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ORIGINAL RESEARCH

Low-Speed Voltage-Input Tracking Control of a DC-Motor Numerically Modelled by a Dynamical System with Stick-Slip Friction

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Abstract Stick-slip vibrations appear during relative motion between contacting surfaces of miscellaneous frictional pairs. They depend on the viscous force, Coulomb force or other velocity-dependent forces. These effects appear in almost all mechanical systems, for instance, in positioning systems like servomechanisms, impulse encoders and stepper motors which operate at, or about zero velocity of relative motion between shafts and sliding bearings. This paper presents numerical modelling of a DC-motor as a dynamical system with stick-slip effect which appears while direction of rotation of its rotor crosses zero velocity speed. These investigations are aimed on some future applications of the control technique serving for explanation of bifurcation phenomena existing in such kind of discontinuous systems. Putting emphasis on nonlinear effects we apply the well-known, but a bit extended sliding-surface method allowing for compensation of frictional effects. A limit cycle on a phase plane as well as time-histories of control inputs and system outputs were obtained using numerical simulations performed in Simulink.

Keywords Numerical approximation · Nonlinear dynamical systems · Control of mechanical systems

Introduction

The natural resistance to relative motion between non-lubricated surfaces of two contacting bodies is called dry friction. In some dynamical systems modelling nonlinearities caused by dry friction a controller has to be designed to avoid steady-state tracking errors or vibrations.

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An adaptive friction compensation to improve performance for tracking errors without Stribeck effect has been proposed in [11]. A new control strategy for compensation of frictional phenomena including Stribeck effect, hysteresis, stick-slip limit cycling, pre-sliding displacement and rising static friction has been particularly described and examined. The proposed compensator could be useful for handling significant nonlinearities in motor controls. Similarly, the Lund-Grenoble model of dynamical friction has been used in [10] to control nonlinear effects that the model captures: the Stribeck and Dahl effects, viscous and Coulomb friction [4]. A new Lyapunov-based continuous dynamic controller has been delivered for a more general class of nonlinear systems. It produced better control of a high-speed precision linear tracking table than some tuned PID controllers without the direct nonlinear effects compensation. As it has been shown also in [21], the conventional feedback control methods cannot ensure good results in the presence of dry (stick-slip) friction even in a one degree-of-freedom DC-motor system. Because of steady-state errors a traditional PD controller will not achieve satisfactory performance. These errors could be reduced by increasing the P gain, but significant instabilities would be reported while driving the motor between some angular velocities or along the desired rapidly changing time history of its angular position. Very good positioning accuracy have been obtained with the use of a new sliding-mode based smooth adaptive robust controller designed for dry friction compensation.

A study of control of a mechanism under the influence of low velocity friction has been conducted in [1]. The theoretical and experimental comparative study of linear (PD, PID) and nonlinear (smooth continuous and piece-wise linear discontinuous) compensation algorithms have been proposed. In the case of modelling a two degree-of-freedom controlled planar manipulator the nonlinear controllers have proved superior in performance to any PD controller for some P and D gains. Moreover, their tracking performance was also superior to the PID controller, but it cost an oscillatory response and jerky torque time-dependency. Stability of smooth controllers was much simpler to demonstrate.

Simple active control of the belt-driven oscillator with stick-slip friction in a control system with feedback loop created by a transducer, frequency filter, phase shifter, amplifier and a shaker attached to the oscillating body has been studied in [9]. The feedback system allowed for suppression of unstable vibrations at high effectiveness insensitive to errors in phase shift and amplification. Similarly in [14], some type of friction driven oscillator controlled by Lyapunov redesign based on delayed state feedback has been numerically investigated. Authors redesigned a continuous controller on the basis of a delayed state feedback to ensure that the nonsmooth friction driven system is ultimately bounded. Moreover, by constructing a Lyapunov-Krasovskii function the sufficient condition of stability for the investigated system was obtained.

Neural networks have the capability of approximating nonlinear functions, therefore they are also demanding in estimation of frictional behaviours. Much work has been done in this subject [12,19]. Work [19], for instance, brings investigations on control of linear motion of motors by means of the learning forward controller that is designed in the discrete state-space. There has been also solved in [8] the problem of discrete-time iterative learning control for position trajectory tracking of multiple-input, multiple-output systems including Coulomb friction, bounds on the inputs and friction coefficients (static and sliding). On the background of a two-link revolute-joint planar robot arm some satisfactory learned angular position time histories (at a decrease of position-tracking error) have been shown. In accordance to linear servo motor control, a novel very interesting approach for designing a wavelet basis function network learning controller for a linear motor control system was considered in [15]. The proposed wavelet net-based controller dealt with viscous friction and force ripples that occur in motion control of linear synchronous motors (Fig.1).

