Nonlinear Dynamics and Systems Theory, **15** (1) (2015) 99–106

Cooperation of One and Multi-Joint Muscles

B. Zagrodny * and J. Awrejcewicz

Lodz University of Technology, Department of Automation, Biomechanics and Mechatronics,
Stefanowskiego Str. 1/15 Lodz, Poland

Received: March 31, 2014;   Revised: January 7, 2015

Abstract: In this paper an analysis of equations describing single and multi-joint muscles cooperation during movement of limb segments is presented. Additionally, the Pareto-optimum problem is considered for the human upper limb in case of movement in sagittal plane. Uncertainty of this problem and some additional physiological restrictions such as angular range of motion or tissue tension are described. Moreover, effects of practical verification based on the video analysis of the volunteers arm movement and its lack of reproducibility are addressed. Examination of the artificial arm prototype shows similar behaviour to the human biological musculo-skeletal system. Furthermore, results of comparison with those obtained by other authors are shown.

Keywords: *biomechanics; muscle cooperation; motion analysis.*

Mathematics Subject Classification (2010): 93A30.

1 Introduction

The structure of biological systems is complex, which causes serious challenging problems in their control. The occurred imperfections, external actions on the objects under investigations or fatigue of musculo-skeletal systems affect trajectories of motion, their speed and precision of repetitions. It is conjectured that a movement of biological system is not determined along one fixed trajectory. Namely, there are infinite number of admissible paths of moving from one point to another one. In order to verify the hypothesis a mathematical model of cooperation of one and multi-joint muscles of the human upper limb during motions is proposed and analysed. This model is used to illustrate the way of cooperation of any number of single and multi-joint flexors and extensors of an arm

---

* Corresponding author: `mailto:bartlomiej.zagrodny@p.lodz.pl`

and forearm. Moreover, inability to determine exactly one trajectory, which is Pareto-optimal solution for the existing biological limitations, is also shown. On the other hand, it is proved that the set of admissible trajectories is narrow. Obtained theoretical results are verified in an experiment based on the repeatability of the human arm movement. In order to record the trajectories of movement a video analysis is used. The presented model is also applied to analysing the human walking on the treadmill and the obtained results are compared with earlier results concerning the issue. More sophisticated models, in this line, can be applied to analysing any system of the repeatable movement of the human body.

2 Mathematical Description of Problem

2.1 Notation and physiological description

Let us introduce a problem of a forearm flexion undergoing following muscle actions. The following notation is applied:

(i) $m,\ r$ – one joint flexors muscles of forearm and arm, respectively;

(ii) n – two joint flexors;

(iii) $o,\ s$ – one joint extensors of forearm and arm, respectively;

(iv) p – two joint extensors muscles.

Figure 1 presents an example of such system for a few of these muscles. Furthermore, the following nomenclature is used:

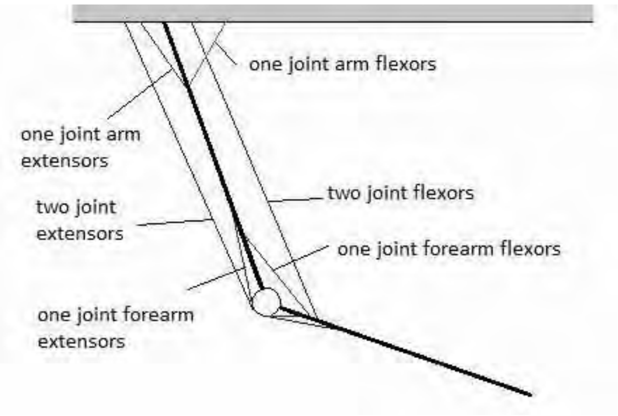

Figure 1: Example of the one and two joint muscles extensors and flexors system.

(i) $z_{(rj)a}$ – one joint arm flexors, $a \in (1, ..., r)$;

(ii) $z_{(pj)b}$ – one joint forearm flexors, $b \in (1, ..., m)$;

(iii) $z_{(2)c}$ – two joint flexors, $c \in (1, ..., n)$;

(iv) $p_{(rj)d}$ – one joint arm extensors, $d \in (1, ..., s)$;

(v) $p_{(pj)e}$ – one joint forearm extensors, $e \in (1, ..., o)$;

(vi) $p_{(2)f}$ – two joint extensors, $f \in (1, ..., p)$.

