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NON-LINEAR DYNAMICS OF TRIPLE PHYSICAL PENDULUM WITH IMPACTS

The triple pendulum dynamics, especially with impacts, is rather rarely investigated in comparison to single pendulums [1]. Dynamics of a triple pendulum rotating around a vertical [2] axis with damping, with external forcing and with impacts is analysed analytically and numerically. In this mechanical system periodic, quasi-periodic and chaotic motions are detected.

The governing differential equations of the system without impacts have the following non-dimensional form:

$$A\cdot\ddot{\psi}+B\cdot\dot{\psi}^2+C\cdot\dot{\psi}+D=F, \qquad (1)$$

where

$$\ddot{\psi}=\begin{Bmatrix}\ddot{\psi}_1\\ \ddot{\psi}_2\\ \ddot{\psi}_3\end{Bmatrix},\ \dot{\psi}^2=\begin{Bmatrix}\dot{\psi}_1^2\\ \dot{\psi}_2^2\\ \dot{\psi}_3^2\end{Bmatrix},\ \dot{\psi}=\begin{Bmatrix}\dot{\psi}_1\\ \dot{\psi}_2\\ \dot{\psi}_3\end{Bmatrix},$$

$$A=\begin{bmatrix}1 & \nu_{12}\cos(\psi_1-\psi_2) & \nu_{13}\cos(\psi_1-\psi_3)\\ \nu_{12}\cos(\psi_1-\psi_2) & \beta_2 & \nu_{23}\cos(\psi_2-\psi_3)\\ \nu_{13}\cos(\psi_1-\psi_3) & \nu_{23}\cos(\psi_2-\psi_3) & \beta_3\end{bmatrix},$$

$$B=\begin{bmatrix}0 & \nu_{12}\sin(\psi_1-\psi_2) & \nu_{13}\sin(\psi_1-\psi_3)\\ -\nu_{12}\sin(\psi_1-\psi_2) & 0 & \nu_{23}\sin(\psi_2-\psi_3)\\ -\nu_{13}\sin(\psi_1-\psi_3) & -\nu_{23}\sin(\psi_2-\psi_3) & 0\end{bmatrix},$$

$$C=\begin{bmatrix}c_1+c_2 & -c_2 & 0\\ -c_2 & c_2+c_3 & -c_3\\ 0 & -c_3 & c_3\end{bmatrix},\quad F=\begin{Bmatrix}q_1\cos(\omega_1 t+\varphi_1)\\ q_2\cos(\omega_2 t+\varphi_2)\\ q_3\cos(\omega_3 t+\varphi_3)\end{Bmatrix},$$

$$D = \begin{Bmatrix} \sin\psi_1 - \omega^2\cos\psi_1\left(\kappa_1\sin\psi_1 + \upsilon_{12}\sin\psi_2 + \upsilon_{13}\sin\psi_3\right) \\ \sin\psi_2 - \omega^2\cos\psi_2\left(\upsilon_{12}\sin\psi_1 + \kappa_2\sin\psi_2 + \upsilon_{23}\sin\psi_3\right) \\ \sin\psi_3 - \omega^2\cos\psi_3\left(\upsilon_{13}\sin\psi_1 + \upsilon_{23}\sin\psi_2 + \kappa_3\sin\psi_3\right) \end{Bmatrix}.$$

In the case of impact in a system of more than one-degree-of-freedom the main problem is to assign the motion of a system just after an impact [3]. In such systems it is advisable to define the obstacle as such unilateral constrain like this for our triple pendulum:

$$f(\psi_1, \psi_2, \psi_3) \geq 0. \qquad (2)$$

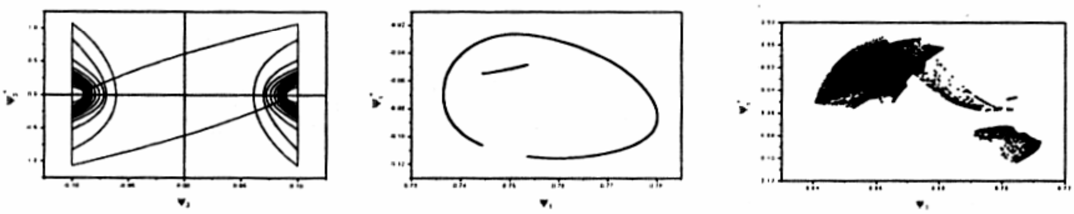
The impact appears if $f(\psi_1, \psi_2, \psi_3) = 0$. The generalized impact law [3] for the case of triple pendulum may be written as follows:

$$\begin{bmatrix} n^T \\ t_1^T \cdot A \\ t_2^T \cdot A \end{bmatrix} \cdot \begin{Bmatrix} \dot{\psi}_1^+ \\ \dot{\psi}_2^+ \\ \dot{\psi}_3^+ \end{Bmatrix} = \begin{bmatrix} -e\, n^T \\ t_1^T \cdot A \\ t_2^T \cdot A \end{bmatrix} \cdot \begin{Bmatrix} \dot{\psi}_1^- \\ \dot{\psi}_2^- \\ \dot{\psi}_3^- \end{Bmatrix}, \qquad (3)$$

where e - restitution coefficient ; n - normal vector to the surface $f(\psi_1, \psi_2, \psi_3) \equiv 0$,

and $n^T \cdot t_1 = 0$, $n^T \cdot t_2 = 0$, $t_1^T \cdot t_2 = 0$. (4)

The written program allows to set different cases of obstacles, and some exemplary (for special case of the triple pendulum, i.e. consisting of three identical rods) results are shown in Fig. 1.



a) b) c)

Fig. 1. Some exemplary solutions: a trajectory (a) and the Poincaré sections (b, c). The impact law for triple pendulum has been introduced and some exemplary solutions for such system with impacts, with damping and external forcing have been shown. The calculations exhibit very rich variety of periodic, quasi-periodic and chaotic solutions, are characteristic for the investigated system.

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