The considerations presented by the exemplary articles were motivated by examination of control approaches, but one should mention even on their stability. With regard to subject of the paper, an interesting reference [7] presents analysis of global stability of linearising control with a new robust nonlinear observer induction motors. Authors used the traditional induction model of Park in a stator fixed reference frame related to the stator given by [17]. They designed a control algorithm based on feedback linearisation [18]. After assumption of parameters of the induction motor a detailed scheme of the nonlinear control with an observer has been done in Simulink. The new robust observer based on a nonlinear control scheme offered advantage of only one tuning parameter. The global stability of the whole system consisting of motor, controller and observer was established by means of the precise Lyapunov function that kept the observer's dynamics free. More on the initial strategy on input-output linearisation could be found in [6], but on the global stability of the process-observer-controller system in [16].

Deeper survey through the cited literature provides many references to theoretical derivations and practical implementations confirming permanent interest in control of nonsmooth systems. Basically, control strategies depend on the aim, the friction law, the system at hand and its field of application.

Problem Statement

This study is concerned on a numerical simulation of compensation of frictional effects present in a real system designed for observations and experimental estimation of friction force characteristics, see [2]. It consists of a DC-motor driving a wide transmission belt on which a rigid body vibrates being in frictional contact with surface of the belt. For instance, to find bifurcations of sliding solutions [3] after relative motion observed between contacting surfaces of the investigated coupling it is required to precisely realise some desired function of changes of angular velocity of the DC-motor that drives the belt. Therefore, the rotational velocity of the driving motor should vary in a periodic cycle tracking exactly the desired time-dependent characteristics (triangular, sinusoidal, etc.), but with regard to the existing nonlinear friction characteristics it does not occur. From the point of control theory it states a problem of providing a robust tracking control of rotational velocity of the DC-motor. Therefore, some close to ideal generation of regular input signal-excitation of the belt—would be possible after application of some tracking control technique that was implemented, for instance, to control robot manipulators [13]. Inputs to the method would be a control errors $e(t) = \varphi(t) - \varphi_d(t)$ and $\varepsilon(t) = \dot{\varphi}_d(t) - \lambda e(t)$, where: φ and $\dot{\varphi}$ are respectively, the angular position and velocity of the motor's rotor driving the base by means of a non-stretchable transmission toothed belt; index d denotes desired values of corresponding variables - along with the desired phase trajectory.

Particular investigation will be focused on examination of influence of frictional contacts existing during rotation of rotor of a DC-motor. One can distinguish there a phenomenon of stick-slip friction that mostly affects accuracy of positioning. Friction of that type was investigated in [4] by authors of the paper, and may result from the following: Coulomb friction that represents a maximum static friction $T_{sm} \operatorname{sgn} \dot{\varphi}(t)$ at a slip phase and $T_{sm}(1 - \operatorname{sgn} |\dot{\varphi}(t)|)$ at a stick phase during which an input torque generated by a system driven by the motor's rotor could by applied, exponential friction described by the Stribeck curve $T_{Stm}(1 - \exp(-T_0|\dot{\varphi}(t)|) \operatorname{sgn} \dot{\varphi}(t))$, a viscous friction $T_{vm}\dot{\varphi}(t)$, and a position-dependent friction $T_{1m} \sin(T_2\varphi(t) + T_3) \operatorname{sgn} |\dot{\varphi}(t)|$ as proposed in [20], where: $\operatorname{sgn} \dot{\varphi}$ denotes sign of value of angular velocity, φ is an angular displacement, T_{sm} is the maximum static friction torque, T_{Stm} and $T_0 > 0$ are the parameters of Stribeck curve, T_{vm} is the coefficient of viscous friction, T_{1m}, T_2 and T_3 are constants. Mechanical part of the reduced dynamical system of ordinary differential equations used for modelling dynamics of rotational motion of a DC-motor holds:

$$J_m\ddot{\varphi}(t) + \left(\frac{c_b c_m}{R_a} + T_{vm}\right)\dot{\varphi}(t) - T_{Stm}\left(1 - e^{-T_0|\dot{\varphi}(t)|}\right) \operatorname{sgn}\dot{\varphi}(t) + T_{1m}\sin(T_2\varphi(t) + T_3)\operatorname{sgn}|\dot{\varphi}(t)| + T_{sm}(1 - \operatorname{sgn}|\dot{\varphi}(t)| + \operatorname{sgn}\dot{\varphi}(t)) = c_m\psi_m(t)\,, \tag{1}$$