It is well known that contraction of the flexors causes flexion of the limb and contraction of the extensors causes extension assuming that they act stronger then its antagonists. For a precise movement they have to cooperate in an appropriate way (see [2-9]).

2.2 Formulation of the mathematical problem

Let us consider a forearm flexion. In this particular case the muscles z_{pj} and z_2 have to contract, whereas the muscles p_{pj} and p_2 have to reduce their tension. If the contraction is strong enough and not compensated by p_2, then we will observe also an arm flexion, what is undesirable in this case. To prevent this movement a contraction of p_{rj} muscles has to occur. This will compensate a momentum of two joint muscles yielding a movement of a forearm only. Similarly, such cooperation will happen in the case of other limbs movements. In what follows we analyse this problem in the case of arm and forearm movement in the sagittal plane. In the considered model it is assumed that each muscle contributes to the creation of the momentum acting on the joint.

We define it as follows:

$$M_i = r_i \cdot F_i, \tag{1}$$

where r means forces arm acting with respect to the joint with axis of rotation (see Figure 2). This arm is defined as a function of muscle length and limb flexion angle. Observe that it is different for each muscle.

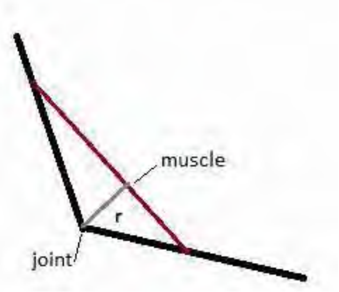

Figure 2: Example of defining forces arm acting on a joint.

The following forces momentum are acting on the elbow joint:

(i) flexing forearm: $M_{zp} = \sum_{i=1}^{r} M_{(zpj)i} + \sum_{i=1}^{n} M_{(z2)i}$;

(ii) straightening forearm: $M_{pp} = \sum_{i=1}^{o} M_{(ppj)i} + \sum_{i=1}^{p} M_{(p2)i}$.

On the shoulder joint the following forces momentum are acting:

(i) flexing arm: $M_{zr} = \sum_{i=1}^{r} M_{(zrj)i} + \sum_{i=1}^{n} M_{(z2)i}$;

(ii) straightening arm: $M_{pr} = \sum_{i=1}^{o} M_{(prj)i} + \sum_{i=1}^{p} M_{(p2)i}$.

Moments of the straightening forces are defined as the product of force F_i and constant radius block d.

Hence it follows, that sum of momentum acting on the joint are:

(i) for elbow joint: $M_l = M_{zp} + M_{pp}$;

(ii) for arm join: $M_r = M_{zr} + M_{pr}$.

A movement in one joint is caused by muscles lengths and tensions (forces) change involved in the movement, which belong to the physiological phenomenon ([2, 4, 6]). This means that we have to consider these equations as a system. Only in this way we can correctly and completely describe the problem.

Introducing the following notation:

$$\begin{gathered}\sum_{i=1}^{r} M_{(zpj)i} = Z_p; \sum_{i=1}^{n} M_{(z2)i} = Z_2; \sum_{i=1}^{r} M_{(zrji)} = Z_r;\\ \sum_{i=1}^{o} M_{(ppj)i} = P_p; \sum_{i=1}^{p} M_{(p2)i} = P_2; \sum_{i=1}^{s} M_{(prj)i} = P_r.\end{gathered} \tag{2}$$

The system dynamics is governed by the following system of equations:

$$\begin{cases} Z_p + Z_2 + P_p + P_2 = 0,\\ Z_r + Z_2 + P_r + P_2 = 0.\end{cases} \tag{3}$$

This yields:

$$Z_p + P_p = Z_r + P_r. \tag{4}$$

It can be observed that two joint muscles do not compensate a movement in an adjacent joint. Transforming this equation into a quotient form and assuming according to physiological behaviour that $Z_r + P_r \neq 0$, we obtain:

$$\frac{Z_p + P_p}{Z_r + P_r} = 1. \tag{5}$$

From that we can observe that when the flexor forces increase, the force P_p have to decrease or $Z_r + P_r$ have to increase.

