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CALCULATING LYAPUNOV EXPONENTS FROM AN INTERPOLATED TIME SERIES

In this paper our attention will be focused on calculation of Lyapunov exponents from a time series [4, 5] which was interpolated by Lagrange polynomial scheme. The time series was obtained from a dynamical system with friction. With respect to accuracy of solution in phase space, the stick-slip system was solved by Runge-Kutta method with variable time step evolution. An 'exact' solution based on the Hénon's method [2] with small step is used to the Lagrange interpolation. The interpolating polynomial of degree n-1 through the n points $f_1=f(t_1)$, …, $f_1=f(t_n)$ is given by Lagrange's classical formula [3]:

$$L_n(t)=\Sigma_{i=0}^{n} f_i \frac{(t-t_0)\ldots(t-t_{i-1})(t-t_{i+1})\ldots(t-t_n)}{(t_i-t_0)\ldots(t_i-t_{i-1})(t_i-t_{i+1})\ldots(t_i-t_n)}, \tag{1}$$

where: f_i – 'exact' or known value in time t_i, L_n – nth searched value in time t interpolated from points t_0, …, t_n . After this procedure we obtain an adequate interpolated trajectory. Evidently, this interpolation is driven only from one dimension of our system. For each dimension we apply the introduced procedure separately. Later this way will be clearly illustrated.

Let's take a dynamical self-excited two-degree-of-freedom system with friction [1]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \left(-x_1 + x_3 + \left(x_1^3 - x_3^3\right) + \left(x_4 - x_2\right) + F, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = \left(-x_3 + x_1 - x_3 + \left(x_3^3 - x_1^3\right) + \left(x_2 - x_4\right.\right. . \end{cases} \tag{2}$$

where: $F=\begin{cases} -\operatorname{sgn}(V_{rel})\dfrac{1+x_3}{1+|V_{rel}|} & \text{for } V_{rel}\neq 0, \\ 1+x_3 & \text{for } V_{rel}=0. \end{cases}$, $V_{rel}=x_2-V_b$ is the relative velocity, V_b is a velocity of a belt on which one of two masses is lying. An example trajectory and its adequate Lagrange interpolation of this simplified dynamical system is illustrated in Figure 1.

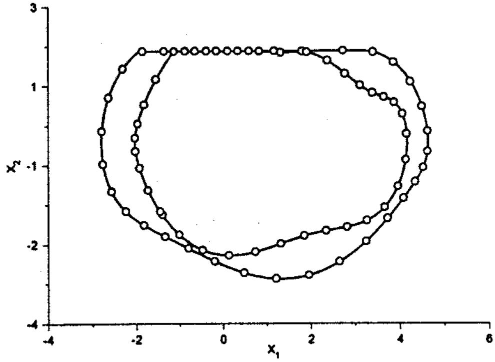

Figure 1. Example two periodic trajectories (black line) and its Lagrange interpolation (open circles).

The 2-periodic trajectory shown in Figure 1 was obtained in two steps. In the first step, two time series for displacement x_1 and velocity x_2 have been interpolated. In the second step, phase space using this interpolated points with its original trajectory has been plotted. It is very interesting that after the simple derived construction the proper image of the Lagrange interpolation could be obtained.

After all of previous operations the special time series with fixed time step was used in numerical algorithm for spectrum calculation of the Lyapunov exponents from time series. Some of the results for 2-periodic (see Figure 1), a quasi-periodic and a chaotic trajectories are listed below.

The Lyapunov exponents for some exemplary trajectories are as follows: 2-periodic trajectory - ± 0.000111; -0.034891; -0.079654; -1.729909, a quasi-periodic trajectory - $+0.001887$; $+0.000048$; -0.002914; -0.029456, a chaotic trajectory - $+0.017218$; -0.000485; -0.015889; -0.420986. The values of the above Lyapunov exponents have been calculated in 4000 iterations.

Interpolation of the standard time series with a variable step size have allowed to decrease number of used points about 400 times. It means, that shorter time history is stored in computer memory. From a point of view of the introduced method the assumed time history can be significantly elongate.

References

[1] Awrejcewicz, J., Olejnik, P., "Stick-slip dynamics of a two-degree-of freedom system", Proceedings of DETC' 01, 2001 ASME Design Technical Conferences, September 9-12, Pittsburgh, PA, USA, 2001, 9 pages.

[2] Hénon, M., "On the numerical computation of Poincaré maps", Physica D 5, 412-414, 1982.

[3] Stoer, J., Bulirsch, R., "Introduction to Numerical Analysis", Springer-Verlag, 1980.

[4] Van Wyk, M. A., Steeb, W. –H., "Chaos in Electronics", Kluwer Academic Publishing, 1997.

[5] Wolf, A., et al, "Determining Lyapunov Exponents From a Time Series", Physica 16D, 285-317, 1985.