau)-\psi(\tau)\big)+\cos\big(\varphi(\tau)-\psi(\tau)\big)\ddot{\varphi}(\tau)\Big)-\\ w_3^2\big(1+z(\tau)\big)\sin\big(\varphi(\tau)-\psi(\tau)\big)\big(\dot{\varphi}(\tau)\big)^2+2w_3^2\cos\big(\varphi(\tau)-\psi(\tau)\big)\dot{z}(\tau)\dot{\varphi}(\tau)=\\ f_3\cos(p_3\tau),$$

where $L=L_0+mg/k$, k denotes the spring stiffness, L_0 is its length, when it is unstretched, g is the gravitational acceleration, $\omega_1^2=k/m$, $\omega_2^2=g/L$, $\omega_3^2=Sg/I_A^2$, I_A is the radius of the body inertia with respect to the axis, which passes through point A and is perpendicular to the plane of motion:

$$z=Z/L,\quad s=S/L,\quad \tau=\omega_2 t,\quad w_1^2=\omega_1^2/\omega_2^2,\quad w_3^2=\omega_3^2/\omega_2^2,$$

$$c_1=\frac{C_1}{m\omega_2},\quad c_2=\frac{C_2}{Lm\omega_2},\quad c_3=\frac{C_3S}{I_A^2Lm\omega_2},\quad f_1=\frac{F_0}{Lm\omega_2^2},\quad f_2=\frac{M_2}{L^2m\omega_2^2},\quad f_3=\frac{M_3}{m\omega_2^2I_A^2},$$

$$p_1=\frac{\Omega_1}{\omega_2},\quad p_2=\frac{\Omega_2}{\omega_2},\quad p_3=\frac{\Omega_3}{\omega_2}.$$

The above presented equations describing the dynamical behaviour of the pendulum are solved by the multiple scales method (MSM).

The behaviour of the nonlinear system can be very complicated, especially near the resonance, when several resonances will occur simultaneously. The MSM application allows determining the condition under which external and internal resonances occur in the system. The differential equations of amplitude and phase modulation were obtained for several combinations of simultaneously occurring external resonances. The equations were solved numerically due to their very complex form. The amplitude responses as function of detuning parameters were received taking into account the steady-state conditions.

References

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