However, this equation, with these conditions is not marked – there is infinitely many solutions. This is reflected in the actual behaviour of the muscular system. During movement it do not realise the ideal of one operating model. It can be explained by the imperfection of the nervous system, muscle fatigue or other physiological or environmental reasons.

In order to obtain the best solution, optimisation methods are used, such as minimising the cost function of energy, assuming that individual muscles can not exceed the maximum physiological tension. It is also possible to consider optimisation problem based on minimising the tension σ of a muscle.

Let us consider the following Pareto minimisation problem:

$$\{\sigma_i\},\ \sigma_i = \frac{F_i}{S_{i\ anat}}, \tag{6}$$

where σ_i means i-th muscle tension and F_i denotes force generated by i-th muscle, $S_{i\ anat}$ means anatomical cross-section area of i-th muscle.

We denote by J the following objective function:

$$J : \mathbb{R}_+^{r+m+s+o} \Longrightarrow \mathbb{R}_+^4, \tag{7}$$

$$J(\sigma_1, ..., \sigma_n) = \left(\sum_{i=1}^{r} \sigma_i,\ \sum_{i=r+1}^{r+m} \sigma_i,\ \sum_{i=r+m+1}^{r+m+s} \sigma_i,\ \sum_{i=r+m+s}^{l} \sigma_i \right),\ l = r + m + s + o. \tag{8}$$

We minimise the function J with the following conditions:

$$\begin{aligned} &0 \leq \sigma_i,\ \sigma_i \leq \sigma_{max},\ i = 1, ..., l, \\ &\sum_{i=r+1}^{r+m+s} S_i \sigma_i r_i(\alpha) = \sum_{i=1}^{r} S_i \sigma_i r_i(\alpha) + \sum_{i=r+m+s}^{l} S_i \sigma_i r_i(\alpha), \end{aligned} \tag{9}$$

where

$$\sum_{i=r+1}^{r+m+s} S_i \sigma_i r_i(\alpha) - \sum_{i=r+m+s}^{l} S_i \sigma_i, r_i(\alpha) = M, \tag{10}$$

and M is the momentum generated by the flexors. Moreover:

$$\begin{aligned} &\sigma_1 = \sigma_2 = ... = \sigma_r; \\ &\sigma_{r+1} = \sigma_{r+2} = ... = \sigma_{r+m}; \\ &\sigma_{r+m+1} = \sigma_{r+m+2} = ... = \sigma_{r+m+s}; \\ &\sigma_{r+m+s+1} = \sigma_{r+m+s+2} = ... = \sigma_l. \\ &M > 0. \end{aligned} \tag{11}$$

Even if we know $\sigma_{i\ max},\ i = 1, ..., l,\ \alpha \in (0, \pi),\ S_i$, the problem still is indeterminate, that is there exist infinitely many solutions. For obvious reasons some of them are more important for us than others. To choose them an additional criterion have to be added, that is a scalar function have to be supplemented to the objective function (for example, such as the function of the cost of energy $E(\sigma)$, which limits the number of Pareto-optimal solutions). Moreover, it is also possible to enter additional, physiological restrictions (see Section 3), so the set of admissible solutions becomes narrower.

3 Physiological Movement Restrictions

First of these restrictions concerns the limbs range of motion, and is provided in Table 1.

This introduces restrictions on the angles of abduction/adduction, flexion/extension and reversion/inversion of the limb or its segments. It can be combined with a stress distribution in the individual muscle. Another physiological condition is the phenomenon of the optimal length of the muscles and increase of tissue resistance (see for example Figure 3) in the extreme positions of the limb or change in limb muscle an its moments (see Figure 4). It can be seen that muscles not always work in optimal range, which moreover does not always cover an optimal moment arm.

Table 1: International Standard Orthopaedic Measurements Norm for an upper limb.

