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MODELING AND ANALYSIS OF THERMAL PROCESSES IN MECHANICAL FRICTION CLUTCH — NUMERICAL AND EXPERIMENTAL INVESTIGATIONS*

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Thermal processes occurring in the mechanical clutch or brake systems have a great influence on the strength of elements of these systems as well as on their dynamics. The contact problems exhibited by such systems include heat generated by dry friction contact surfaces. The contact dynamics in general depends on many system parameters, and it attracted attention of many researches focused on analysis of the mentioned phenomena in different kinds of mechanical systems like clutches, brakes, and others. In this work a mathematical model describing the processes of heat generation and its propagation in the mechanical friction clutch is presented. The presented model takes into account the unequal distribution of flux density of produced heat in the clutch, the thermal conductivity of materials of friction linings, and the heat transfer between the friction linings of clutch and its environments. The analyzed object is described by a set of algebraic linear homo- and heterogeneous equations, and it is derived using a computer numerical method. Many interesting numerical and experimental results are obtained, illustrated and discussed. Presented numerical results coincide with experimental data.

Keywords: Clutch; thermal processes; heat generation.

1. Introduction

Thermal phenomena associated with the generation, conduction and heat transfer through a direct contact between various solid bodies can be found in many mechanical systems. The mentioned phenomena are becoming essential in such mechanical/mechatronical objects like clutches or brakes. Heat generation in these systems is associated with the transformation from a mechanical to thermal energy as a result of

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friction phenomenon occurring on the contact surface between clutch or brake friction linings rubbing themselves. Heat transfer is accompanied by thermal energy propagation inside the bodies, and one may distinguish the following three kinds of heat transfer: conduction, convection and radiation. While solving thermal issues the law of additive properties of heat transfer is applied. Calculated specific amount of energy associated with the conduction, convection and radiation is summed together.

The heat generated in the clutch is yielded by friction during sliding of friction linings. Next, the heat propagates within the material of the mentioned friction linings. In addition, the heat transfer occurs between friction materials and its environments. In the transient states of clutches also transient thermal processes appear, which contribute to a nonuniform temperature distribution at the interface between their working surfaces. Therefore, in general in these systems there is uneven and unsteady temperature distribution occurred on the working surfaces, which essentially changes the tribological friction and wear processes.

In the studies related to thermal phenomena in clutches (or brakes) various simplified mathematical models for their description have been used and applied. Namely, heat simple models describing processes of the generation and propagation of heat have been used, assuming a uniform temperature on the contact surfaces and ignoring the unequal temperature distribution on the surface. We are aimed on a more accurate mathematical description of the considered thermal processes. Interestingly the mathematical model allowing determination (at any time instant) the uneven temperature distribution on the contact surfaces of the clutch system has been proposed. Results obtained in this way should allow for a better forecast of the clutch systems real behavior.

Main scientific objectives of this paper are the mathematical modeling, numerical analysis and experimental verification of the proposed mathematical model governing thermal processes occurring at the interface between the surfaces of friction materials of mechanical friction clutches (or brakes). The brief work outline follows. First section contains a brief introduction to the discussed issues related to the thermal phenomena occurring at the contact surface of friction clutch. The next one focuses on presentation of a few mathematical models often used and applied by many researches. Moreover, a review of scientific state-of-art of this research field focused on numerical investigations is carried out. In Sec. 3, the mathematical model governing thermal processes occurring in the friction clutch or brake is proposed. A detailed mathematical description, analysis and numerical algorithms of the dimensionless solution are presented. In Sec. 4 the numerical analysis is performed. Section 5 describes the experimental studies, whereas conclusions of our research are given in the last Sec. 6.

2. State-of-Art of the Problem

Thermal processes are objects of interest of many researches working in mechanical engineering. Different friction laws have been applied in various machine members like gears, sprockets, bearings, clutches, brakes, and others. In general, mentioned processes are very difficult for the proper and reliable analyses, and in addition they usually may depend on many parameters. Reference 21 describes how to determine the surface temperature of the clutch taking into account the whole studied object as well as the temperature of friction surfaces, and the results are briefly presented below. The amount of heat Q_d supplied to the clutch per an hour can be described by the relation[21]

$$Q_d = L_t m_w, \tag{1}$$

where L_t is the work of the friction force during a single association of clutch, while m_w is the number of starts of clutch per an hour. The heat Q_o dissipated from the clutch in one hour is equal to

$$Q_o = A_{sp} \lambda_s \Delta t, \tag{2}$$

where A_{sp} is the outer surface of the clutch used to remove the heat to the surrounding environment, λ_s is the coefficient of heat transfer between the clutch and the environment, and Δt denotes the average difference of temperature between the outer surface of the clutch and the surrounding environment. A comparison of supplied heat Q_d to the clutch and heat Q_o dissipated from this one within an hour, we can estimate the average temperature difference

$$\Delta t = \frac{L_t m_w}{A_{sp} \lambda_s}, \tag{3}$$

and as a result the temperature of the clutch. These calculations are only approximate, because not all of the work L_t of friction force is converted to the increase of the clutch temperature (part of this work is related to wear), and it is difficult to estimate the heat transfer coefficient λ_s between the clutch and the surrounding environment.

In addition to calculating the clutch as a whole mechanical system, approximate calculations of the temperature of its working surfaces are also carried out. In this case, as a model the plate with unlimited diameter and being heated from both sides by a uniform heat flux is considered. Increase of the working surface temperature of the clutch is

$$\Delta t_p = \frac{q_c}{\lambda_t} \left(\frac{2 c_t \tau_w}{d} + \frac{d}{c_t} \right), \tag{4}$$

where the heat flux density q_c has the following form:

$$q_c = \frac{L_t}{\tau_w A_\tau}, \tag{5}$$

where τ_w is the time of one switching of clutch, A_τ is the total friction surface area, λ_t is the coefficient of thermal conductivity of the plates material, c_t is the temperature coefficient of transmission, and d denotes the plates thickness.

