

**STICK-SLIP DYNAMICS OF A TWO-DEGREE-OF-FREEDOM SYSTEM**

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Two-degree-of-freedom autonomous system with friction is analyzed numerically [1]. The friction coefficient has been smoothed using arc tan function. The standard, but slightly modified chaos identification tools have been applied for the analyzed discontinuous system. Some interesting examples of stick-slip regular and chaotic dynamics have been illustrated and discussed.

An idea to apply arc tan in order to smoothen Coulomb type friction is not new, but recently has been reconsidered by Van de Vrande et al. [3]. A simple linear system has been considered with friction and a validity of approximations has been discussed. We use a similar like approach by introduction arc tan instead of sgn. This approximation is especially useful during calculations of the Lyapunov exponents where a similar like approach to Oseledec [2] can be applied. The following approximation to the sgn function is applied

$$-\text{sgn } v_{w,i} = -\frac{2}{\pi} \arctan(\epsilon v_{w,i}),$$

where  $\epsilon > 0$ . The estimation of the approximation given by Eq. (4) is shown in Fig. 2.

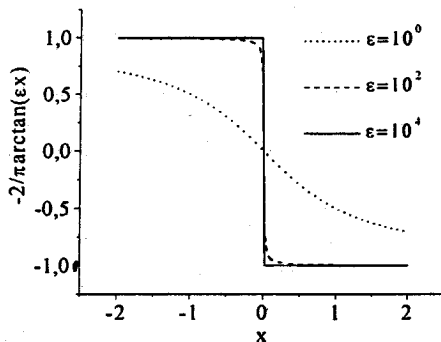


Figure 2. Approximation governed by Eq. (4).

Taking into account the approximation in Eq. (4) we get

$$F_i = -\frac{2F_{s,i} \arctan(\epsilon v_{w,i})}{\pi(1 + \delta|v_{w,i}|)} \quad i = 1, 2,$$

which is further used.

**REFERENCES**

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