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DYNAMIKA REGULARNA I CHAOTYCZNA W UKŁADZIE O DWÓCH STOPNIACH SWOBODY
REGULAR AND CHAOTIC DYNAMICS OF THE TWO-DEGREE-OF-FREEDOM SYSTEM

In this paper our attention is focused on stick-slip regular and chaotic dynamics of a two-degree-of-freedom system.

The analysed mechanical system with two-degree-of-freedom and the Duffing type stiffness is shown in *Figure 1*. The masses m_1 and m_2 are oscillating on a driving belt. The driving belt is moving with the constant velocity υ_{dr}. The moving bodies with masses m_i $(i=1,2)$ are linked by linear (k_0, k_1) and non-linear (k_2) stiffness and damping (c_0, c_1, c_2). The masses are influenced by dry frictions F_i $(i=1,2)$ acting on each of masses, correspondingly. The displacements of m_1 and m_2 are denoted by x_1 and x_2, respectively.

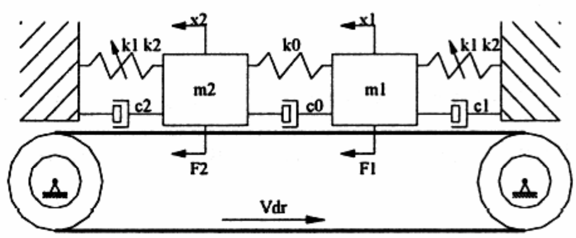


Figure 1. The considered system

Two-degree-of-freedom stick–slip oscillator is governed by the following second order differential system of equations:

$$
\begin{aligned}
m_1\ddot{x}_1 + (c_0 + c_1)\dot{x}_1 - c_0\dot{x}_2 + (k_0 - k_1)x_1 + k_2x_1^3 - k_0x_2 &= F_1, \\
m_2\ddot{x}_2 + (c_0 + c_2)\dot{x}_2 - c_0\dot{x}_1 + (k_0 - k_1)x_2 + k_2x_2^3 - k_0x_1 &= F_2.
\end{aligned} \tag{1}
$$

Friction forces to the right-hand sides of equations (1) read:

$$
\begin{aligned}
&|F| \le \mu_0 F_N = F_s && \text{for } \upsilon_{rel} = 0, \\
&F = -\mu F_N \operatorname{sgn}\upsilon_{rel} = -(\mu\mu_0^{-1})F_s \operatorname{sgn}\upsilon_{rel} && \text{for } \upsilon_{rel} \ne 0,
\end{aligned} \tag{2}
$$

where: F is the friction force, μ_0 is the constant static friction coefficient, F_N is the normal force, F_s is the maximum static friction force, υ_{rel} is the relative velocity, and the dynamic friction coefficient is given by $\mu = \mu_0(1+\delta|\upsilon_{rel}|)$.

The non-dimensional form of equation (1) can be written as follows:

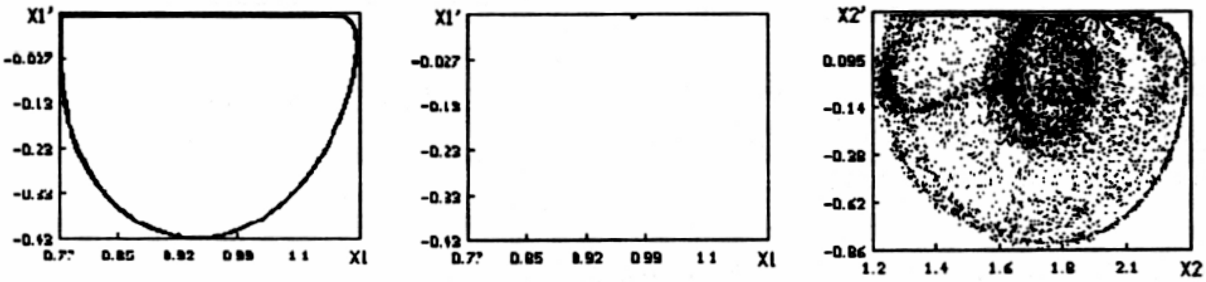
$$\xi_1 \ddot{y}_1 + \alpha_0(\dot{y}_1 - \dot{y}_2) + \alpha_1 \dot{y}_1 + (1 - \beta_1) y_1 + \beta_2 y_1^3 - y_2 = \hat{F}_1,$$
$$\xi_2 \ddot{y}_2 + \alpha_0(\dot{y}_2 - \dot{y}_1) + \alpha_2 \dot{y}_2 + (1 - \beta_1) y_2 + \beta_2 y_2^3 - y_1 = \hat{F}_2, \qquad (3)$$