In order to obtain in a more accurate manner a description of the processes of heat generation and propagation associated with friction in the clutch or brake, general relationships and interactions of these processes should be used. One of the basic equations describing the mentioned phenomena is the heat conduction equation. For example, in Ref. 4 these equations were used to determine the temperature of the shaft of cylindrical bearing, while in Ref. 5 the same authors have used them to determine the temperature of the spherical bearing. In Ref. 20 heat conduction equation is used to determine the temperature at the interface between two thermo-elastic semi-spaces pressed together by a normal loading, and application of this equation to describe the thermal phenomena occurring in the aircraft carbon-carbon composite multi-disc clutch (similar to an aircraft brake) can be found in Ref. 25. In general, the heat flux density $q(t)$ generated on the clutch working surfaces can be determined as a part of the work of friction force generated per unit time, and penetrated through the unit area. The heat flux density has the form[6]

$$q(t) = (1 - \chi)\mu |V_r(t)| P(t), \tag{6}$$

where χ is a part of the work of friction force not converted into heat (this part of the work is associated with wear), μ is the coefficient of friction, $V_r(t)$ is the relative sliding velocity, and $P(t)$ is the contact pressure distribution at the interface of bodies. Equation (6) of the heat flux density is used in Refs. 17 and 18 for a three-dimensional thermo-elastic contact problem with frictional heat generation. This formula has also been used in Ref. 10, where the finite element method analysis of thermal phenomena in the friction clutch with ceramic disc is carried out. The presented relationship of the heat flux density is also applied in Refs. 3, 20 and 22 for various friction approximations. Heat conduction in the material is governed by the Fourier's law of the following form:

$$q(t) = -k^{(p)} \mathrm{grad}\, T', \tag{7}$$

or in the scalar form

$$q(t) = -k^{(p)} \frac{\partial T'}{\partial n}, \tag{8}$$

where $k^{(p)}$ is the thermal conductivity coefficient (thermal conductivity) of the material, and $\mathrm{grad}\, T'$ is the gradient of temperature T' (in the scalar form $\partial T'/\partial n$ is a derivative of the temperature in the direction n perpendicular to the isothermal surface). The heat flux density exchanged at the border between the body and its surroundings environment is

$$q(t) = \lambda (T' - T'_{ot}), \tag{9}$$

where λ is the coefficient of heat transfer between the body and its surroundings environment, T' is the temperature of the body at the border of its surroundings, and T'_{ot} is the ambient temperature. Equation (9) describes the mechanism of heat exchange between body and environment known as the second type boundary condition. An extensive literature review on scientific papers devoted to heat various types can be traced in Refs. 13–15. In Ref. 12, papers published in recent years devoted to the analysis and calculations of thermal phenomena occurred in different kinds of friction brakes are also presented, illustrated and discussed.

Temperature, wear and contact pressure distributions have the significant influence on the strength of elements of clutch or brake systems and their dynamics during the exploitation of these machine members. In general, the heat flux density generated at an interface between working surfaces of these systems is distributed on an uneven manner. Material processing and manufacturing may induce geometric imperfections perturbing the otherwise uniform temperature and pressure distributions at the interface. These perturbations can cause severe thermo-mechanical damages in clutches or brakes with respect to sufficiently large sliding speeds, i.e. black patches known as hot spots develop on the surface of steel plates abrading the friction material and reducing its working life. It is related to the occurrence of frictionally excited thermo-elastic instability known to cause localized decrease of a frictional coefficient, overheating, carbonization of the frictional materials and torsion vibrations in many cases.[1,2] The heat flux density is not uniform involving nonuniform thermal distortions of the contact interface, which, in turn, affect the contact pressure distribution. This feedback between interface frictional heat and thermo-mechanical deformation can be unstable leading to the phenomenon known as frictionally-excited thermo-elastic instability.[8] Frictionally-excited thermo-elastic instability in such systems as brakes or clutches might lead to the formation of hot spots and small regions (where high temperature and pressure are experienced) being responsible for an increase in wear coefficient, thermo-mechanical damage, noise and vibrations.[7,16,19,23] The first analytical model for estimating the occurrence of thermo-elastic instability have been proposed by the authors of Ref. 9, where two thermo-elastic half-planes have been considered in sliding contact along their common interface. Since then, various methods to assess the thermo-elastic stability in clutches and brakes have been developed, for instance using finite element method.[11,24] In this paper, thermo-elastic instability problems in clutch are not considered, however, the presented mathematical model allows to determine the temperature distribution, which affects the thermo-elastic instability in this type of friction interactions.

3. Mathematical Modeling of Thermal Processes in Friction Clutch

In this section a mathematical model governing processes of the heat generation and its propagation in the mechanical friction clutch is presented. The model takes into account the unequal flux density distribution of the produced heat in a clutch system, the thermal conductivity of materials of linings, and the heat transfer between the linings and its environments. The analyzed system is described by a set of algebraic linear homogeneous and heterogeneous equations.

3.1. *Model of the system*

Figure 1 shows a cross-section of the friction linings of a mechanical friction clutch. The figure also shows computing grid (deposited on the clutch cross-section divided into m equal sections of length ΔR along the radius R) with the nodes, where the appropriate temperatures are calculated.

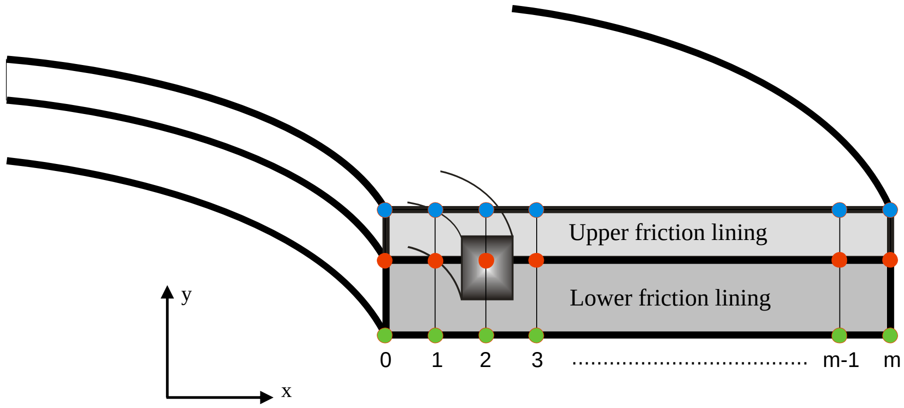


Fig. 1. Mechanical friction clutch linings cross-section and computing grid with nodes.

Thickness of the upper friction lining and the lower one are H_1 and H_2, respectively. The coefficients of thermal conductivity (thermal conductivities) of the individual linings are $k_1^{(p)}$ and $k_2^{(p)}$. Heat transfer coefficients between the upper (lower) friction material and the upper (lower) shield are λ_1 and λ_2, respectively. Moreover, the heat transfer coefficients between the upper and lower friction material and the surrounding environment are λ_3 and λ_4, respectively. Specific heat capacities per unit mass and temperature of friction materials are c_{w1} and c_{w2}, while their mass densities are ρ_1 and ρ_2, respectively.

