Ninth Conference On Nonlinear Vibrations, Stability, and Dynamics of Structures

July 28 - August 1, 2002

Program

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BIFURCATIONS OF PERIODIC SOLUTIONS AND IRREGULAR VIBRATIONS IN TRIPLE PHYSICAL PENDULUM WITH BARRIERS

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In this work the triple pendulum with damping, external forcing and with impacts is investigated. The governing equations of motion are given in the non-dimensional form as the set of differential equations and the set of algebraic inequalities representing unilateral constraints imposing restrictions on the position of the system.

The fourth order Runge-Kutta method is used for integration of the system of differential equations between two successive impacts and the impact times are detected (by halving the integration step until obtaining required precision). In each discontinuity point a state vector of the system is transformed due to the extended Newton's rule (the so called restitution coefficient rule) with assumption that there is no dry friction between the rod and the obstacle [2].

The algorithms for calculation of Lyapunov exponents, for finding periodic solutions and following their branches, analyzing of stability of periodic solutions and their bifurcations have been developed. All these methods base on calculation of fundamental solution matrices. In our system exhibiting discontinuities, the time evaluation of fundamental solution matrix undergoes jumps called also 'saltations'. The transition condition for the vector of perturbation (and fundamental solution matrix) in the discontinuity points is calculated using the theory of Aizerman and Gantmakher [3].

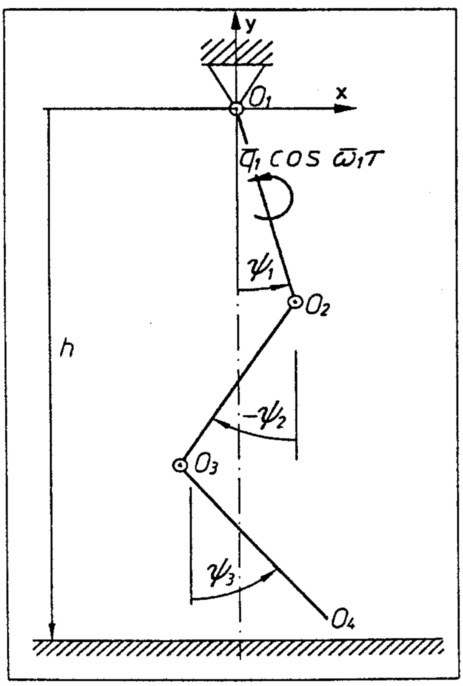

Fig.1. The investigated system.

Here we show results of numerical investigation of three coupled (with damping) identical rods, with a horizontally situated barrier, and external harmonic forcing acting on the first pendulum (Fig. 1).

As an example of results of numerical simulations we present the continuous Neimark-Sacker bifurcation leading to quasiperiodicity (Fig. 2a). When the non-dimensional amplitude of the external excitation q_1 is changed, two conjugated Floquet multipliers cross the unit circle on the complex plane (Fig. 2b) - the periodic solution looses its stability and the stable invariant torus (quasiperiodic attractor) is generated. In the region of quasiperiodicity we can also observe a lot of windows of periodic solutions (Fig. 2a) situated on the torus.

The exemplary Lyapunov exponents spectrum for the quasiperiodic attractor (q_1 = 0.7878) is (0.000, 0, -0.006, -0.074, -1.264, -1.764, -1.778).

The calculations have been performed for the following real parameters: $\bar{q}_1^* = 2.5 mgl\, q_1^*$, $\bar{\omega}_1 = \sqrt{15/14} \cdot \sqrt{g/l}$, $\bar{c} = 0.2 m/2 \cdot \sqrt{35/6} \cdot \sqrt{g l^3}$, $h = 2.5l$, $e = 1$, where $\bar{c}$ is the coefficient of damping in all joints of the pendulum, e is the restitution coefficient, l and m are the length and the mass of each rod (more details are given in [1]).

Another example is given in Fig. 3, when the hyperchaotic attractor for q_1 = 0.75 is shown. The spectrum of Lyapunov exponents is (0.06, 0.01, 0, -0.67, -0.94, -1.48, -2.10).

This system exhibits very rich non-linear dynamics: in addition to examples reported in this abstract the coexisting stable and unstable periodic solutions and their various bifurcations have been found; various quasiperiodic chaotic and hyperchaotic attractors (sometimes coexisting) have been also detected.

In the next future we are going to improve our algorithms in order to continue the investigations of classical and non-classical bifurcations in dynamical systems with impacts. In addition the investigations of more practical example: the piston-connecting rod-crankshaft system as a self-excited triple pendulum with impacts are in progress.