where: prime denotes differentiation with respect to non-dimensional time $\tau = \omega_1 t$, t is time, $\omega_1 = k_1^{0.5} m_1^{-0.5}$, $y_i = k_0 x_i F_{S_1}^{-1}$. Other non-dimensional coefficients are defined as follows: $\xi_1 = m_1 \omega_1^2 k_0^{-1}$, $\xi_2 = m_2 \omega_1^2 k_0^{-1}$, $\alpha_0 = c_0 \omega_1 k_0^{-1}$, $\alpha_1 = c_1 \omega_1 k_0^{-1}$, $\alpha_2 = c_2 \omega_1 k_0^{-1}$, $\beta_1 = k_1 k_0^{-1}$, $\beta_2 = k_2 F_{S_1}^2 k_0^{-3}$, $\beta = F_{S_2} F_{S_1}^{-1}$, and the forces are given by:

$$\left|\hat{F}_1\right| \le 1 \qquad \textit{for} \quad \hat{\upsilon}_{rel,1} = 0,$$
$$\hat{F}_1 = \operatorname{sgn} \hat{\upsilon}_{rel,1}\left(1 + \gamma \left|\hat{\upsilon}_{rel,1}\right|\right) \qquad \textit{for} \quad \hat{\upsilon}_{rel,1} \ne 0, \qquad (4)$$

$$\left|\hat{F}_2\right| \le \beta \qquad \textit{for} \quad \hat{\upsilon}_{rel,2} = 0,$$
$$\hat{F}_2 = -\beta \operatorname{sgn} \hat{\upsilon}_{rel,2}\left(1 + \gamma \left|\hat{\upsilon}_{rel,2}\right|\right) \qquad \textit{for} \quad \hat{\upsilon}_{rel,2} \ne 0, \qquad (5)$$

respectively, where: $\hat{\upsilon}_{rel,i} = \dot{y}_i - \hat{\upsilon}_{dr}$ $(i=1,2)$, $\hat{\upsilon}_{dr} = (k_1 m_1)^{0.5} \upsilon_{dr} F_{S_1}^{-1}$, $\gamma = F_{S_1} \delta (km)^{-0.5}$. Examples of the investigated system are shown in *Figure 2*.



a) b) c)

Figure 2. Phase plot (a), Poincaré sections (b, c) for the parameters:
$\xi_1 = 0.5$, $\xi_2 = 0.1$, $\alpha_0 = 0$, $\alpha_1 = 0.02$, $\alpha_2 = 0.1$, $\beta_1 = 0.1$, $\beta_2 = 0.6$, $\beta = 1$, $\gamma = 3$, $\hat{\upsilon}_{dr} = 0.07$ *(a, b), and*
$\xi_1 = 0.1$, $\xi_2 = 0.77$, $\alpha_0 = 0$, $\alpha_1 = 0.02$, $\alpha_2 = 0.09$, $\beta_1 = 0.22$, $\beta_2 = 0.12$, $\beta = 0.54$, $\gamma = 2$, $\hat{\upsilon}_{dr} = 0.32$ *(c).*

References

1) J. Awrejcewicz, "Deterministic oscillations of discrete systems", WNT, Warsaw 1996, in Polish.