3.2. *Mathematical modeling*

Let $T_i'(t)$ denote temperatures at the interface of friction linings at the points (red mesh nodes) with a radius equal to $R_1 + i\Delta R$, while ${T'}_i^{(1)}(t)$ and ${T'}_i^{(2)}(t)$ denote the temperatures at the upper (blue mesh nodes) and lower (green mesh nodes) of the corresponding friction linings at the points with a radius equal to $R_1 + i\Delta R$. Let further $Q_{gen,i}(t)$ denote the heat generated as a result of dry friction in the clutch during time Δ_t at the working surface. Next, let $Q_{c,i}(t)$, $Q_{c,i}^{(1)}(t)$, $Q_{c,i}^{(2)}(t)$ denote the values of heats, which at the time Δ_t are converted to heats (relevant passages of annular friction discs with cross-section are shown in Fig. 1). $Q_{1,i}(t)$, $Q_{1,i}^{(1)}(t)$, $Q_{1,i}^{(2)}(t)$, $Q_{2,i}(t)$, $Q_{2,i}^{(1)}(t)$, $Q_{2,i}^{(2)}(t)$, $Q_{3,i}(t)$, $Q_{3,i}^{(1)}(t)$, $Q_{3,i}^{(2)}(t)$, $Q_{4,i}(t)$, $Q_{4,i}^{(1)}(t)$ and $Q_{4,i}^{(2)}(t)$ denote the heat exchanged with adjacent elements as a result of heat conduction in the material or as a result of heat transfer between the body and its environment. $Q_{1,i}(t)$, $Q_{1,i}^{(1)}(t)$ and $Q_{1,i}^{(2)}(t)$ denote the heats exchanged in the direction of the y-axis, $Q_{2,i}(t)$, $Q_{2,i}^{(1)}(t)$ and $Q_{2,i}^{(2)}(t)$ denote the heats exchanged in the opposite direction of the y-axis, $Q_{3,i}(t)$, $Q_{3,i}^{(1)}(t)$ and $Q_{3,i}^{(2)}(t)$ denote the heats exchanged in the opposite direction of the x-axis, while $Q_{4,i}(t)$, $Q_{4,i}^{(1)}(t)$ and $Q_{4,i}^{(2)}(t)$ denote the heats exchanged in the direction of the x-axis. The so far introduced mathematical model presents the heat balance equations for all mesh points. The system of $3(m+1)$ algebraic equations has been obtained in this way, where each equation has only one variable, i.e. the temperature at the same point and in the next step time. This approach is known as an *explicit* method.

The flux heat density generated in the clutch in time instant Δ_t on the annular contact surface equal to $2\pi(R_1 + i\Delta R)a_i\Delta R$ has the form

$$Q_{gen,i}(t) = (1-\chi)\mu P_i(t)|\Omega_r(t)|(R_1 + i\Delta R)\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R, \tag{10}$$

where μ is the frictional coefficient, and the coefficients a_i have the form: $a_0 = a_m = 1/2$, $a_i = 1$ and fulfill the same role as factors of the method of trapezium. For each mesh point equations described the values of heats (taking into consideration physical parameters of individual components) can be obtained. We assumed that each node of the mesh is a source of heat; positive value of this heat is at the working surface (red nodes), whereas in other mesh nodes these values are zero (blue and green nodes). Heats $Q^{(1)}_{c,i}(t)$, $Q^{(2)}_{c,i}(t)$ and $Q_{c,i}(t)$ converted into increase of temperatures of masses around the respective mesh points are

$$Q^{(1)}_{c,i}(t) = \rho_1 2\pi(R_1 + i\Delta R)a_i\Delta R\frac{1}{2}H_1 c_{w1}(T'^{(1)}_i(t+\Delta_t) - T'^{(1)}_i(t)), \tag{11}$$

$$Q^{(2)}_{c,i}(t) = \rho_2 2\pi(R_1 + i\Delta R)a_i\Delta R\frac{1}{2}H_2 c_{w2}(T'^{(2)}_i(t+\Delta_t) - T'^{(2)}_i(t)), \tag{12}$$

$$\begin{aligned} Q_{c,i}(t) = {} & \rho_1 2\pi(R_1 + i\Delta R)a_i\Delta R\frac{1}{2}H_1 c_{w1}(T'_i(t+\Delta_t) - T'_i(t)) \\ & + \rho_2 2\pi(R_1 + i\Delta R)a_i\Delta R\frac{1}{2}H_2 c_{w2}(T'_i(t+\Delta_t) - T'_i(t)). \end{aligned} \tag{13}$$

Heats $Q^{(1)}_{1,i}(t)$, $Q^{(2)}_{1,i}(t)$, $Q_{1,i}(t)$, $Q^{(1)}_{2,i}(t)$, $Q^{(2)}_{2,i}(t)$, $Q_{2,i}(t)$, $Q^{(1)}_{3,i}(t)$, $Q^{(2)}_{3,i}(t)$, $Q_{3,i}(t)$, $Q^{(1)}_{4,i}(t)$, $Q^{(2)}_{4,i}(t)$ and $Q_{4,i}(t)$ have the form

$$Q^{(1)}_{1,i}(t) = \lambda_1(T'^{(1)}_i(t) - T'_{ot})\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{14}$$

$$Q^{(2)}_{1,i}(t) = k^{(p)}_2 \frac{T'^{(2)}_i(t) - T'_i(t)}{H_2}\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{15}$$

$$Q_{1,i}(t) = k^{(p)}_1 \frac{T'_i(t) - T'^{(1)}_i(t)}{H_1}\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{16}$$

$$Q^{(1)}_{2,i}(t) = k^{(p)}_1 \frac{T'^{(1)}_i(t) - T'_i(t)}{H_1}\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{17}$$

$$Q^{(2)}_{2,i}(t) = \lambda_2(T'^{(2)}_i(t) - T'_{ot})\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{18}$$

$$Q_{2,i}(t) = k^{(p)}_2 \frac{T'_i(t) - T'^{(2)}_i(t)}{H_2}\Delta_t 2\pi(R_1 + i\Delta R)a_i\Delta R. \tag{19}$$

$$Q^{(1)}_{3,0}(t) = \lambda_3(T'^{(1)}_0(t) - T'_{ot})\Delta_t 2\pi R_1\frac{1}{2}H_1. \tag{20}$$

$$Q_{3,i}^{(1)}(t) = k_1^{(p)} \frac{T'^{(1)}_i(t) - T'^{(1)}_{i-1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_1, \quad i = 1, 2, \ldots, m. \tag{21}$$

$$Q_{3,0}^{(2)}(t) = \lambda_4(T'^{(2)}_0(t) - T'_{ot})\Delta_t 2\pi R_1 \frac{1}{2}H_2. \tag{22}$$

$$Q_{3,i}^{(2)}(t) = k_2^{(p)} \frac{T'^{(2)}_i(t) - T'^{(2)}_{i-1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_2, \quad i = 1, 2, \ldots, m. \tag{23}$$

$$Q_{3,0}(t) = \lambda_3(T'_0(t) - T'_{ot})\Delta_t 2\pi R_1 \frac{1}{2}H_1 + \lambda_4(T'_0(t) - T'_{ot})\Delta_t 2\pi R_1 \frac{1}{2}H_2. \tag{24}$$

$$\begin{aligned} Q_{3,i}(t) &= k_1^{(p)} \frac{T'_i(t) - T'_{i-1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_1 \\ &\quad + k_2^{(p)} \frac{T'_i(t) - T'_{i-1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_2, i = 1, 2, \ldots, m. \end{aligned} \tag{25}$$

