Nonlinearity of muscle stiffness

Jan Awrejcewicz,[a)] G. Kudra,[b)] and B. Zagrodny[c)]
Department of Automation and Biomechanics, Technical University of Lodz, Lodz 90-924, Poland

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Abstract In this letter, a comparison between three types (two linear and one nonlinear) of models of skeletal muscle stiffness is shown. Results are compared with experimental data for biceps brachii in the case of muscle stretching and with the Hill equation for a biological muscle. It is shown that results for nonlinear stiffness model in case of length-force relationship fits to the experimental data.
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Keywords muscle modeling, muscle stiffness, biological and muscle model properties

Testing and muscle modelling are very important aspects of biomechanics. The first mathematical model of muscle was due to Hill, who created it in the twenties of the twentieth century. Since that time a huge progress in understanding of muscle behaviour has been observed.[1–5] Most experiments show that biological muscles have nonlinear behaviour.[1–3] It has been observed in experiments that stretching force depends nonlinearly on elongation, muscle internal force depends nonlinearly on velocity of shortening, and velocity of muscle movement as a function of load (called Hill curve) is also nonlinear (shown as Figs. 1 and 2). These graphs have been confirmed repeatedly in experimental studies by many authors for many types of muscles of many species (inter alia: frog, cat, human).[1,3,5] In the paper muscle models of stiffness are presented. They are investigated and numerical simulations are carried out. It turns out that nonlinear model fits to the experimental data from the literature in contrary to linear ones.

Proposed muscle model is built from six parts: two tendons, insertions, venter lower middle, upper part and surrounding tissues. Each part of venter is built from actin and myosin. Based on that for each muscle part, muscle properties (stiffness and damping) are proposed.

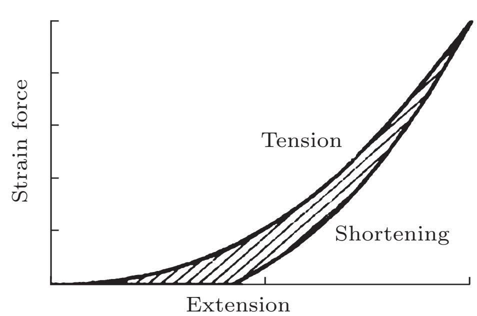

Fig. 1. Value of external stretching force as a function of elongation,[2] where the values are normalised.

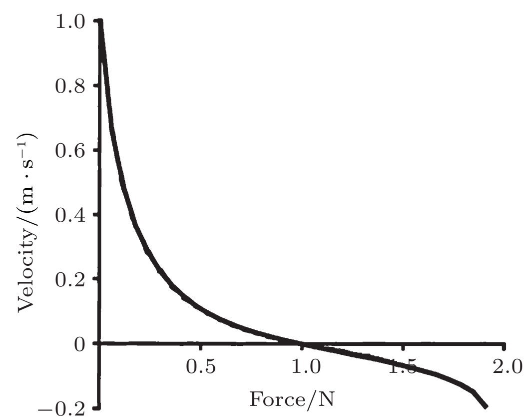

Fig. 2. The Hill's curve (obtained from the Hill equation) exhibiting the value of muscle shortening or elongation velocity as a function of load.[3]

Mathematical description of the model presented in Fig. 3 follows

$$
\begin{aligned}
& m_1\ddot{x}_1 + K_1P_kx_1 + C_1\dot{x}_1 - K_2(x_2 - x_1) - \\
& \quad C_2(\dot{x}_2 - \dot{x}_1) = 0, \\
& m_2\ddot{x}_2 + K_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) - \\
& \quad K_3(x_3 - x_2) - C_3(\dot{x}_3 - \dot{x}_2) = F_{2,3}, \\
& m_3\ddot{x}_3 + K_3(x_3 - x_2) + C_3(\dot{x}_3 - \dot{x}_2) - \\
& \quad K_4(x_4 - x_3) - C_4(\dot{x}_4 - \dot{x}_3) = -F_{2,3}, \\
& m_4\ddot{x}_4 + K_4(x_4 - x_3) + C_4(\dot{x}_4 - \dot{x}_3) - \\
& \quad K_5(x_5 - x_4) - C_5(\dot{x}_5 - \dot{x}_4) = F_{4,5}, \\
& m_5\ddot{x}_5 + K_5P_k(x_5 - x_4) + C_5(\dot{x}_5 - \dot{x}_4) - \\
& \quad K_6(x_6 - x_5)^3 - C_6(\dot{x}_6 - \dot{x}_5) = -F_{4,5}, \\
& m_6\ddot{x}_6 + K_6(x_6 - x_5) + C_6(\dot{x} - \dot{x}_5) - \\
& \quad K_7(x_7 - x_6) - C_7P_c(\dot{x}_7 - \dot{x}_6) = F_{6,7}, \\
& m_7\ddot{x}_7 + K_7(x_7 - x_6) + C_7(\dot{x}_7 - \dot{x}_6) - \\
& \quad K_8(x_8 - x_7) - C_8(\dot{x}_8 - \dot{x}_7) = -F_{6,7}, \\
& m_8\ddot{x}_8 + K_8(x_8 - x_7) + C_8(\dot{x}_8 - \dot{x}_7) + \\
& \quad K_zx_8 + C_z\dot{x}_8 = F_{\text{external}}.
\end{aligned} \tag{1}
$$

