

# BOUNDARY CONDITIONS AND CONTROL PARAMETERS INFLUENCE ON THIN SHELLS THERMO-DYNAMICS.

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We consider dynamical stability of a shell, where the Young's modulus and the linear thermal expansion coefficient do not depend on the temperature. The problem is governed by the following dimensionless equations

$$\frac{1}{12(1-\nu^2)} \left( \lambda^{-2} \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \lambda^2 \frac{\partial^4 w}{\partial y^4} \right) - \nabla_{\kappa}^2 F - L(w, F) + \kappa \left( \frac{\partial^2 w}{\partial \tau^2} + \xi \frac{\partial w}{\partial \tau} \right) = q, \quad (1)$$

$$\lambda^{-2} \frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \lambda^2 \frac{\partial^4 F}{\partial y^4} + \nabla_{\kappa}^2 w + \frac{1}{2} L(w, w) = 0, \quad (2)$$

where:  $\nu$  - Poisson's coefficient;  $\lambda = a/b$ ;  $a, b$  - length and width of a shell;  $w$  - normal displacement of the mean surface;  $F$  - stress function;  $L$  - linear partial operator;  $\tau = \alpha t/h^2$ ;  $t$  - time;  $\alpha$  - thermal expansion coefficient;  $h$  - thickness of a shell;  $\kappa = a^2 b^2 \gamma \alpha / (g E h^6)$  - physical-geometrical parameter;  $\gamma$  - specific gravity;  $g$  - acceleration of gravity;  $E$  - Young modulus;  $\xi$  - medium damping coefficient;  $q$  - transversal load intensity,

$$\nabla_{\kappa} = K_x \frac{\partial^2}{\partial y^2} + K_y \frac{\partial^2}{\partial x^2}.$$

On the internal shell surface a unstationary heat convection occurs according to the Newton's law. The external and lateral surfaces are isolated.

The boundary conditions for a heat conduction have the form

$$\begin{aligned} \frac{\partial T}{\partial z} + \text{Bi}(T - T_s) &= 0 & \text{for } z = \frac{1}{2}, \\ \frac{\partial T}{\partial z} &= 0 & \text{for } z = -\frac{1}{2}, \end{aligned} \quad (3)$$

where:  $T$  - temperature;  $\text{Bi} = \frac{\alpha h}{\lambda_g}$ ;  $\alpha$  - corresponds to the surface  $z = 1/2$ ;  $\lambda_g$  - thermal expansion coefficient;

$T_s$  - is a temperature surrounding medium.

The heat conduction equation has the form

$$\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial z^2} \quad (4)$$

with the initial conditions

$$T = T_0 \quad \text{for } \tau = 0. \quad (5)$$

For a positive temperature value  $\theta$  a shell can work in a dynamical heating conditions, whereas for a negative value it can cooling conditions. Both of the mentioned condition are often met in practice.

In this work we analyse an influence of the different boundary and loading conditions as well as the dimensionless Biot parameter  $\text{Bi}$  for a shell stress-deformation state and its dynamical stability. Many interesting results are discussed and illustrated.