and the remaining unknown model parameters read: R_a and ψ_m denote respectively the armature resistance and the armature current, J_m is the moment of inertia of the rotor, c_b is a constant of the back electromotive force, and c_m is the motor torque constant. One rewrites Eq. (1) in a form scaled with respect to c_m as follows:

$$J\ddot{\varphi}(t) + B\dot{\varphi}(t) + \tau(t) = \psi(t)\,, \tag{2}$$

where $\tau(t) = T_v\dot{\varphi}(t) - T_{St}(1 - \exp(-T_0|\dot{\varphi}(t)|))\operatorname{sgn}\dot{\varphi}(t) + T_1\sin(T_2\varphi(t) + T_3)\operatorname{sgn}|\dot{\varphi}(t)| + T_s(1 - \operatorname{sgn}|\dot{\varphi}(t)| + \operatorname{sgn}\dot{\varphi}(t))$ is the scaled friction force, and J, B, T_v, T_s, T_1, T_{St} are equal $\frac{J_m}{c_m}$, $\frac{c_b}{R_a}$, $\frac{T_{vm}}{c_m}$, $\frac{T_{sm}}{c_m}$, $\frac{T_{1m}}{c_m}$, $\frac{T_{Stm}}{c_m}$, respectively.

It is not possible to exactly describe the friction and to correctly assume all values of parameters. A tracking control that is a point of the study should also correct any inaccuracies caused by an imprecise system modelling.

Control Strategy

The task of control is to design an adaptive controller that would allow to change angular velocity of rotation of the motor's rotor according to some desired function $\varphi_d(t)$. Let us begin from the so-called sliding surface method [20]. On its background the control error $e(t) = \varphi(t) - \varphi_d(t)$, an auxiliary variable $\varepsilon(t) = \dot{\varphi}_d(t) - \lambda e(t)$ and the definition of sliding surface $r(t) = \dot{\varphi}(t) - \varepsilon(t) = 0$, where variables with index d denote corresponding desired values, $\lambda > 0$ is for more general multidimensional case a positive definite main diagonal matrix. Having introduced variables of the sliding surface method let us propose derived from Eq. (2) a control law in the form:

$$\psi(t) = \hat{J}\dot{\varepsilon}(t) + \hat{D}\varepsilon(t) - \hat{T}_{St}\left(1 - e^{-\hat{T}_0|\dot{\varphi}(t)|}\right)\operatorname{sgn}\dot{\varphi}(t) + \hat{T}_s\operatorname{sgn}\dot{\varphi}(t) + \hat{T}_s(1 - \operatorname{sgn}|\dot{\varphi}(t)|)\,u_s(t) - u_b(t)\,, \tag{3}$$

where $u_b(t)$ is a condition of bounding function, $u_s(t) = 1 - \operatorname{sgn}|r(t)|$ at term describing sticking phase is a function introduced with respect to definition of sliding surface $r(t) = 0$, $\hat{D} = \hat{B} + \hat{T}_v$, circumflex ˆ above symbols marks estimates of corresponding parameters.

Equation (3) will be used for adaptation of unknown estimates in a scheme, where $\psi(t)$ is put to system (1) to compensate for linear and nonlinear forces included in it. Such adaptive feed-forward control loop is good to compensate linear friction forces like Coulomb and viscous ones [21]. Nonlinear friction forces like Stribeck effect and the angular position-dependent friction force cannot be controlled in the loop but some adaptation law based on a robust compensator to learn an upper bounding function has to be used [13]. The following bounding function is assumed

$$u_b(t) = k_D r(t) + \hat{\rho}k_T\tanh(r(t)(a + bt))\,, \tag{4}$$

where a, b and k_D are positive constants, and $k_T > 1$. If parameter $\hat{\rho}$ is an estimate of the upper bound of the nonlinear residual terms, $u_b(t)|_{r(t)=\lambda e}$ behaves as a proportional gain robustly compensating nonlinear friction forces. It inputs to the control law (3) a torque greater than the maximum static friction allowing for compensation.

