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Geometry and dynamics of low dimensional mechanical systems with constraints

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ABSTRACT

In this paper we make use of Riemannian geometry to analyze dynamics of low dimensional mechanical systems with constraints. The presented approach has been recently applied to study chaos in Hamiltonian systems [1,2]. It is believed that this kind of geometrical approach may provide a good explanation (as an alternative one to homoclinic intersections) of the onset of chaos in Hamiltonian systems. It has been shown that other mechanical systems can be described in this manner, e.g. mechanical systems with velocity-dependent potentials (these systems are described in Finslerian geometry) [3]. Since we make use of Riemannian geometry, we have two Riemannian spaces at our disposal, namely a configuration space manifold endowed with the Jacobi metric and an enlarged space-time manifold endowed with the Eisenhart metric. The dynamics is analyzed by means of the Jacobi-Levi-Civita (JLC) equation and its solutions [2]. In this paper, two simple mechanical systems with constraints are studied, namely a spherical pendulum with a perturbed potential and a double physical pendulum. It is well known, that for diagonal metrics of Hamiltonian systems using the Eisenhart metric, the JLC equation yields the ordinary tangent dynamics equation (the same that one uses to compute the Lyapunov exponents) [1]. Hence, we focus mainly on the Jacobi metric approach because the JLC equation, in this case, is different from the tangent dynamics one. We show that geometrical approach is in qualitative agreement with the classical techniques analyzing dynamics of mechanical systems.

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[2] Casetti M., Pettini M., Analytic computation of the strong stochasticity threshold in Hamiltonian dynamics using Riemannian geometry. *Physical Review E*, 48(6), 1993, 4320-4332.
[3] Di Bari M., Cipriani P., Geometry and chaos on Riemann and Finsler manifolds. *Plant. Space Sci.*, 46(11/12), 2000, 1543-1555.