where $x_i, i = 1, \cdots, 8$ is location of centre of gravity of the masses of the different muscle parts; $K_i = K_i(x_1, \cdots, x_8)$ and $C_i = C_i(\dot{x}_1, \cdots, \dot{x}_8)$ denote stiffness

[a)]Email: jan.awrejcewicz@p.lodz.pl.
[b)]Email: grzegorz.kudra@p.lodz.pl.
[c)]Corresponding author. Email: b.zagrodny.pl@gmail.com.

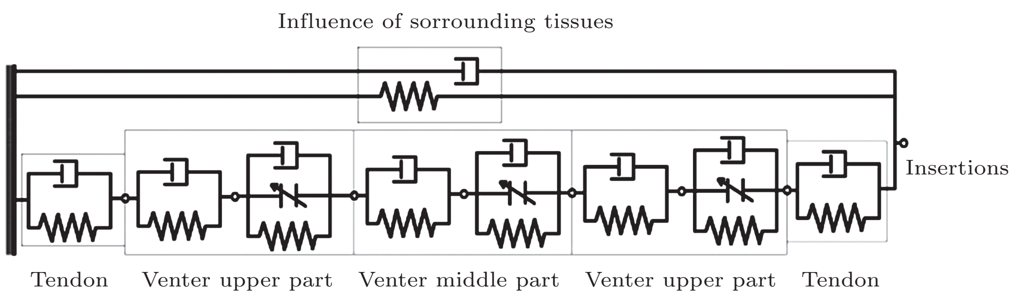

Fig. 3. Scheme of the muscle model.

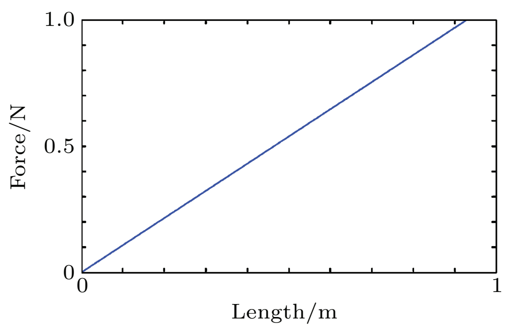

Fig. 4. Value of external stretching force as a function of elongation for linear–constant stiffness parameter (Eq. (2)).

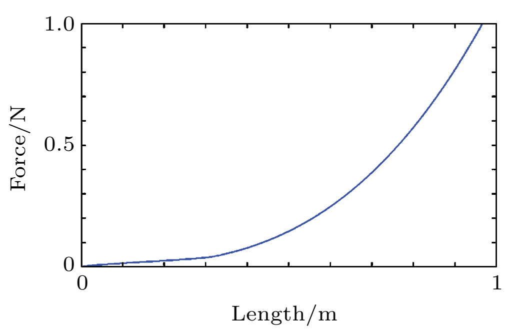

Fig. 5. Value of external stretching force as a function of elongation for nonlinear stiffness parameter (Eq. (3)).

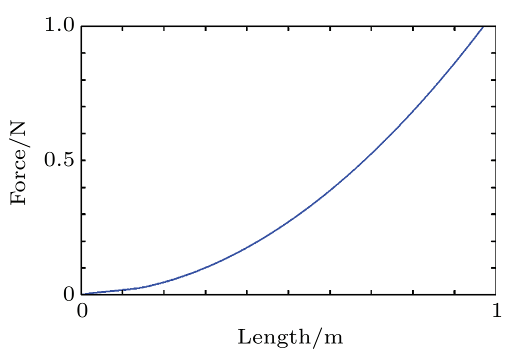

Fig. 6. Value of external, stretching force as a function of elongation for linear stiffness parameter (Eq. (4)).

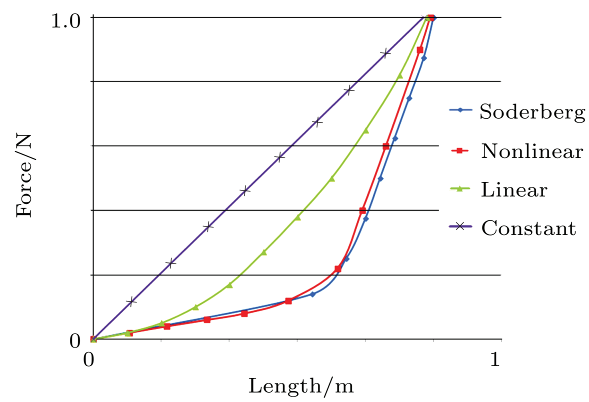

Fig. 7. Results comparison with experimental data obtained by Soderberg (triangles line).