Estimation of Unknown Linear and Nonlinear Parameters

In the sliding surface method the adaptive law to validate all unknowns at each step of integration is based on simple first-order differential equation. Therefore, the following adaptive law [20] to validate estimates of system parameters at linear terms takes the form:

$$\dot{\hat{J}}(t) = -\delta_1 \dot{\varepsilon}(t) r(t)\,, \tag{5}$$

$$\dot{\hat{D}}(t) = -\delta_2 \varepsilon(t) r(t)\,, \tag{6}$$

$$\dot{\hat{T}}_s(t) = \begin{cases} -\ \delta_3 \operatorname{sgn}(\dot{\varphi}(t)) r(t), & \text{slip}\,, \\ \ \ \delta_3 (1 - \operatorname{sgn}|\dot{\varphi}(t)|)\, |r(t), & \text{stick}\,, \end{cases} \tag{7}$$

where $\delta_{1\ldots3}$ are positive constants, $r(t) = \dot{\varphi}(t) - \varepsilon(t)$.

Putting $\psi(t)$ from Eqs. (3) to (2) including $\varepsilon(t) = \dot{\varphi}(t) - r(t)$, $\dot{\varepsilon} = \ddot{\varphi}(t) - \dot{r}(t)$, with $\tilde{T}_{St} = \hat{T}_{St} - T_{St}$ and $\tilde{T}_0 = \hat{T}_0 - T_0$ measuring differences between estimates and their corresponding real values, one gets:

$$J\dot{r}(t) + Dr(t) = \hat{J}\dot{\varepsilon}(t) + \hat{D}\varepsilon(t) + \hat{T}_s \operatorname{sgn}\dot{\varphi}(t) + \omega(t) - u_b(t)\,, \tag{8}$$

where

$$\omega(t) = \left(\hat{T}_{St} e^{-\hat{T}_0|\dot{\varphi}(t)|} - T_{St} e^{-\hat{T}_0|\dot{\varphi}(t)|} e^{-\tilde{T}_0|\dot{\varphi}(t)|} - \tilde{T}_{St} - T_p\right) \operatorname{sgn}\dot{\varphi}(t). \tag{9}$$

Expanding $\exp(\tilde{T}_0|\dot{\varphi}(t)|) = 1 + \tilde{T}_0|\dot{\varphi}(t)| + \tilde{T}_0|\dot{\varphi}(t)|^2/2 + \tilde{R}$ in a Taylor series about $|\dot{\varphi}(t)| = 0$ and using only the first three terms of the expansion with a rest $\tilde{R} \le \exp(\tilde{T}_0|\dot{\varphi}(t)|)$ $T_0^3|\dot{\varphi}(t)|^3/6$:

$$\omega(t) = \left[\hat{T}_{St} e^{-\hat{T}_0|\dot{\varphi}(t)|} - T_{St} e^{-\hat{T}_0|\dot{\varphi}(t)|} \left(1 + \tilde{T}_0|\dot{\varphi}(t)| + \tilde{T}_0 \frac{|\dot{\varphi}(t)|^2}{2} + \tilde{T}_0^3 \frac{|\dot{\varphi}(t)|^3}{6} e^{\tilde{T}_0|\dot{\varphi}(t)|}\right) - \tilde{T}_{St} - T_p\right] \operatorname{sgn}\dot{\varphi}(t) = \rho \operatorname{sgn}\dot{\varphi}(t)\,, \tag{10}$$

where $\rho = -\gamma_1 + \gamma_2 \exp(-\hat{T}_0|\dot{\varphi}(t)|) - \gamma_3|\dot{\varphi}(t)| \exp(-\hat{T}_0|\dot{\varphi}(t)|) - \gamma_4|\dot{\varphi}(t)|^2 \exp(-\hat{T}_0|\dot{\varphi}(t)|)$, $\gamma_1 = \max_{|\dot{\varphi}(t)| \in [0,\infty]}\{|\dot{\varphi}(t)|^3 \exp(-T_0|\dot{\varphi}(t)|) \tilde{T}_0^3 T_{St}/6\} + \tilde{T}_{St} + T_p$, $\gamma_2 = \tilde{T}_{St}$, $\gamma_3 = \tilde{T}_0 T_{St}$, $\gamma_4 = \tilde{T}_0^2 T_{St}/2$. Constants $\gamma_{1\ldots4}$ depend on estimates or on its deflection from real values.