$$Q_{4,m}^{(1)}(t) = \lambda_3(T'^{(1)}_m(t) - T'_{ot})\Delta_t 2\pi R_2 \frac{1}{2}H_1. \tag{26}$$

$$Q_{4,i}^{(1)}(t) = k_1^{(p)} \frac{T'^{(1)}_i(t) - T'^{(1)}_{i+1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_1, \quad i = 0, 1, \ldots, m-1. \tag{27}$$

$$Q_{4,m}^{(2)}(t) = \lambda_4(T'^{(2)}_m(t) - T'_{ot})\Delta_t 2\pi R_2 \frac{1}{2}H_2. \tag{28}$$

$$Q_{4,i}^{(2)}(t) = k_2^{(p)} \frac{T'^{(2)}_i(t) - T'^{(2)}_{i+1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_2, \quad i = 0, 1, \ldots, m-1. \tag{29}$$

$$Q_{4,m}(t) = \lambda_3(T'_m(t) - T'_{ot})\Delta_t 2\pi R_2 \frac{1}{2}H_1 + \lambda_4(T'_m(t) - T'_{ot})\Delta_t 2\pi R_2 \frac{1}{2}H_2. \tag{30}$$

$$\begin{aligned} Q_{4,i}(t) &= k_1^{(p)} \frac{T'_i(t) - T'_{i+1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_1 \\ &\quad + k_2^{(p)} \frac{T'_i(t) - T'_{i+1}(t)}{\Delta R} \Delta_t 2\pi(R_1 + i\Delta R)\frac{1}{2}H_2, \quad i = 0, 1, \ldots, m-1. \end{aligned} \tag{31}$$

Heat balance equations for each grid points have the form

$$\begin{aligned} 0 &= Q_{c,i}^{(1)}(t) + Q_{1,i}^{(1)}(t) + Q_{2,i}^{(1)}(t) + Q_{3,i}^{(1)}(t) + Q_{4,i}^{(1)}(t), \\ 0 &= Q_{c,i}^{(2)}(t) + Q_{1,i}^{(2)}(t) + Q_{2,i}^{(2)}(t) + Q_{3,i}^{(2)}(t) + Q_{4,i}^{(2)}(t), \\ Q_{gen,i}(t) &= Q_{c,i}(t) + Q_{1,i}(t) + Q_{2,i}(t) + Q_{3,i}(t) + Q_{4,i}(t). \end{aligned} \tag{32}$$

On the basis of Eqs. (32), temperatures in time instants $t + \Delta_t$ are governed by the following equations,

$$T'^{(1)}_i(t + \Delta_t) = T'^{(1)}_i(t) + \frac{0 - Q_{1,i}^{(1)}(t) - Q_{2,i}^{(1)}(t) - Q_{3,i}^{(1)}(t) - Q_{4,i}^{(1)}(t)}{\rho_1 2\pi(R_1 + i\Delta R)a_i \Delta R \frac{1}{2}H_1 c_{w1}},$$

$$T'^{(2)}_i(t+\Delta_t) = T'^{(2)}_i(t) + \frac{0 - Q^{(2)}_{1,i}(t) - Q^{(2)}_{2,i}(t) - Q^{(2)}_{3,i}(t) - Q^{(2)}_{4,i}(t)}{\rho_2 2\pi(R_1 + i\Delta R)a_i\Delta R\frac{1}{2}H_2 c_{w2}},$$
$$T'_i(t+\Delta_t) = T'_i(t) + \frac{Q_{gen,i}(t) - Q_{1,i}(t) - Q_{2,i}(t) - Q_{3,i}(t) - Q_{4,i}(t)}{a_i\pi(R_1 + i\Delta R)\Delta R(\rho_1 H_1 c_{w1} + \rho_2 H_2 c_{w2})}. \quad (33)$$

3.3. *Nondimensional form*

The obtained equations and relationships have also been presented in nondimensional form. We introduce coefficients t_*, T_*, Ω_*, P_* and Q'_*, the nondimensional time $\tau = t/t_*$, the nondimensional radius $r = (R - R_1)/(R_2 - R_1)$, the nondimensional geometrical parameter $\rho = R_1/(R_2 - R_1)$, and other nondimensional coefficients:

$$\frac{2\pi t_*\Omega_* P_* R_2^3\Delta_\tau\Delta_r}{Q'_*(1+\rho)^3} = g, \quad \frac{\pi T_* R_2^2\Delta_r\rho_1 H_1 c_{w1}}{Q'_*(1+\rho)^3} = e_1, \quad \frac{\pi T_* R_2^2\Delta_r\rho_2 H_2 c_{w2}}{Q'_*(1+\rho)^3} = e_2,$$
$$\frac{2\pi t_* T_* R_2^2\Delta_\tau\Delta_r k_1^{(p)}}{H_1 Q'_*(1+\rho)^3} = c_1, \quad \frac{2\pi t_* T_* R_2^2\Delta_\tau\Delta_r k_2^{(p)}}{H_2 Q'_*(1+\rho)^3} = c_2, \quad \frac{2\pi t_* T_* R_2^2\Delta_\tau\Delta_r\lambda_1}{Q'_*(1+\rho)^3} = c_1^{(1)},$$
$$\frac{2\pi t_* T_* R_2^2\Delta_\tau\Delta_r\lambda_2}{Q'_*(1+\rho)^3} = c_2^{(2)}, \quad \frac{\pi t_* T_*\Delta_\tau k_1^{(p)} H_1}{Q'_*\Delta_r} = c_3^{(1)}, \quad \frac{\pi t_* T_*\Delta_\tau k_2^{(p)} H_2}{Q'_*\Delta_r} = c_3^{(2)},$$
$$\frac{\pi t_* T_* R_2\Delta_\tau\rho\lambda_3 H_1}{Q'_*(1+\rho)} = d_3^{(1)}, \quad \frac{\pi t_* T_* R_2\Delta_\tau\rho\lambda_4 H_2}{Q'_*(1+\rho)} = d_3^{(2)},$$

and the following nondimensional functions:

$$\frac{T'^{(1)}_i(t_*\tau)}{T_*} = T_i^{(1)}(\tau), \quad \frac{T'^{(2)}_i(t_*\tau)}{T_*} = T_i^{(2)}(\tau), \quad \frac{T'_i(t_*\tau)}{T_*} = T_i(\tau), \quad \frac{T'_{ot}}{T_*} = T_{ot},$$
$$\frac{P_i(t_*\tau)}{P_*} = p_i(\tau), \quad \frac{\Omega_r(t_*\tau)}{\Omega_*} = \omega_r(\tau), \quad \mu = f, \quad \frac{Q_{gen,i}(t_*\tau)}{Q'_*} = q_{gen,i}(\tau),$$
$$\frac{Q^{(1)}_{c,i}(t_*\tau)}{Q'_*} = q^{(1)}_{c,i}(\tau), \quad \frac{Q^{(2)}_{c,i}(t_*\tau)}{Q'_*} = q^{(2)}_{c,i}(\tau), \quad \frac{Q_{c,i}(t_*\tau)}{Q'_*} = q_{c,i}(\tau),$$
$$\frac{Q^{(1)}_{1,i}(t_*\tau)}{Q'_*} = q^{(1)}_{1,i}(\tau), \quad \frac{Q^{(2)}_{1,i}(t_*\tau)}{Q'_*} = q^{(2)}_{1,i}(\tau), \quad \frac{Q_{1,i}(t_*\tau)}{Q'_*} = q_{1,i}(\tau),$$
$$\frac{Q^{(1)}_{2,i}(t_*\tau)}{Q'_*} = q^{(1)}_{2,i}(\tau), \quad \frac{Q^{(2)}_{2,i}(t_*\tau)}{Q'_*} = q^{(2)}_{2,i}(\tau), \quad \frac{Q_{2,i}(t_*\tau)}{Q'_*} = q_{2,i}(\tau),$$
$$\frac{Q^{(1)}_{3,i}(t_*\tau)}{Q'_*} = q^{(1)}_{3,i}(\tau), \quad \frac{Q^{(2)}_{3,i}(t_*\tau)}{Q'_*} = q^{(2)}_{3,i}(\tau), \quad \frac{Q_{3,i}(t_*\tau)}{Q'_*} = q_{3,i}(\tau),$$
$$\frac{Q^{(1)}_{4,i}(t_*\tau)}{Q'_*} = q^{(1)}_{4,i}(\tau), \quad \frac{Q^{(2)}_{4,i}(t_*\tau)}{Q'_*} = q^{(2)}_{4,i}(\tau), \quad \frac{Q_{4,i}(t_*\tau)}{Q'_*} = q_{4,i}(\tau).$$

Then, we obtain the following nondimensional relationships:

$$q_{gen,i}(\tau) = a_i g(1-\chi) f p_i(\tau)|\omega_r(\tau)|(r_i+\rho)^2, \tag{34}$$

$$q_{c,i}^{(1)}(\tau) = a_i e_1 (T_i^{(1)}(\tau+\Delta_\tau) - T_i^{(1)}(\tau))(r_i+\rho), \tag{35}$$

$$q_{c,i}^{(2)}(\tau) = a_i e_2 (T_i^{(2)}(\tau+\Delta_\tau) - T_i^{(2)}(\tau))(r_i+\rho), \tag{36}$$

$$q_{c,i}(\tau) = a_i (e_1+e_2)(T_i(\tau+\Delta_\tau) - T_i(\tau))(r_i+\rho), \tag{37}$$

$$q_{1,i}^{(1)}(\tau) = a_i c_1^{(1)} (T_i^{(1)}(\tau) - T_{ot})(r_i+\rho), \tag{38}$$

$$q_{1,i}^{(2)}(\tau) = a_i c_2 (T_i^{(2)}(\tau) - T_i(\tau))(r_i+\rho), \tag{39}$$

$$q_{1,i}(\tau) = a_i c_1 (T_i(\tau) - T_i^{(1)}(\tau))(r_i+\rho), \tag{40}$$

$$q_{2,i}^{(1)}(\tau) = a_i c_1 (T_i^{(1)}(\tau) - T_i(\tau))(r_i+\rho), \tag{41}$$

$$q_{2,i}^{(2)}(\tau) = a_i c_2^{(2)} (T_i^{(2)}(\tau) - T_{ot})(r_i+\rho), \tag{42}$$

$$q_{2,i}(\tau) = a_i c_2 (T_i(\tau) - T_i^{(2)}(\tau))(r_i+\rho), \tag{43}$$

$$q_{3,0}^{(1)}(\tau) = d_3^{(1)} (T_0^{(1)}(\tau) - T_{ot}), \tag{44}$$

$$q_{3,i}^{(1)}(\tau) = c_3^{(1)} (T_i^{(1)}(\tau) - T_{i-1}^{(1)}(\tau))(r_i+\rho), \quad i = 1,2,\ldots,m, \tag{45}$$

$$q_{3,0}^{(2)}(\tau) = d_3^{(2)} (T_0^{(2)}(\tau) - T_{ot}), \tag{46}$$

$$q_{3,i}^{(2)}(\tau) = c_3^{(2)} (T_i^{(2)}(\tau) - T_{i-1}^{(2)}(\tau))(r_i+\rho), \quad i = 1,2,\ldots,m, \tag{47}$$

$$q_{3,0}(\tau) = (d_3^{(1)} + d_3^{(2)})(T_0(\tau) - T_{ot}), \tag{48}$$

$$q_{3,i}(\tau) = (c_3^{(1)} + c_3^{(2)})(T_i(\tau) - T_{i-1}(\tau))(r_i+\rho), \quad i = 1,2,\ldots,m, \tag{49}$$

$$q_{4,m}^{(1)}(\tau) = \frac{1+\rho}{\rho} d_3^{(1)} (T_m^{(1)}(\tau) - T_{ot}), \tag{50}$$

$$q_{4,i}^{(1)}(\tau) = c_3^{(1)} (T_i^{(1)}(\tau) - T_{i+1}^{(1)}(\tau))(r_i+\rho), \quad i = 0,1,\ldots,m-1, \tag{51}$$

$$q_{4,m}^{(2)}(\tau) = \frac{1+\rho}{\rho} d_3^{(2)} (T_m^{(2)}(\tau) - T_{ot}), \tag{52}$$

$$q_{4,i}^{(2)}(\tau) = c_3^{(2)} (T_i^{(2)}(\tau) - T_{i+1}^{(2)}(\tau))(r_i+\rho), \quad i = 0,1,\ldots,m-1, \tag{53}$$

$$q_{4,m}(\tau) = \frac{1+\rho}{\rho} (d_3^{(1)} + d_3^{(2)})(T_m(\tau) - T_{ot}), \tag{54}$$

$$q_{4,i}(\tau) = (c_3^{(1)} + c_3^{(2)})(T_i(\tau) - T_{i+1}(\tau))(r_i+\rho), \quad i = 0,1,\ldots,m-1. \tag{55}$$

Heat balance equations in nondimensional form for individual grid nodes have the form

$$
\begin{aligned}
0 &= q_{c,i}^{(1)}(\tau) + q_{1,i}^{(1)}(\tau) + q_{2,i}^{(1)}(\tau) + q_{3,i}^{(1)}(\tau) + q_{4,i}^{(1)}(\tau),\\
0 &= q_{c,i}^{(2)}(\tau) + q_{1,i}^{(2)}(\tau) + q_{2,i}^{(2)}(\tau) + q_{3,i}^{(2)}(\tau) + q_{4,i}^{(2)}(\tau),\\
q_{gen,i}(\tau) &= q_{c,i}(\tau) + q_{1,i}(\tau) + q_{2,i}(\tau) + q_{3,i}(\tau) + q_{4,i}(\tau).
\end{aligned}
\tag{56}
$$

