ANALYTICAL AND NUMERICAL INVESTIGATION OF VIBRO-IMPACT DYNAMICS CONTROL

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Abstract

In this work a feedback control of the vibro-impact systems with one and two degrees of freedom have been presented. The applied feedback delay loop improves stability of the vibro-impact periodic orbit and therefore the system becomes more disturbance-resistant. The delay loop is automatically switched off when the vibro-impact periodic attractor is achieved. It starts to act when the disturbances occur.

The following vibro-impact model is assumed:

– its dynamics is described without any constraints between the successive impacts;
– the new boundary conditions are introduced in the movement of each impact, using a restitution coefficient (the Newton hypothesis).

A key point of that control is to define the feedback delay loop coefficients which should satisfy the earlier mentioned requirements.

For the one-degree-of-freedom system and for far from a resonance, an analytical approach to define the mentioned coefficients has been already proposed [1,2].

In this work a similar approach is applied to the systems in the resonance vicinity. In the proposed methods for determination of the delay loop coefficients, the perturbation technique has been used [2]. In addition, a special impact mapping has been applied, which allows to reduce a problem to the consideration of the algebraic equation [3].

To verify the analytical approach applied in this work the numerical calculations using the MATLAB-Simulink environment have been carried out, which fully confirmed the theoretical considerations.

In the case of the two-degree-of-freedom system (Fig. 1) impact-dynamics has been analysed only numerically. The analysed system consists of two masses m_i, stiffnesses k_i, dampings c_i ($i = 1,2$), and the friction coefficient f_0. The exiting harmonic force has the amplitude P_0, and the frequency ω. Finally, φ_1 is the phase and s denotes the constraint.

Contrary to the system with one degree of freedom, we have new possibilities to apply a feedback control loop:

1. The control loop acts only on the mass m_1.
2. The control loop acts only on the mass m_2.
3. Two control loops acting on both masses are applied.

The most general case (with two loops) is presented in Fig. 1b.

The system dynamics is governed by the following equations:

$$\ddot{y}_1 + 2b_1\dot{y}_1 + f_1\,\mathrm{sign}(\dot{y}_1 - \dot{y}_2) + \beta_{11}^2 y_1 - \beta_{12}^2 y_2 = \\ = P\cos(\omega_0\tau + \varphi_1) + p_1\left[y_1(\tau) - y_1(\tau - \bar{T})\right] + q_1\left[\dot{y}_1(\tau) - \dot{y}_1(\tau - \bar{T})\right],$$

$$\ddot{y}_2 + 2b_2\dot{y}_2 - f_2\,\mathrm{sign}(\dot{y}_1 - \dot{y}_2) + y_2 - y_1 = \\ = p_2\left[y_2(\tau) - y_2(\tau - \bar{T})\right] + q_2\left[\dot{y}_2(\tau) - \dot{y}_2(\tau - \bar{T})\right].$$

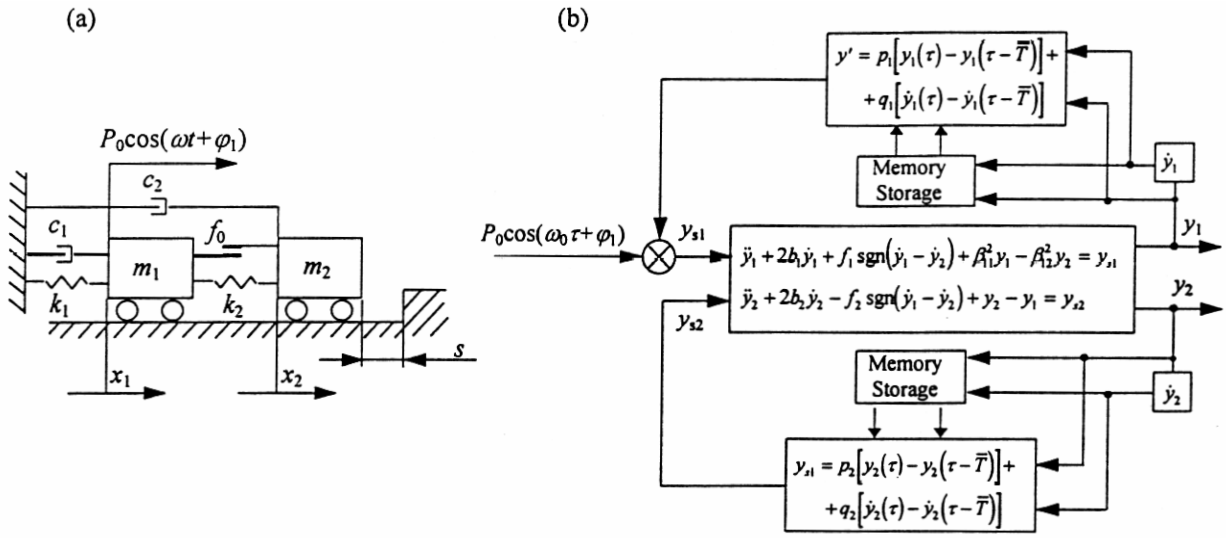

Fig. 1. Schematic view of the two-degree-of-freedom vibro-impact system (a) and scheme of its control (b)

where: $y_i = x_i/x_{st}$ are the nondimensional coordinates, and

$$b_1 = \frac{c_1}{2m_1}\sqrt{\frac{m_2}{k_2}},\quad b_2 = \frac{c_2}{2\sqrt{k_2 m_2}},\quad f_1 = \frac{m_2}{m_1}\frac{k_1}{k_2}\frac{f_0}{P_0},\quad f_2 = \frac{k_1}{k_2}\frac{f_0}{P_0},\quad \beta_{11}^2 = \frac{k_1+k_2}{k_2}\frac{m_2}{m_1},\quad \beta_{12}^2 = \frac{m_2}{m_1},$$

$$\beta_{11}^2 = \frac{k_1+k_2}{k_2}\frac{m_2}{m_1},\quad \beta_{12}^2 = \frac{m_2}{m_1},\quad P = \frac{m_2}{m_1}\frac{k_1}{k_2},\quad \omega_0 = \omega\sqrt{\frac{m_2}{k_2}}.$$

When the mass m_2 meets the barrier, we have:

$$\begin{cases} y_{2+} = y_{2-} \\ \dot{y}_{2+} = -R\dot{y}_{2-} \end{cases} \quad \text{for } y_2 \geq \bar{s},$$

where $\bar{s} = s/x_{1st}$ denotes the nondimensional distance of the mass m_2 to the barrier.

In this work three earlier mentioned control methods have been applied using the MATLAB-Simulink package.

Among others, it has been shown that the suitable our application of one loop may sufficiently decrease "stabilisation time" either for one or two of the masses being stabilised. The addition of the second loop sometimes does not only improve the system stabilisation, but can lead to a change for the worse.

Finally, we point out that the proposed control may change qualitatively a motion of the analysed vibro-impact system. Instead of the vibro-impact motion, a periodic non-impact dynamics can occur.

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