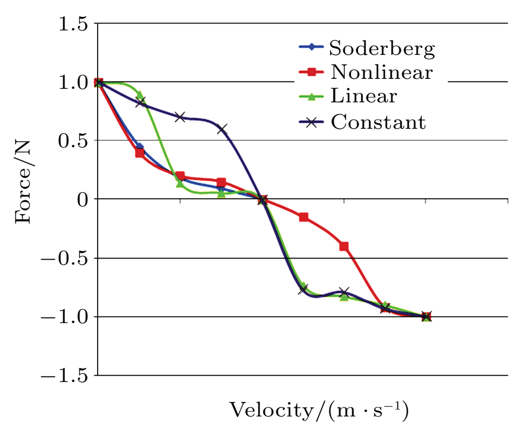

Fig. 8. Results comparison with Hill equation (shown as Fig. 2).

and damping parameter of i-th element, respectively, and the last equation describes influence of surrounding tissues on muscle behaviour.

Presented model was simulated with three different types of stiffness functions. The simulation time was 50 seconds and the tensile force was applied according to the formula $F_{\text{external}}(t) = 5[N/s]t$.

Muscle was divided into parts with masses follows: $m_1 = m_8 = 0.01$ kg, $m_2 = m_7 = 0.02$ kg, $m_3 = m_6 = 0.05$ kg, $m_4 = m_5 = 0.07$ kg, as a result giving a mass of an average male biceps brachii.

First simulation was done with the following stiffness parameters:

$$K_i := k_i(x_i - x_{i-1})^2, i = 1, 2, \cdots, 8, \tag{2}$$

where $k_1 = k_7 = 3\times10^4$ N/m^3, $k_2 = k_6 = 5\times10^3$ N/m^3, $k_3 = k_4 = k_5 = 4 \times 10^3$ N/m^3.

This type of stiffness coefficient was adjusted by the authors of this letter to the presented model. Second simulation was done with constant stiffness parameters

$$K_i := k_i := \text{ const}, i = 1, 2, \cdots, 8, \tag{3}$$

where $k_1 = k_7 = 3 \times 10^4$ N/m, $k_2 = k_6 = 5 \times 10^3$ N/m, $k_3 = k_4 = k_5 = 4 \times 10^3$ N/m.

The last simulation was done with stiffness given by

$$K_i := k_i(x_i - x_{i-1}),\ i = 1, 2, \cdots, 8, \tag{4}$$

where $k_1 = k_7 = 3\times10^4$ N/m^2, $k_2 = k_6 = 5\times10^3$ N/m^2, $k_3 = k_4 = k_5 = 4 \times 10^3$ N/m^2.

Because most of the published in literature data are normalised, authors decided to normalise obtained results, which allows its comparison. Following results were obtained for previously defined stiffness.

As it can be seen in Fig. 4 muscle characteristic is linear, whereas the stiffness coefficient is constant. However, experimental data suggests (shown as Figs. 1 and 2) that it should be nonlinear. It is easy to observe that graph in Fig. 5 fits to the graph in Fig. 1 in the best way. The graph presented in Fig. 6 has similar shape as that in Fig. 5. However, comparing normalised results in one figure (shown as Fig. 7) with experimental data[2] (Soderberg curve for biceps brachii), it is clearly seen that curves from Figs. 4 and 6 fit worse to the experimental curve than that presented in Fig. 5. The standard deviations of the differences between the experimental data and those from the simulations are of the order: 0.004 5 for the first model — nonlinear; 0.058 for the third model — linear; 0.089 for the second model — linear (constant). Similarly, nonlinear parameters are better in case of comparison with the Hill equation (shown as Fig. 8). In that case the standard deviations of the differences between the experimental data and those from the simulations are of the order: 0.004 for the first model-nonlinear; 0.021 for the third model–linear; 0.026 for the second model-linear (constant). It can be expected that our model with nonlinear parameters can be more adequate in other cases.

Computation of a model with nonlinear parameters is much more complicated and more time consuming than a simple, linear model. However, nonlinear models are better biocompatibile, because biological tissues (like muscles) are characterized by nonlinear parameters. Another problem is to find an adequate function, which will describe correctly the dependence (in this case length–force dependence). In this case quadratic function form of stiffness was taken under consideration, because many authors describe muscle characteristics as an inverse parabolic characteristics.[2]

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