On the basis of Eqs. (56), temperatures in time instants $\tau + \Delta_\tau$ are obtained in the following way:

$$
\begin{aligned}
T_i^{(1)}(\tau + \Delta_\tau) &= T_i^{(1)}(\tau) + \frac{0 - q_{1,i}^{(1)}(\tau) - q_{2,i}^{(1)}(\tau) - q_{3,i}^{(1)}(\tau) - q_{4,i}^{(1)}(\tau)}{a_i e_1 (r_i + \rho)},\\
T_i^{(2)}(\tau + \Delta_\tau) &= T_i^{(2)}(\tau) + \frac{0 - q_{1,i}^{(2)}(\tau) - q_{2,i}^{(2)}(\tau) - q_{3,i}^{(2)}(\tau) - q_{4,i}^{(2)}(\tau)}{a_i e_2 (r_i + \rho)},\\
T_i(\tau + \Delta_\tau) &= T_i(\tau) + \frac{q_{gen,i}(\tau) - q_{1,i}(\tau) - q_{2,i}(\tau) - q_{3,i}(\tau) - q_{4,i}(\tau)}{a_i (e_1 + e_2)(r_i + \rho)}.
\end{aligned}
\tag{57}
$$

4. Numerical Investigations

Numerical calculations are carried out using a so called *explicit* method and own numerical algorithms for solving the set of algebraic linear homo- and heterogeneous equations. In all our calculations we take the geometrical parameter $\rho = 0.5$ and time step $\Delta_\tau = 0.01$. At the beginning the uniform contact pressure distribution on the entire working surface has been considered ($p(r, \tau) = \text{const.}$). We take the following initial nondimensional parameters: $\chi = 0.4$, $f = 0.1$, $\omega_r = 10$, $g = 0.02$, $e_1 = e_2 = 50$, $c_1 = c_2 = 1.0$, $c_1^{(1)} = c_2^{(2)} = 1$, $c_3^{(1)} = c_3^{(2)} = 1$ and $d_3^{(1)} = d_3^{(2)} = 1$. Here we take $m = 100$, however satisfactory results can also be obtained for $m = 10$.

Figure 2 shows the nondimensional temperature distributions $T(R, \infty)$ in the steady state. Numerical calculations have been obtained for various parameters $c_1 = c_2$.

In this case the temperature distribution reaches a steady state due to the setting up of thermal equilibrium between the heat generated in the friction material of the clutch and the heat transmitted to the surrounding environment. Inside the friction contact surface the temperature increases linearly with the radius r, while at the borders of this surface the temperature is much lower. This decrease in temperature within the borders of contact materials is a result of the heat exchange between the friction materials and the surrounding environment (air). In addition, for larger values of c_1 and c_2 responsible for the thermal conductivity of friction linings, steady

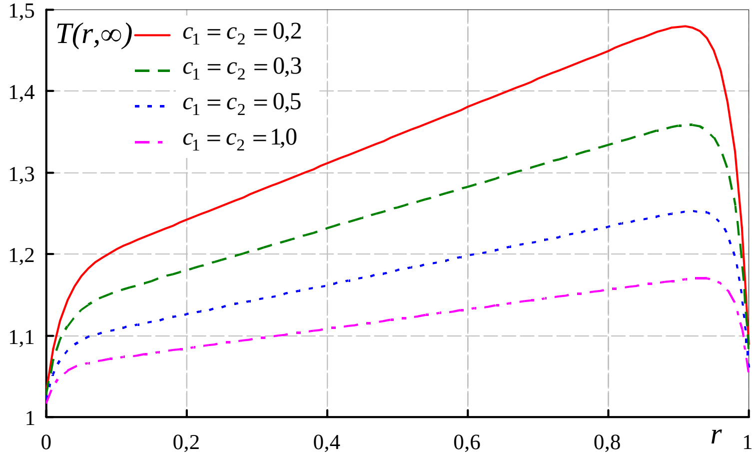
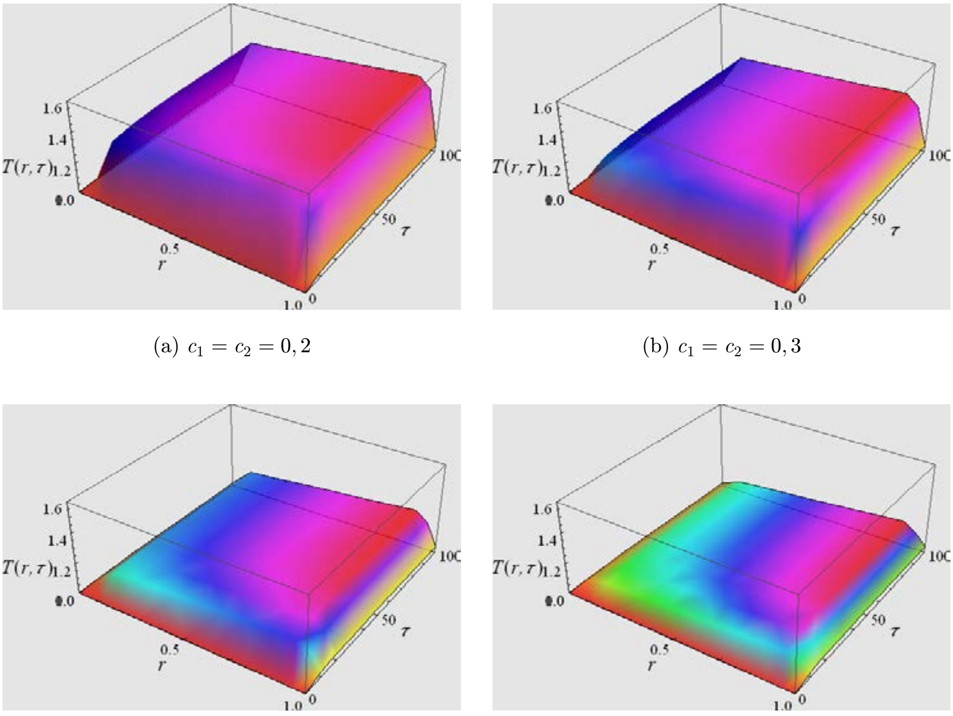


Fig. 2. Steady states contact friction materials surface temperature distributions.



(a) $c_1 = c_2 = 0,2$

(b) $c_1 = c_2 = 0,3$

(c) $c_1 = c_2 = 0,5$

(d) $c_1 = c_2 = 1,0$

Fig. 3. Time evolutions of the interface temperature distribution.

