# Stability analysis of a multi-body system with rigid unilateral constraint and its application

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The work includes modeling of three joined with viscous damping flat physical pendulums, with arbitrarily situated barriers imposed on the position of the system, where each of a body can be externally excited. The system is governed by the set of ordinary differential equations together with the set of algebraic inequalities representing rigid limiters of motion. In the system the impacts are possible (modelled by the use of generalized Newton's law based on the restitution coefficient rule) as well as the sliding states with permanent activity of some obstacles. The system is simulated by numerical integration of smooth ordinary differential equations (ODE's) or differential-algebraic equations (DAE's) (in the case of sliding) between two successive events (impact, start or end of sliding) with detection of these events.

Then the linear stability is numerically investigated using the special rules for handling with perturbed solution in points of discontinuity ('saltation matrices') based on the theory of Aizerman and Gantmakher. This approach is used then to seek and follow branches constructed through determination of stability and bifurcation analysis of periodic solutions (Floquet multipliers), and to calculate the Lyapunov exponents of attractors in the investigated piece-wise smooth (PWS) system.

In the next step, we show some examples of analysis of a special case of the investigated system: three identical coupled rods, where the first rod harmonically excited. We show coexisting stable and unstable periodic solutions, quasi-periodic, chaotic and hyper-chaotic attractors. Some bifurcations of periodic solutions with impacts are shown, i.e. classical (for example the Neimark-Sacker bifurcation leading to quasi-periodicity) as well as non-classical (grazing bifurcations).

In the end we show the possible application of the developed model: the piston-connecting rod-crankshaft system of a mono-cylinder four-stroke combustion engine, modelled as an inverted triple pendulum. The obtained self-excited system can be only treated as a first step in more advanced modelling of real processes (since some very important technological details are neglected), but after taking account of some technological details, the proposed model can be useful in impact dynamics analysis of the piston in a cylinder barrel.