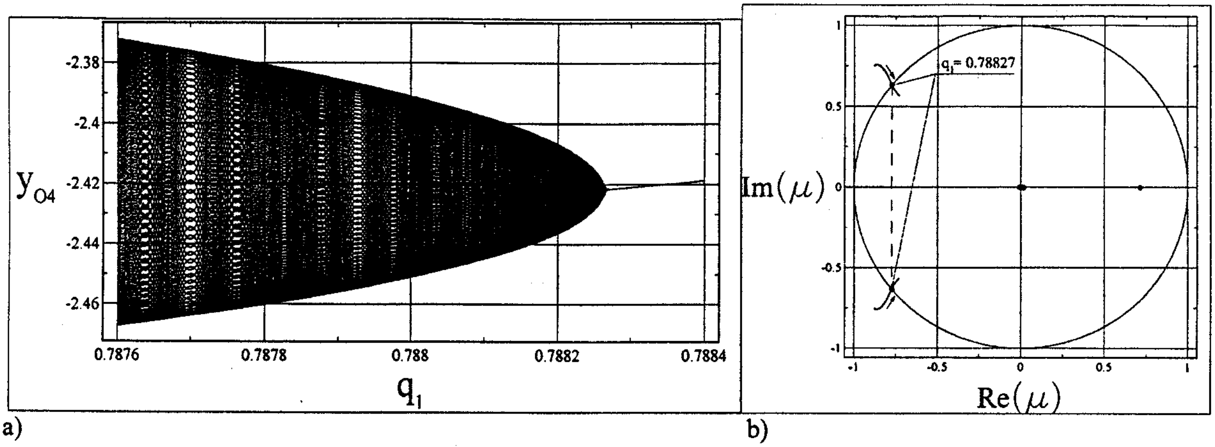

Fig.2. The Neimark-Sacker bifurcation (for the parameter $q_1 = q_1^* = 0.78827$): a) bifurcation diagram (y_{O4} is the vertical position of the end of the third rod); b) the complex plane of the Floquet multipliers.

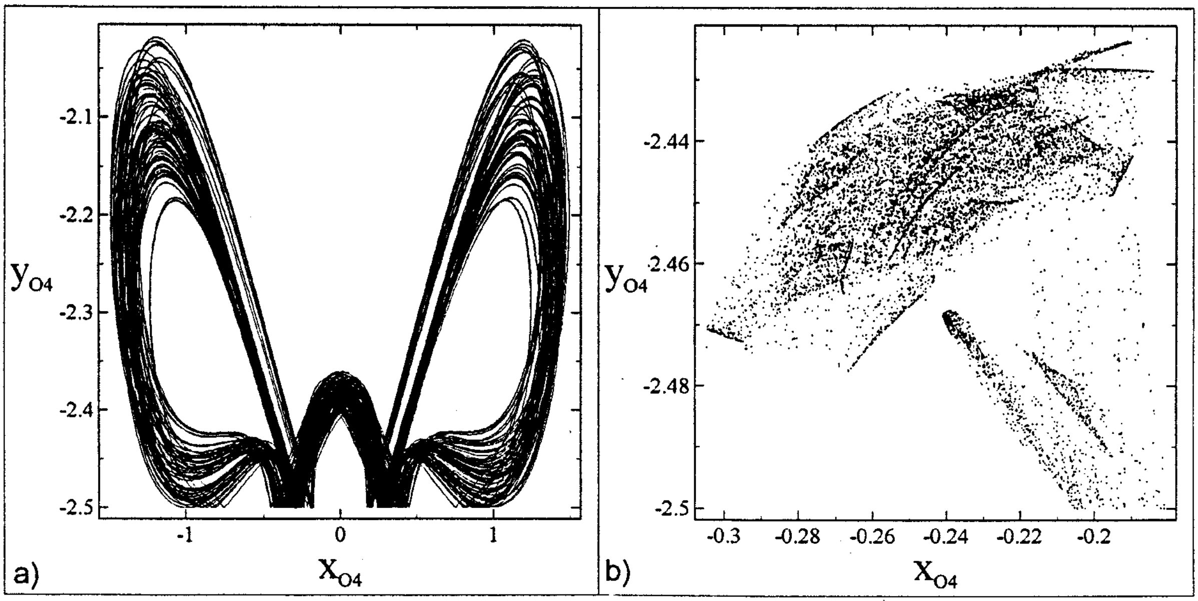

Fig. 3 Example of hyperchaotic attractor: trajectory (a) and Poincaré map (b); x_{O4} and y_{O4} are the co-coordinates of the third rod's end position.

References

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