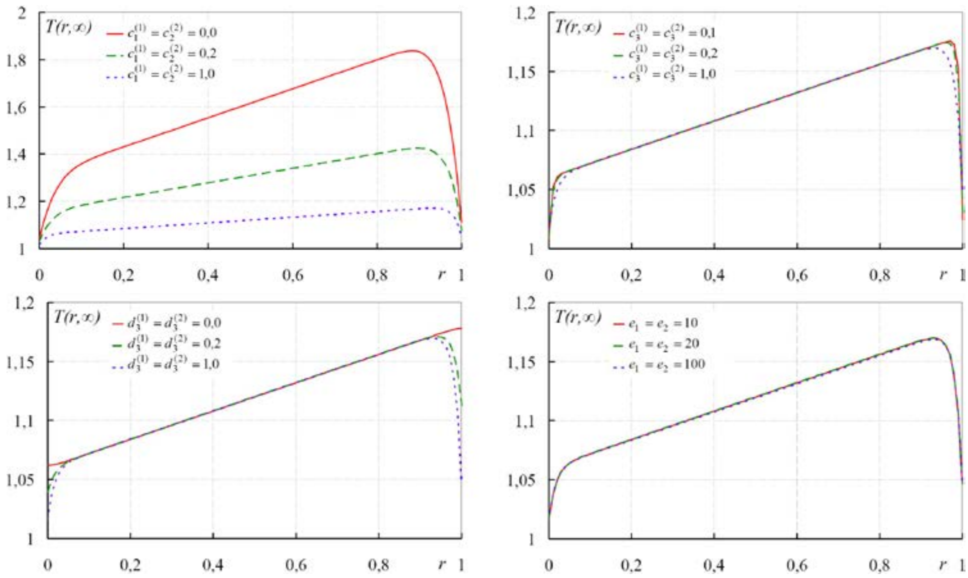


Fig. 4. Temperature distributions in the steady state.

state temperatures are smaller. This is a result of the higher speed heat propagation in friction materials, which increases the outflow of the heat from the inside interface to its borders, where the heat is transferred into the environment. Figure 3 shows time evolutions of the obtained earlier temperature distributions.

In the initial time instant the temperature distribution is uniform on the entire surface of the contact materials. With increase of the time, in individual points of the contact surface temperatures are increasing, finally reaching a steady state,

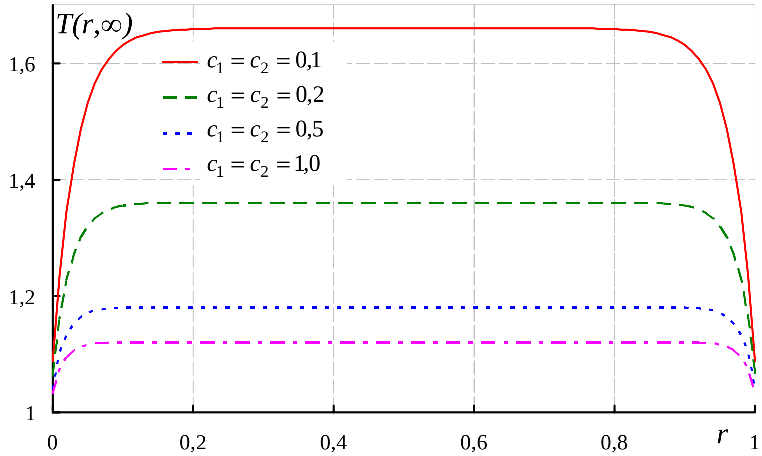


Fig. 5. Steady state temperature distributions on the contact friction surface.

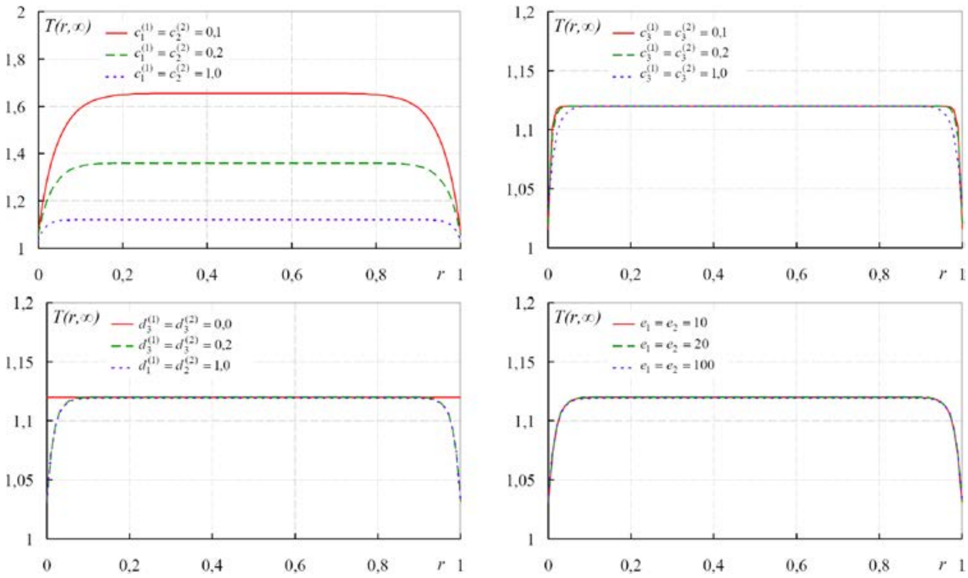


Fig. 6. Steady state temperature distributions for different parameters.

depending on the parameters c_1 and c_2. Other temperature distributions on the friction contact surface steady states for different values of the parameters are reported in Fig. 4. Numerical analysis of mathematical model describing the considered thermal phenomena in the friction clutch has been also carried out for the case of non-uniform contact pressure distribution on the contact surface ($p(r, \tau) = A/r, A = \text{const.}$). Figure 5 shows the nondimensional temperature distributions $T(R, \infty)$ in the steady state for various parameters $c_1 = c_2$. Temperature distribution reaches a steady state due to the setting up of thermal equilibrium between the heat generated in the friction material of the clutch and the heat transmitted to the surrounding environment. Inside the friction contact surface the temperature is constant, while at the borders of this surface it is much lower. Other temperature distributions on the contact surface of friction in a steady state for different values of the parameters describing the considered system are shown in Fig. 6. The obtained relations show the influence of individual parameters on the temperature distribution in the steady state.

5. Experimental Investigations

Presented numerical simulations are verified experimentally. Frictional linings used to study are made of the agglomerated cork, which is a natural material used as a frictional material in friction clutches and brakes. Figure 7 shows measuring system with the infrared camera for determining the temperature distribution on the surface

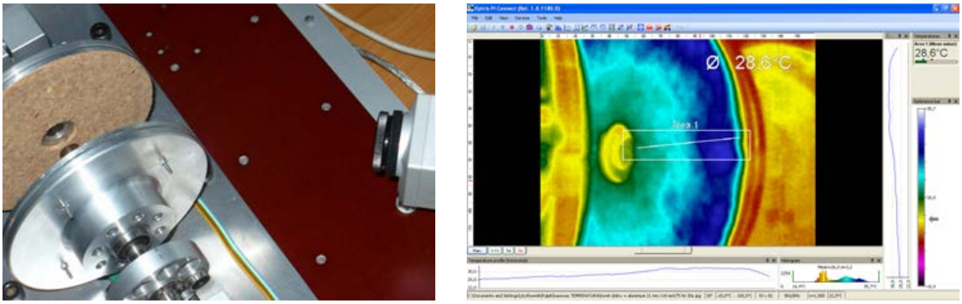


Fig. 7. Diagram of thermal imaging camera settings for determination of the temperature distribution on the surface of the clutch friction shield, and a computer program to handle the thermal imager with an exemplary temperature distribution.

of the frictional shield as well as a computer program with a thermal imaging camera sample image made of the temperature distribution.