At this point let us back to Eq. (4) containing an unknown estimate $\hat{\rho}$. To get in Eq. (9) cancellation of reminders not dependent on $r(t)$, $\dot{r}(t)$, $\varepsilon(t)$ and $\dot{\varepsilon}(t)$, $\omega(t) - \hat{\rho}(t)\omega_r(t) \to \infty$, where $\omega_r(t) = k_T \tanh(r(t)(a + bt))$ as introduced in [5]. Therefore $\rho \operatorname{sgn}\dot{\varphi}(t) \to \hat{\rho} k_T \tanh(r(t)\ (a + bt))$ and if $u_b(t)$ is the upper bounding function of $\omega(t)$ then

$$\hat{\rho}(t) = -\hat{\gamma}_1 + \hat{\gamma}_2 e^{-\hat{T}_0|\dot{\varphi}(t)|} - \hat{\gamma}_3|\dot{\varphi}(t)| e^{-\hat{T}_0|\dot{\varphi}(t)|} - \hat{\gamma}_4|\dot{\varphi}(t)|^2 e^{-\hat{T}_0|\dot{\varphi}(t)|} \tag{11}$$

Table 1 System and tuning parameters for the numerical procedure

	Notation	Value	Unit
Motor torque constant	c_m	0.5	N m/A
Constant of the back electromotive force	c_b	0.011	V/rpm
Armature resistance	R	1.1	Ω
Moment of inertia of the rotor	J_m	2	kg m^2
Armature inductance	L_a	10^{-3}	H
Coefficient of viscous friction	T_v	8	N m s/rad
Max. static friction torque on Stribeck curve	T_{Stm}	0.5	N m
Max. static friction torque	T_{sm}	1.5	N m
Position-dependent friction torque	T_{1m}	0.35	N m
Constant of Stribeck curve	T_0	10	–
Constants of position-dependent friction	$[T_2, T_3]$	[1, 0.5]	–
Constants of adaptation laws	$\delta_{i=1\ldots7}$	$9 \times i$	–
Tuning factors: P gain	k_1	0.21×10^3	–
Tuning factors: D gain	k_2	1.40×10^3	–

states the estimate of ρ. Similarly to construction of Eq. (8) we can calculate the remaining estimate $\hat{\rho}(t)$ by solving the following system of equations:

$$\begin{aligned}
\dot{\hat{\gamma}}_1(t) &= \delta_4|r(t)|\,,\\
\dot{\hat{\gamma}}_2(t) &= -\delta_5|r(t)|e^{-\hat{T}_0|\dot{\varphi}(t)|}\,,\\
\dot{\hat{\gamma}}_3(t) &= -\delta_6|r(t)||\dot{\varphi}(t)|e^{-\hat{T}_0|\dot{\varphi}(t)|}\,,\\
\dot{\hat{\gamma}}_4(t) &= -\delta_7|r(t)||\dot{\varphi}(t)|^2e^{-\hat{T}_0|\dot{\varphi}(t)|}\,,
\end{aligned} \tag{12}$$

where $\gamma_{1\ldots4}$ are positive constants.

Voltage Input for Control of Rotational Velocity of the DC-Motor

In a real application one would require to source the motor not with the electrical current, but with a voltage of known function. In this situation the following full system has to be taken into the analysis:

$$J\ddot{\varphi}_f(t) + B\dot{\varphi}_f(t) + \tau_f(t) = \psi_f(t)\,, \tag{13}$$

$$L_a\dot{\psi}_f(t) + R_a\psi_f(t) + c_b\dot{\varphi}_f(t) = v_f(t)\,, \tag{14}$$

where index f is used to denote full three-dimensional dynamical system, L_a is the armature inductance, $v_f(t)$ is a time-dependent function of voltage required to realise some desired task of control.

One assumes that Eqs. (1) and (2) describe mathematically dynamics of the motor of which electrical and mechanical parameters will be taken according to an existing direct current commutation motor $PZTK\ 60 - 46\ J$ suitable to use it in cross-feed drives of numerically controlled machines (Table 1).

Voltage control in the full electromechanical system requires to regard to (13). If the current-input control of the DC-motor works correctly, then the best solution would be to maximally reduce influence of the second equation. In the full system it provides these unwanted disturbances influencing the optimal current input. The most obvious would be to apply to Eq. (14) the voltage input $v_f(t)$ calculated on the basis of $\psi(t)$ that is estimated after solution of only the reduced mechanical system (2) given by control law (3). Therefore, a voltage input necessary to cancel produced by (14) dynamical disturbances of the complete three-dimensional electromechanical system is expected in the form:

$$v_f(t) = L_a\dot{\psi}(t) + R_a\psi(t) + c_b\dot{\varphi}(t) + d(t)\,, \tag{15}$$

with a limitation that $\psi(t)$ ensures proper tracking current-input control of the reduced model (2), and $\dot{\varphi}$ is the angular velocity resulting from that control. Function $d(t)$ is a compensator of dynamical differences between state variables of Eqs. (14) and (15).

