# Numerical analysis of displacements in an elastic half-space subjected to impulse type perturbations

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**Abstract**

An axially symmetric problem of an impulse type perturbation acting on a half-space elastic surface is considered. A solution is formulated using the inverse Hankel-Laplace transformation. Then, a numerical analysis of the problem is carried out.

The integral transformations belong to suitable tools for solution of many problems devoted to wave propagation analysis. In a case of axially symmetric problems of theory of elasticity, the Hankel transformation is applied very often along a radial coordinate, whereas the Laplace transformation is related to time. In result, instead of the partial differential equations, the ordinary differential equations are further investigated [1]. Solving the obtained equations, the Hankel-Laplace transformation of a being sought solution $\mathbf{U}^{LH}$ is applied. Owing to the Riemann-Mellin rule, the inverse Hankel-Laplace transformation has the following form

$$\mathbf{U}(r,z,t)=\frac{1}{2\pi i}\int_{c_0-i\infty}^{c_0+i\infty}\left(\int_0^{\infty}\mathbf{U}^{LH}(k,z,s)J_n(kr)k\,dk\right)e^{st}ds. \qquad (1)$$

Let in the plane $z=0$, being a boundary of the elastic half-space, a puls function load along $z$ axis is applied. Owing to this load action, a stress-strain state is generated, as well as the longitudinal (dilatation), transversal (distortion) and surface (Rayleigh) waves appear.

The stated problem reduces to a solution of linear equations of motion in the cylindrical coordinates [1] with the following boundary

$$\sigma_{zz}=-Z(r)f(t),\ \ \sigma_{zr}=0,\ \text{for}\ z=0,\quad u,\ w\to 0,\ \text{for}\ z\to\infty, \qquad (2)$$

and initial conditions

$$u=\partial u/\partial t=w=\partial w/\partial t=0,\ \text{for}\ t<0, \qquad (3)$$

where: $u(r,z,t)$, $w(r,z,t)$ are components of the displacement $\mathbf{U}$ in the directions $r$ and $z$, correspondingly. In addition, $\sigma_{rr}$, $\sigma_{zz}$ are components of the normal stresses in the cylindrical coordinates, $\sigma_{zr}$ is a component of the tangential stress in the cylindrical coordinates, $f(t) = At\exp(-\nu_0 t)H(t)$, $Z(r) = (2P/\pi r_*^4)(r_*^2 - r^2)H(r_* - r)$, and $H(t) = 1,\ t > 0$; $H(t) = 0, t < 0$.

Accounting both positions of branch points of the integral functions (1) and poles, the integration contour in the inversed Laplace transformation is modified, and then the appropriate numerical calculations are carried out.

The latter ones are supported by the results included in reference [2], and the Gauss quadrature formulas are additionally applied. During the numerical analysis of the inversed Hankel transforms the occurred integrals are replaced by the sum of finite integrals with the limits defined by the roots of the appropriate Bessel's function. The series is an alternating series, and the summation can be carried out using the $\varepsilon$-algorithm of Wynn.

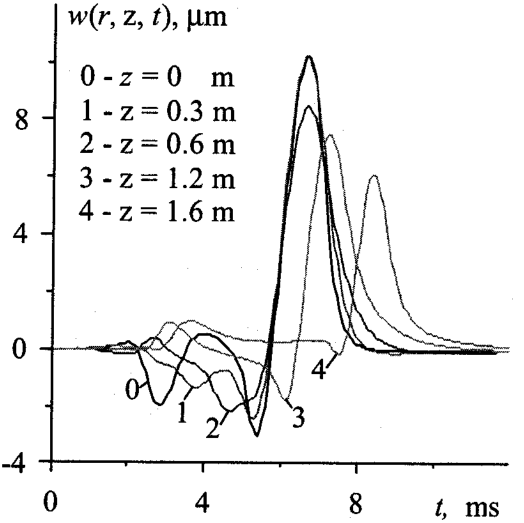

Fig. 1: The dependence of displacement $z$ for various distances from the half-space surface

The numerical computations are carried out for different distances from the load action place. A ground with the parameters: $c_1 = 800$ m/s, $c_2 = 320$ m/s, $\rho = 1600$ kg/m$^3$ is analysed. The following additional parameters are fixed: $r_* = 0.3$ m, $P = 50$ kN, $\nu_0 = 5000$ s$^{-1}$. Time histories of displacements in axis $z$ direction for $r = 1.6$m and for different values of distances measured from the half-space surface are reported in Figure 1.

## References


[1] W. Nowacki. *Theory of Elasticity*. PWN, Warszawa, 1973, in Polish.

[2] J. Awrejcewicz., Yu. Pyryev. De Saint-Venant Principle and an Impact Load Acting on an Elastic Half Space. *Journal of Sound and Vibration*, 2003 (to appear)