The main measuring device is the infrared camera allowing us to determine accurately the surface temperature of the clutch shields. Appropriate software of the infrared camera allows recording the temperature distribution as a colored image. Measurements consisted of heating the working surfaces at a specified time, constant relative sliding velocity and constant force pressed shields. One shield was a metal plate made of aluminum, while the second disc is also made of aluminum covered with a friction lining of natural cork. The temperature distribution was measured on the surface of the second lining.

Figure 8 shows temperature distributions on the surface of the new unused frictional lining (for which it may be assumed uniform contact pressure distribution) with dimensions of $R_1 = 11\,\text{mm}$ and $R_2 = 59\,\text{mm}$ heated to increasingly higher temperatures. The presented images show that the temperature plane distributions of the contact surfaces are not uniform. For each presented cases on the inner part of the lining temperature is lower (yellow color), but higher is on the border of outwardly facing (red color). Figure 9 (on the left) shows the temperature distribution and temperature profile along the radius of the friction lining for uniform contact pressure distribution, and the friction shield has dimensions: $R_1 = 20\,\text{mm}$ and $R_2 = 59\,\text{mm}$. Also this case clearly shows that the temperature distribution on the contact surface facing the friction is not homogeneous. The temperature rises approximately linearly along the radius of the lining. Deviations from the linear relationship appear mainly due to the fact that in reality the exchange of heat between the lining and its surroundings on the borders of the contact, hence the less is the temperature at these sites lining.

Figure 9 (on the right) shows the temperature distribution and temperature profile along the radius of lining on the same disc as above. However, it has been run for the lining, which reached (approximately) fixed contact pressure distribution. In

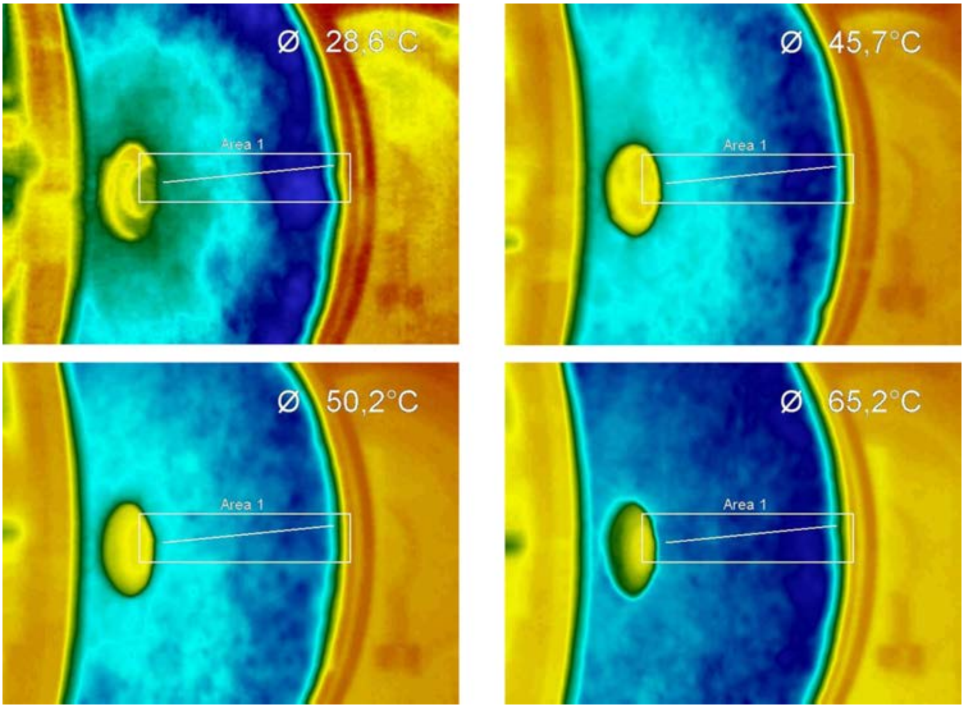


Fig. 8. Various temperature distributions on the contact surface of friction lining.

this case it can be seen that the temperature distribution is approximately uniform over the entire surface of the lining. According to the presented mathematical model of the temperature distribution the obtained results would be also helpful. In a real system one deals with slightly lower temperatures both on the inside and outside due to heat exchange between the linings and its surroundings. Even for this case one may confirm that the presented experimental results are roughly consistent with those obtained numerically. This observation applies both to the case under consideration as well as that previously described.

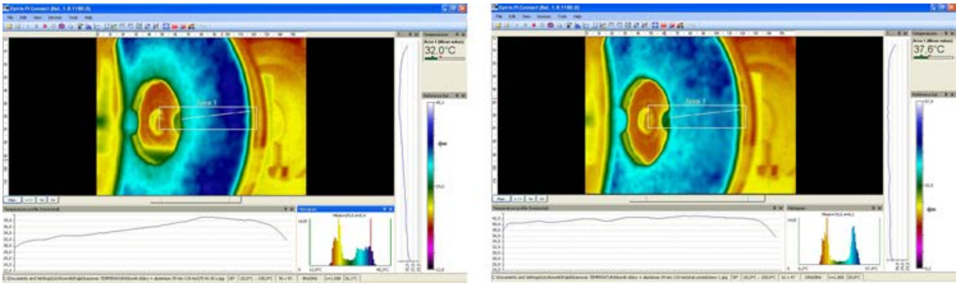


Fig. 9. Temperature distribution and temperature profile at the contact surface linings and the distribution and temperature profile on the surface of the clutch friction linings.

6. Conclusion

This paper has been devoted to mathematical modeling of thermal processes occurring at the interface between the surface linings of the mechanical friction clutch. The proposed model takes into account the unequal distribution of flux density of produced heat in clutch, thermal conductivity of materials of friction linings, and the heat transfer between them and its environments. Work carried out in experimental studies were designed to confirm the proposed mathematical model. Performed a simple qualitative experimental verification of the model indicates a relatively good qualitative agreement with numerical solutions to the experimental results. Relatively simple experiments carried out revealed that the proposed mathematical model describes the phenomenon well enough heat present in the real friction clutches. The model can be used for numerical simulations in a wider range of changes of parameters, which was also presented in the paper.

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