ANALYTICAL INVESTIGATION OF STRONGLY NONLINEAR DYNAMICAL SYSTEMS

Igor V. Andrianov
Pridneprovye State Academy of Civil Engineering and Architecture,
24a Chernyshevskogo St., Dnepropetrovsk 320005, Ukraine.
Jan Awrejcewicz
Technical University of Lodz,
Division of Automatics and Biomechanics,
1/15 Stefanowski St., 90-924 Lodz, Poland,
T: (48-42) 31-22-25, F: (48-42) 31-22-25, E: awrejcew@ck-sg.p.lodz.pl.

A new asymptotic investigation of strongly nonlinear dynamical systems is presented. First a construction of periodic oscillations in an autonomous system is proposed.

We consider the homogeneous nonlinear differential equation

$$\ddot{x} + x^n = 0, \tag{1}$$

and we are going to find a periodic solution with the following initial conditions

$$x(0) = 1, \quad \dot{x}(0) = 0. \tag{2}$$

Using the small δ method, equation (1) is transformed into the following one

$$\ddot{x} + x^{1+2\delta} = 0. \tag{3}$$

Taking into account the following series

$$(x^2)^\delta = 1 + \delta \ln(x^2) + \frac{\delta^2}{2}[\ln(x^2)]^2 + \ldots \tag{4}$$

the solution to equation (3) is sought in the form

$$x = \sum_{n=0}^{\infty} \delta^n x_n. \tag{5}$$

Besides, we introduce a standard change of time following the formula

$$t = \tau / \omega, \tag{6}$$

and the introduced parameter ω is defined by the series

$$\omega^2 = 1 + \alpha_1 \delta + \alpha_2 \delta^2 + \ldots \tag{7}$$

Substituting (5) - (7) into equation (3) and taking into account (4), we get (after splitting with respect to δ) the recurrent system of equations which is solved analytically.

Then a construction of periodic solutions to partial differential equations with nonlinear boundary conditions is presented.

The following equation governing vibration of a string or longitudinal vibration of a rod with nonlinear boundary conditions is considered:

$$u_{tt} = u_{\alpha\alpha} \tag{8}$$

$$u(0,t) = 0, \tag{9}$$

$$u(1,t) + \varepsilon u(1,t) + \varepsilon u^3(1,t) = 0. \tag{10}$$

Introducing in an artificial way the parameter δ, the boundary condition (10) can be formulated in the following manner

$$u + \alpha u + \alpha u^{1+2\delta} = 0. \tag{11}$$

A solution to the boundary problem governed by (8), (9) is sought as a series of the parameter δ. After splitting the initial boundary value problem (8) - (10) with regard to the powers of the small parameter δ, the recurrent system of linear boundary value problems is obtained, which is solved further analytically.

In both cases considered the obtained results are close to exact values.