Modeling, numerical response and response sensitivity of three coupled links with rigid limiters of motion

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1. Introduction

The work is devoted to numerical modeling of the flat triple physical pendulum, with arbitrarily situated barriers imposed on the position of the system, where each of a body can be externally excited [1]-[3]. Such a model must include the impact model as well as a model of a system of pendulums sliding along some obstacles. The model of changing the set of permanently active constraints also must be developed. But the response is a one way of numerical analysis of a system. Much more information can be acquired by the use of response sensitivity analysis. This enables the Lyapunov exponents calculation for non-periodic attractors and stability of periodic orbits calculation. Periodic orbits stability gives possibility to find unstable periodic solutions (not possible in common simulations) and to numerical analysis of periodic orbits bifurcations. In the investigated system the classical bifurcations can be found as well as a non-classical ones. But it should be noted that analysis of classical bifurcations in such a system requires special tools because of discontinuities. This work is still in progress and now the tools for analysis of classical bifurcations was developed.

The investigated system is interesting not only because of richness of dynamical phenomena that it can exhibit. It also important from technical and physical point of view since it can model many real objects. The example is the developed in the frame of this projects the piston - connecting rod – crankshaft system model as a special case of the model of triple physical pendulum with barriers [2].

2. Modeling and results

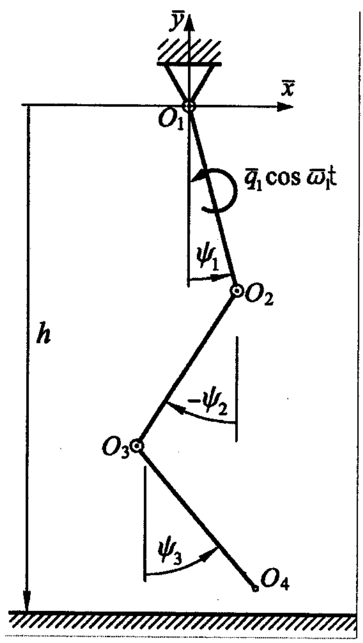

Fig. 1. The special case of investigated system.

The pendulum in its special case is presented in Fig. 1. Since we assume that the constraints are perfect, the system between each two discontinuity points is governed by the following differential-algebraic equations (DAEs):

$$\mathbf{M}(\mathbf{q},t)\ddot{\mathbf{q}} = \mathbf{f}_q(\mathbf{q},\dot{\mathbf{q}},t) + \left(\frac{\partial \mathbf{h}_{act}(\mathbf{q},t)}{\partial \mathbf{q}^T}\right)^T \boldsymbol{\lambda}_{act},$$
$$0 = \mathbf{h}_{act}(\mathbf{q},t) \qquad (1)$$
$$0 = \dot{\mathbf{h}}_{act}(\mathbf{q},t) = \frac{\partial \mathbf{h}_{act}(\mathbf{q},t)}{\partial \mathbf{q}^T}\dot{\mathbf{q}} + \frac{\partial \mathbf{h}_{act}(\mathbf{q},t)}{\partial t},$$

together with the conditions

$$\boldsymbol{\lambda}_{act} > 0, \qquad (2)$$
$$\mathbf{h}_{inact} > 0,$$

where $q=[\psi_1, \psi_2, \psi_3]^T$ is a vector of generalized coordinates, h_{act} is a vector of constraints active, λ_{act} is a vector of corresponding Lagrange multipliers and the h_{inact} is a vector of constraints inactive on a given time interval. The discontinuity points are detected by the zero crossing detection of at least one element of vectors (2). In the discontinuity points the impact may take place or the end of permanent activity of some obstacle. In each discontinuity point the set of permanently active constraints is rearranged by the use of special algorithm [2]-[3]. The single impact model is the generalized impact law based on restitution coefficient [4]. More details on model of triple physical pendulum can be found in [1]-[3].

Multiple impacts in the sense of few impacts in the same instant are treated as a subsequent single impacts. But theoretically an exception is possible when the sequence of impact is crucial and impossible to be determined. The developed program is then stopped. This case have not been encountered however in our simulations so far. The correctness of the algorithm have been strongly tested when simulating the piston - connecting rod – crankshaft of a mono-cylinder four-stroke combustion engine, modeled as an inverted triple pendulum [2].

The stability is investigated numerically by the use of classical approach with special modifications for handling with perturbed solution in points of discontinuity ('saltation matrices') based on the theory of Aizerman and Gantmakher [5]. This approach allows to develop algorithms for Lyapunov exponents computation and for stability of periodic orbits computation but not for the periodic orbits with degenerated contact with obstacles like for example grazing orbits.

In Fig. 2 we present exemplary chaotic attractor of the system presented in Fig. 1 (with horizontal barrier and with the first body harmonically excited). The trajectory of the end point of the third link (a) and the corresponding Poincaré section (b) are presented. The corresponding Lyapunov exponents' spectrum is (0.10, 0, -0.65, -0.74, -1.69, -∞, -∞). One exponent is positive (indicating chaotic nature of the attractor). We can also see two exponents in minus infinity due to some parts of trajectory with permanent contact between the system and the obstacle.

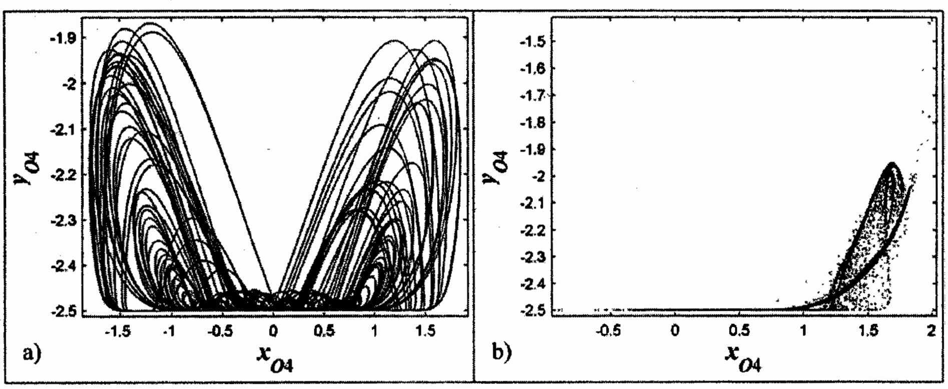

Fig. 2. Projections of chaotic attractor: trajectory (a) and Poincaré section (b).

Acknowledgements

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3. References

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