Stability and bifurcation of periodic solutions in mechanical systems with rigid barriers

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ABSTRACT

The scope of the work includes modelling of mechanical systems with rigid and frictionless limiters of motion, their numerical simulation and stability analysis of periodic solution by the use of numerical tools.

The system is modelled as a hybrid one since its dynamics is treated as a composition of the continuous dynamics and discrete events. The continuous dynamics between each of two successive events is governed by the differential-algebraic equations (DAEs), where algebraic equations results from the permanently active constraints (on the given time interval) and the normal reactions from the active obstacles are introduced to the differential equations in the form of Lagrange multipliers. At each event, the continuous dynamics description for the next time interval is altered (the set of permanently active constraints changes) and the impulsive change in the system state may take place due to the impact phenomenon [2].

The linear stability is investigated numerically by the use of classical approach based on integration of the variational equations with special modifications [1,3] (Aizerman and Gantmakher theory) due to the discontinuities in the perturbations ('saltation matrices') at the impacts instances.

The examples of the use of introduced methods in the analysis of periodic solutions of the triple physical pendulum with rigid horizontal barriers have been already reported in reference [1]. We show examples of stable and unstable periodic solutions, found by the use of shooting method. We present also few examples of classical bifurcations of periodic solutions: saddle-node bifurcation, period doubling bifurcation, symmetry breaking bifurcation and secondary Hopf bifurcation. The characteristic feature of this work is the stability analysis of the periodic solutions with some segments of permanent contact between the system and the obstacle.

[1] Awrejcewicz J., Kudra G., Lamarque C.-H., Investigation of triple pendulum with impacts using fundamental solution matrices. *International Journal of Bifurcation and Chaos*, 14(12), 2004, 4191-4213.
[2] Brogliato B., Nonsmooth Mechanics. *Springer*, London, 1999.
[3] Muller P.C., Calculation of Lyapunov exponents for dynamic systems with discontinuities *Chaos, Solitons and Fractals*, 5, 1995, 1671-1691.