PACKETS OF PERIODIC OSCILLATIONS AND CHAOS IN A NONLINEAR OSCILLATOR WITH DELAY

J. AWREJCEWICZ

(Technical University,

Division of Dynamics and Control/ K-13/,

B. Stefanowskiego 1/15, 90-924 Łódź, Poland)

The following Van de Pol-Duffing's oscillator with delay has been considered:

$$\ddot{x} - \alpha(1 - x^2)\dot{x} + \omega_0^2 x + \beta x^3 = kx(t - \tau_0) + F\cos\omega t \tag{1}$$

The initial functions are as follows: $x(t) = 1$ for $-\tau_0 \leq t < 0$ and $x(0) = x$, and $\dot{x}(t) = 0$ for $-\tau_0 \leq t \leq 0$. First, a case $F = 0$ has been analyzed. We have found using a numerial experiment that the greater is the value of delay the longer is the transient state. However, for certain parameter sets the packets of periodically repeating oscillations have been obtained. We have observed the occurrence of such oscillations with the change of the amplification coefficient k as well as with the change of initial functions.

In the second part of research the full equation (1) has been analyzed. An averaging Van der Pol method has been used to obtain two amplitudes nonlinear differential equations of the first order. Then, the condition of the occurrence of the Hopf bifurcation curve in the τ_0, ω parameters plane for averaged equations has served to examine numerically the forced oscillator (1). We have proved numerically that in the vicinty of the Hopf bifurcation curve a slightly exhibited chaotic behaviour can be found. After the decreasing of damping coefficient the full chaotic motion has been developed.

References

[1] J. Awrejcewicz, "Bifurcation and chaos in simple dynamical system", World Scientific, 1989.

[2] J. Awrejcewicz, "Bifurcation and chaos in coupled oscillators", World Scientific, 1991.