After substitution of v_f given by (15) to (14) all dynamical terms in Eq. (13) have their counterparts cancelling them, but some occurring differences are expected to be compensated by $d(t)$ which if disregarded makes the substitution working incorrectly and some significant oscillations about zero value are observed. To increase effectiveness of the control strategy it is proposed to apply a two-dimensional proportional control with a feedback from plant described by full dynamical system of the modelled motor. Therefore, applying

$$d(t) = k_1(\varphi_d(t) - \varphi_f(t)) + k_2(\dot{\varphi}_d(t) - \dot{\varphi}_f(t)) \tag{16}$$

to Eq. (15) to be used in (13) the following equation of dynamical equilibrium is found:

$$\begin{aligned} L_a\left(\dot{\psi}_f(t) - \dot{\psi}(t)\right) + R_a\left(\psi_f(t) - \psi(t)\right) + c_b\left(\dot{\varphi}_f(t) - \dot{\varphi}(t)\right) = \\ k_1\left(\varphi_d(t) - \varphi_f(t)\right) + k_2\left(\dot{\varphi}_d(t) - \dot{\varphi}_f(t)\right)\,, \end{aligned} \tag{17}$$

where to get the demanding cancellation of Eq. (14), tuning factors k_1 and k_2 should ensure equality of both sides of Eq. (17), but in each time, solution $\psi_f(t)$ have to be updated in Eq. (13), φ_d and $\dot{\varphi}_d$ are the desired coordinates of the phase trajectory of rotor's motion. Having this condition met, solution $\varphi_f(t)$ of Eq. (13) should track the optimal solution $\varphi(t)$ of Eq. (2). During the tracking control the time history of $v_f(t)$ can be saved and used as an input to drive a complete electromechanical dynamical system of the DC-motor along with desired phase trajectory, angular velocity or angular position of its rotor.

Numerical Simulation

Efficiency of the two-stage control method is checked in numerical simulations performed for a model of DC-motor $PZTK\ 60-46\ J$ with stick-slip friction occurring in contact zones placed between the rotor's shaft and bearings. Rotational velocity of the DC-motor is required to follow the desired trajectory $\varphi_d(t)$ drawn with a dashed line in Fig. 2. Time-history of this velocity is formed according to the scheme: it increases from 0 to $0.2\,rad/s$ in $0.2\,s$, is then held at this value for $0.6\,s$, it decreases to 0 in $0.2\,s$ and without a delay changes its value to negative achieving symmetrically (in second half of period) the same thresholds and times of presence as for positive values. After $2\,s$ the cycle is repeated (see the dashed line in Fig. 2).

Initial values of parameter estimates at linear terms: $\hat{J}^{(0)} = 1$, $\hat{D}^{(0)} = 1$, $\hat{T}_s^{(0)} = 0.2$, $\hat{T}_{St}^{(0)} = 1$, $\hat{T}_0^{(0)} = 1$. Initial value of parameter estimate at nonlinear terms $\hat{\rho}^{(0)} = 0$, initial values of state variables $\varphi^{(0)} = \dot{\varphi}^{(0)} = 0$, $\psi_f^{(0)} = 0$.

Important to observe, is that at the beginning of simulation exact values of some motor parameters are not known, but are represented by initial values of their counterpart estimates.

