SYNTHESIS AND OPTIMISATION OF PLATE AND SHELL SURFACES WITH REQUIRED DYNAMICAL CHARACTERISTICS

Jan Awrejcewicz[†] and Vadim A. Krysko[‡]

Technical University of Łódź, Division of Control and Biomechanics (I-10)
1/15 Stefanowskiego St., 90-924 Łódź, POLAND.
Phone: (4842) 312225; Fax: (4842) 361383; E-mail: awrejcew@ck-sg.p.lodz.pl

[‡]Departament of Mathematics, Saratov State University
B. Sadovaya 96/77, 410054 Saratov, Russia

In this work we consider a problem of the shell volume synthesis using an assumption that the thickness $h(x,y)$ as well as the surface Ω of a shell are treated as the control functions [1].

A problem of the free shell oscillations is reduced to the following eigenvalue problem.

$$L[h]u - \omega^2 \mu[h]u = 0; \;\; u = u(x,y), \;\; (x,y) \in \Omega, \tag{1}$$

where the differential operators $L[h]$ and $\mu[h]$ are related to the potential energy of deformations and the mass distribution, and have the form depending on the applied kinematic model of a shell (Kirchhoff - Love, Timoshenko, etc.)

We are going to construct a shell with a given series of N eigenfrequencies $\left\{\alpha_k\right\}_{k=1}^{N}$ by a suitable choice of $h(x,y) = h^*(x,y)$ and $\Omega = \Omega^*$. Because the problem is oriented on synthesis, therefore it is not known if a shell with the given manifolds $h(x,y) \in U_{\partial 1}$ and $\Omega \in U_{\partial 2}$ and the requirements $\omega_k^2 = \theta_k = \alpha_k \quad (k = 1,2.....N)$ exists. Therefore, by a solution to the mentioned problem we mean a construction very closely related to the required one.

A close neighbourhood related to the spectral characteristics is defined by the following (close to zero) function.

$$R(N,\vec{\varepsilon}) = \sum_{k=1}^{N} \frac{1}{\varepsilon_k} \left|\theta_k - \alpha_k\right|^2. \tag{2}$$

Thus, a problem of synthesis (optimisation) can be formulated in the following manner: define $h^*(x,y) \in U_{\partial 1}$ and $\Omega^* \in U_{\partial 2}$ which minimise the functional

$$J(h,\Omega) = \sum_{k=1}^{N} \frac{1}{\varepsilon_k} \left|\theta_k - \alpha_k\right|^2 + I(h,\Omega), \tag{3}$$

where $\{\alpha_k\}_{k=1}^{N}$ is the required frequency spectrum, $\{\theta_k\}_{k=1}^{N}$ is the spectrum defined by the governing equations of shells with the thickness h and the surface Ω, ε_k is the penalty for the constraint $\theta_k = \alpha_k$ violation, $I(h,\Omega)$ is the positive functional for all $h \in U_{\partial 1}$, $\Omega \in U_{\partial 2}$ which include the additional construction shell constraints. However, the described problem is incorrect. In order to realise the synthesis (3) we are going to parametrize the function h and the space Ω. It means that we change the earlier problem of synthesis in the following way.

For the given vector $\{\alpha_k\}_{k=1}^{N}$ and the constructional constraints

$$\Phi_0(\vec{\beta}) = 1(h_\beta, \Omega_{\beta)} \geq 0, \quad \forall \vec{\beta} \in U_\partial \tag{4}$$

we are going to find $\vec{\beta}^* \in U_\partial \subset R^{m+p}$ realising the minimum of the $m+p$ variables function

$$\Phi(\vec{\beta}) = \sum_{k=1}^{N} \frac{1}{\varepsilon_k} \left| \theta_{k\beta} - \alpha_k \right|^2 + \Phi_0(\vec{\beta}), \tag{5}$$

where $\theta_{k\beta}$ denotes the second power of the shell free vibration frequency, and h_β and Ω_β are defined by (1).

Now the problem of synthesis is correct. It is expressed by the following theorem (which is proved).

Theorem : If $F(x,y,\vec{\beta})$ guarantees a continuos regular deformation of the initial space and the function $\Phi_0(\vec{\beta})$ is continuos on U_∂ (which is bounded manifold in R^{m+p}), then $\forall\{\alpha_k\}_{k=1}^{N} \subset K$ (K is the closed bounded manifold R^N). Function (5) achieves its exact low boundary on the pair of elements $\theta^* = \{\theta_k \beta^*\}_{k=1}^{N}$, $\vec{\beta}^*$, where $\vec{\beta}^* \in U_\partial$ is the stable solution to the synthesis problem, whereas $\vec{\theta}^*$ is the vector with components being the N first eigenvalues of (1) for $h = h_{\beta^*}$, $\Omega = \Omega_{\beta^*}$.

A suitable algorithm on the basis of the finite element method is proposed. A wide class of problems is considered.

References

[1] Awrejcewicz J., Krysko V.A., *Techniques and Methods of the Shells and Plates Vibration Analysis*. Technical University of Łódz, Series of Monographs, Łódz, 1996 (in Polish).