Joint	Plane of motion	Norm International Standard Orthopedic Measurements [deg]
Shoulder	S	50-0-170
	F	170-0-0
	T	30-0-135
	R(F90)	90-0-80
	R(F0)	60-0-70
Elbow	S	0-0-150
Forearm	R	90-0-80
radio -carpal	S	50-0-60
	F	20-0-30

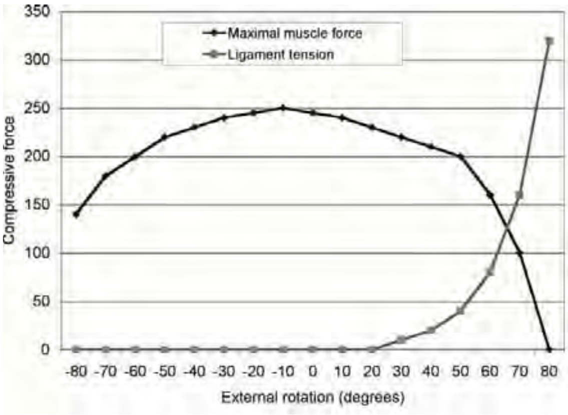

Figure 3: Corelation beetween muscular and capsular tension during arm movement [10].

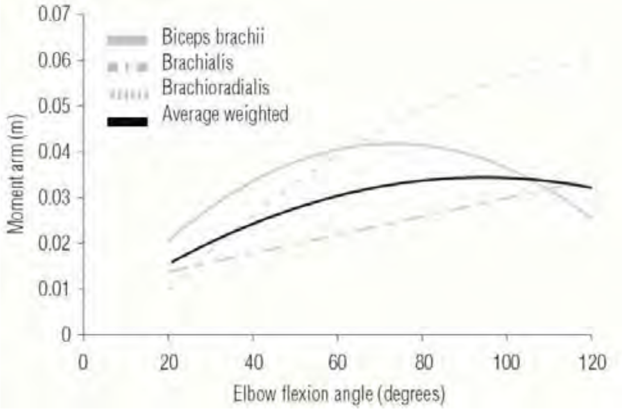

Figure 4: Moment arm for main forearm muscles and respective weighted mean moment arm [11].

Overlapping of the conditions leads to a reduction of possible solutions. However, as mentioned, the last of restrictions should make the solution possible. Not cost-effective (energy or due to excessive stretching of tissue) will be eliminated by the central nervous system. So there will be some narrow family of solutions. This is confirmed by the experience, the results of which are presented in Figure 5. It is also shown experimentally, using a prototype of artificial arm, that this argumentation is correct.

4 Practical Verification of the Problem

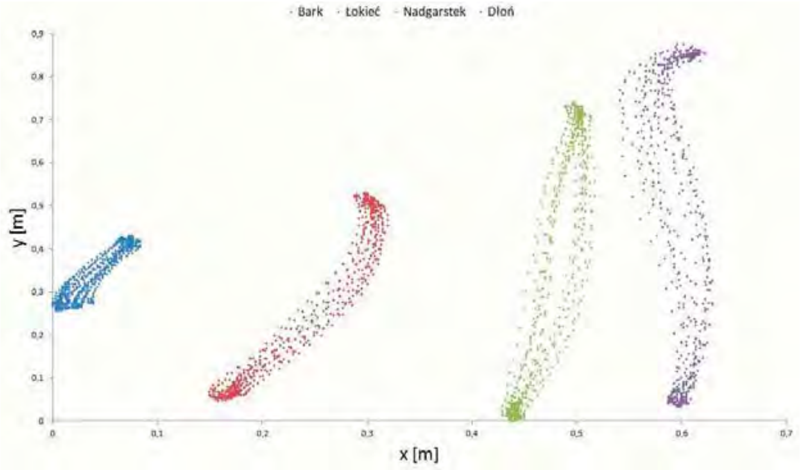

Figure 5: Repeatability of arm and forearm trajectories during subsequent cycles. Shoulder (blue), elbow (red), wrist (green) and palm (violet) trajectories (an example) [1] (with permission).