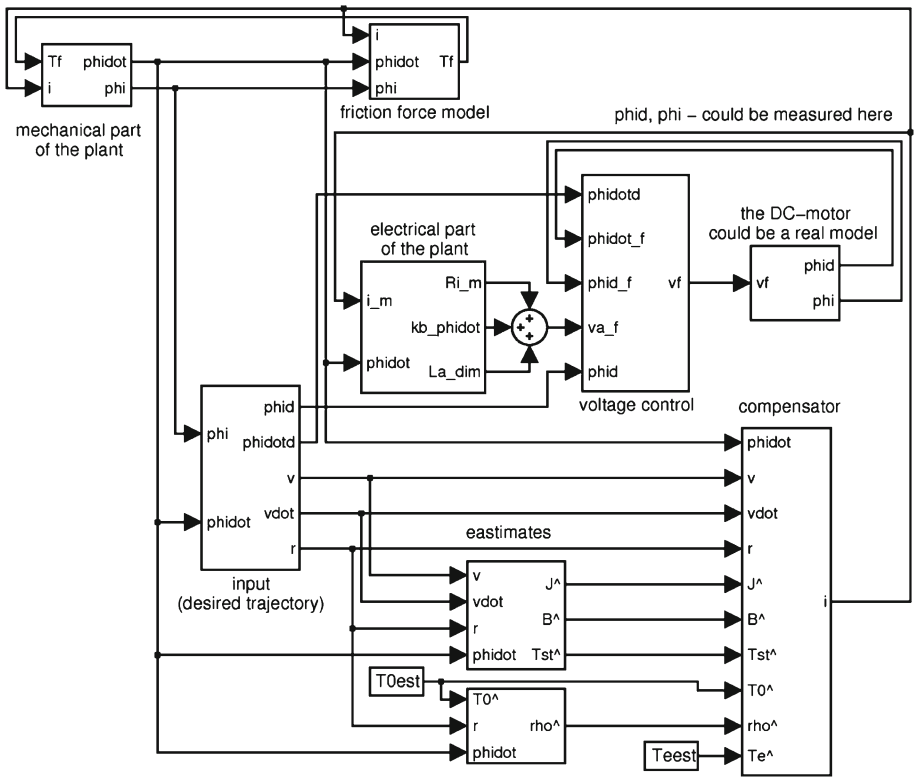

Fig. 1 Schematic diagram of the control system

Besides the uncertainty of parameters there exist also some strong influence of discontinuous terms represented by friction forces described earlier.

It is seen in Fig. 2 that at first occurrence of the threshold of constant angular velocity ($0.2\,rad/s$) the system response is inaccurate. Such transient behaviour results from the model and tuning parameters that are not correctly estimated at the corresponding time. The response changes over time to produce at the beginning of second period (at $2\,s$) acceptable overlapping of both trajectories. At subsequent $\pm 0.2\,rad/s$ thresholds the systems' step response of the model is well damped smoothly fitting edges of the desired shape. Figures 2b, d, f bring a clear comparison of three solutions: oscillating, over-dumped and the most accurate which could be also subject to some small improvement to get faster convergence to the steady state velocity.

The phase trajectory visible in Fig. 3 gives another view on the desired trajectory. It should take a shape of closed curve bounded between $\dot{\varphi} = \pm 0.2$. To achieve the demanding effect of control the voltage input should be applied accordingly to the time history shown in Fig. 4. Amplitude of the demanding voltage control input changes impulsively after crossing $\dot{\varphi} = 0$, for $t = 1, 2, \ldots, n[s]$.

Conclusions

The proposed strategy of control ensures robust adaptation, works correctly and can be applied in solution of other control tasks regarded to shaping of time-histories of responses

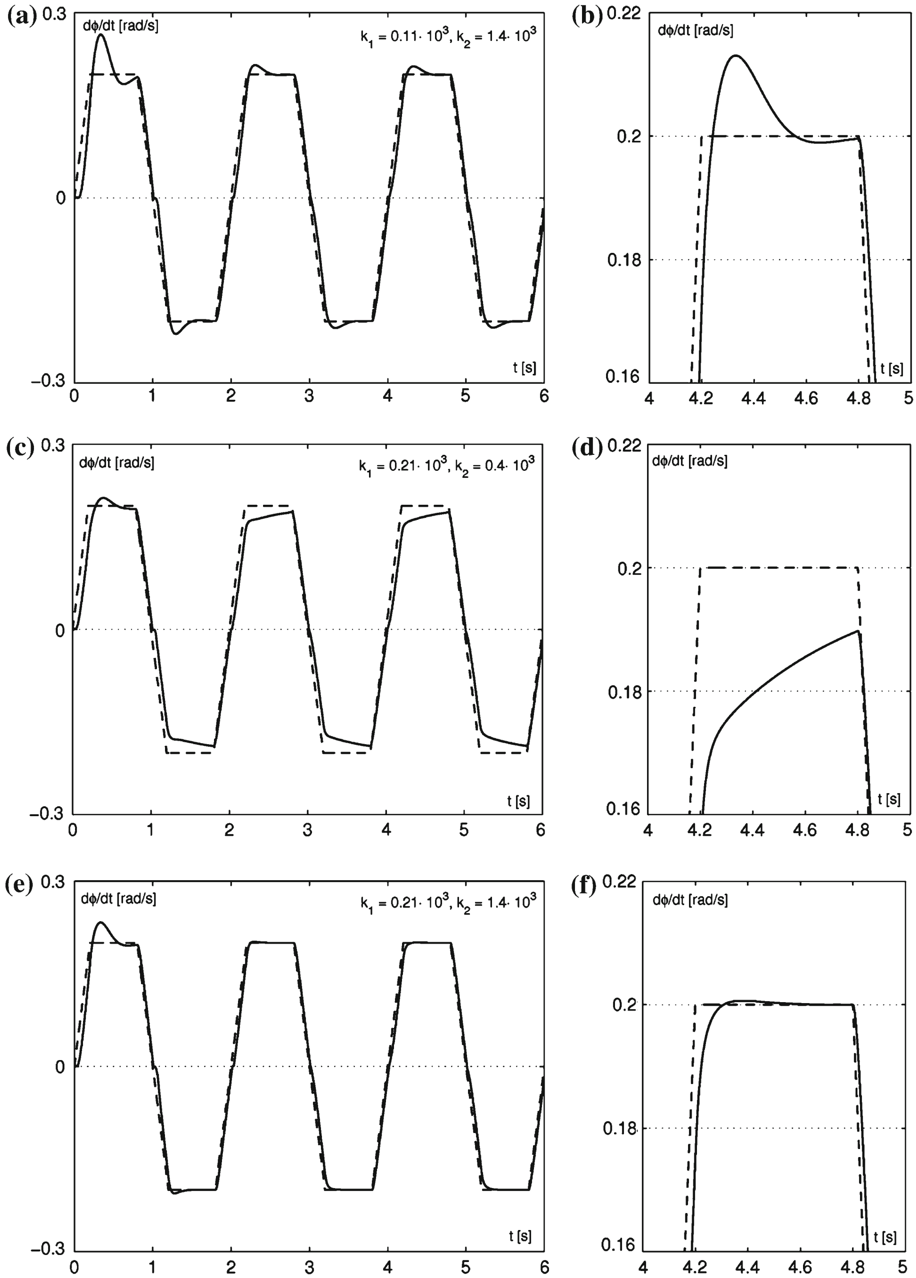

Fig. 2 Desired time-history of angular velocity $\dot{\varphi}_d(t)$ (*dashed line*) and the corresponding response $\dot{\varphi}(t)$ (*solid line*) of the analysed voltage-controlled numerical model of DC-motor defined by the assumed set of model parameters, PD tuning variables k_1 and k_2, and initial values of state variables

of some group of discontinuous systems. After many attempts of tuning, parameters k_1 and k_2 of the second stage of control have been estimated. They significantly affect local step response (appearing while going on the constant angular velocity thresholds). Moreover, on

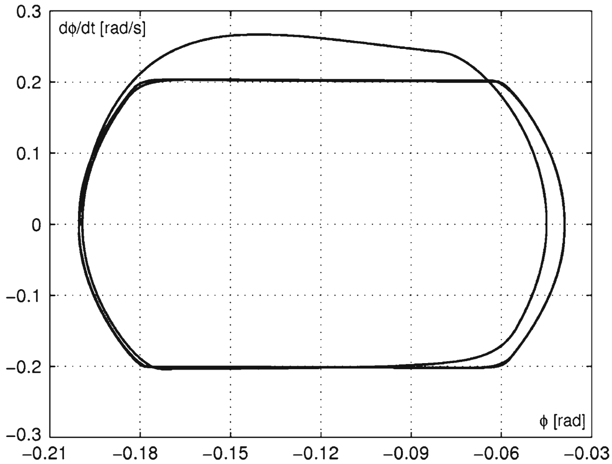

Fig. 3 Projection of phase trajectory of the controlled system on the plane $\dot{\varphi}(\varphi)$

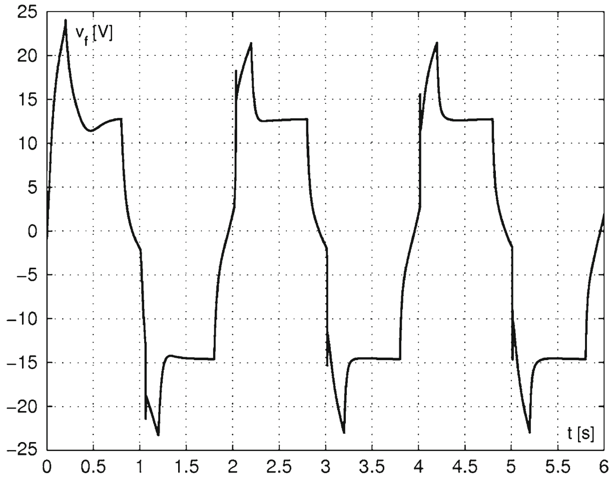

Fig. 4 Voltage input $v_f(t)$ applied to the controlled DC-motor

the basis of sliding-mode based smooth adaptive robust controller for compensation of frictional effects there was proposed useful and easy applicable extension of this method for a numerical tracking control of DC-motors by means of voltage input. A kind of drawback or an inconvenience in application of the control strategy is the requirement of estimation of the upper bounding function for stick-slip friction in order to guarantee closed-loop stability.

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