Coordinates of the upper limb joints that during the test were marked with reflective markers were determined by analysing software, specially developed for this purpose. Markers were illuminated coaxially to the optical axis of the lens to obtain maximum of reflectivity. The obtained results show that in studied biological systems there are no fixed trajectories. Five volunteers were examined. Their task was to raise their arm in the following manner:

(i) start from point on the level of their knees;
(ii) finish at a specific point above their heads;
(iii) complete the motion while sitting, without standing up;
(iv) complete the motion ten times.

Points were marked on the rack. No other restriction in arm movements were applied. Luminescent markers were placed on the shoulder joint, elbow, wrist and small finger. It is shown that the biological system, which is the upper limb human, does not have well-determined trajectory.

5 Conclusions

Due to many factors (imperfections of central nervous system phenomena, onset of muscle fatigue and other human and environmental factors), each trajectory was different. These observations confirmed the study published in the references [8] and [9]. In the first one, trajectory has been studied as the dependence of the position of the wrist the and in the second one movement were analysed in the transverse plane. It was also found that for each of movement the strategy was different (different phases of motion at different speeds, different bending angles of the limb). This means another work of muscles each time, which can be an illustration of an uncertainty muscles cooperation problem (see also [1]). For each cycle, volunteer make the movement in a slightly different manner (different speeds, limb flexion angles, etc.). There are as many solutions as combinations that meet a specified target. In this particular case, there are also some biological and physiological constraints such as maximum bending angles of the joints, the maximum force that can generate muscle action. When analysing the results, it was hypothesised that the differences between the two depend on the state of the musculo-skeletal injuries or illnesses completed. Paper [2] confirms the observations of muscle cooperation during movement. The authors have presented a measurement of EMG signals, which shows that in the case of movement in one or two joints, an activation of muscles that spanning a stationary joint was observes.

Acknowledgements

Authors want to thank Ph.D. Michał Ludwicki for permission to use his motion analysis programm. This paper have been presented during the 12 th Conference on Dynamical Systems - Theory and Applications.

References

[1] Andrysiak, T., Awrejcewicz, J., Ludwicki, M. and Zagrodny, B. On the human arm motion camera tracking system. *Vibrations in Physical Systems* **XXV**, Poznań (2012) 41–46.

[2] Gribble, P. L. and Ostry, D. J. Compensation for Interaction Torques During Single- and Multijoint Limb Movement. *Journal of Neurophysiology* **82** (1999) 2310–2326.

[3] Fick, R. *Handbuch der Anatomie des Menschen*. Jena, Gustav Fisher, vol. 2, 1910.

[4] Reicher, M. *Human Anatomy. General Anatomy, Bones, Joints and Ligaments, Muscles*. Warsaw, Medical Publisher PZWL, 1976. [Polish]

[5] Siemieski, A. *Inverse Optimisation Problem of Interacting Skeletal Muscles*. Wrocaw, Studies and Monographs of the Academy of Physical Education in Wrocaw, nr. 87, 2007. [Polish]

[6] Sokoowska-Pituchowa, J. ed. *Human Anatomy*. Warsaw, Medical Publisher PZWL, 1989. [Polish]

[7] Wojnicz, W. *Modelling and Simulation of the Skeletal Arm-forearm System Behaviour*. Ph.D. thesis. Gdask-d, 2008.

[8] Wang, X. Three-dimensional kinematic analysis of influence of hand orientation and joint limits on the control of arm postures and movements. *Biological Cybernetics* **80** (1999) 449–463.

[9] Okadome, T. and Honda, M. Kinematic construction of the trajectory of sequential arm movements. *Biological Cybernetics* **80** (1999) 157–169.

[10] Frederick, A., Matsen, M.D. and Chebli, C. Principles for the Evaluation and Management of Shoulder Instability. *Lippitt, MDThe Journal of Bone and Joint Surgery* **88** (3) (2006) 647–659.

[11] Loss, J. and Candotti, C. Comparative study between two elbow flexion exercises using the estimated resultant muscle force. *Brazilian Journal for Physical Therapy*, Sao Carlos **12** (6) (2008) 502–510.