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Optimal design of ring-stiffened cylindrical shells using homogenization approach

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Abstract: We propose a modification of the homogenization method for computational model of an axially symmetric cylindrical shell supported by rings having different stiffness properties governed by arbitrary analytical functions. The mentioned functions serve as control for an associated inverse problem. The latter is solved through a zero-order approximation corresponding to the structurally orthotropic solution being formulated for the first-order approximation of the location of discrete rings.

Keywords: asymptotic series, homogenization, optimality condition, singular problem, ring stiffening

1 INTRODUCTION

There are a number of publications devoted to the analysis of an optimal support of plates and shells, since such type of problems play a key role in various applications in civil engineering, ship industry, design of aircrafts and rockets, etc. In general, the mentioned problems have been reduced to an appropriate choice of the ratio of rings and shell stiffness [**1**, **2**], sizing, shape and topology [**3**], material properties, geometric size and different material coefficients [**4**], shells' variable thickness [**5**], geometrical forms of rings [**6**], as well as thickness of rings on a torospherical head [**7**]. An open cylindrical shell reinforced with a quasi-regular set of discrete longitudinal ribs was studied by Lugovoi [**8**]. Stability and vibrations of cylindrical shells discretely reinforced with quasi-regular rings were analysed by Abramovich and Zarutskii [**9**]. However, the least studied case is that of the investigation of increased loading capacity of the reinforced shell rings having different properties. The lack of progress in this field is raised by limitations in getting results suitable for optimization of solutions of direct problems for the so far mentioned structures.

The widely used methods for modelling and computations of reinforced plates and shells can be divided into two directions. The first one is associated with the discretization of constructions through, for instance, the Finite Element Method [**10**, **11**] or the Finite Difference Method [**12**]. The second one relies on the application of various homogenization procedures to reinforced plates and shells governed by PDEs [**13**]. Observe that efficiency of the latter method increases essentially with an increase of the number of rings N. Based on the homogenization procedure [**14**] analysed, the deformable state of an axially symmetric cylindrical shell is reinforced by rings of different stiffness properties. They showed, among the others, that this way of the shell's support improved its load-carrying ability.

A similar approach can also be applied to the inverse problems together with the application of mathematical programming. However, difficulties in the numerical computations increase with the increase of N. Further on, for a non-homogeneously

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reinforced shell, N corresponds to the number of design parameters. Therefore, in many cases, it is more suitable to apply various methods of optimal control, which have been developed for the problems of mechanical engineering [**15**, **16**].

The main aim of this study is to propose a modification of the averaging method [**13**] devoted to computations of cylindrical shells non-uniformly supported by stringers.

This article is organized as follows. In Section 1, we describe the applied direct problem and the averaged equation of axially symmetric deformation of a cylindrical shell non-uniformly supported by stringers. In addition, a correction term describing the influence of the stringers is derived.

The inverse problem for the averaged equation is solved in Section 2. In Section 3, we present a solution for the optimization of a singular problem. In Section 4, the influence of ring positions is investigated, and then also a numerical example is provided in Section 5. Finally, in Section 6, we briefly discuss the results.

2 DIRECT PROBLEM

A differential equation governing deflection of the skin between the rings has the following form [**17**]

$$\frac{d^4 w}{dx^4} + \beta w = q \tag{1}$$

where $\beta = 12(1-\nu^2)/(Rh)^2$; $q = P(x)/D$; $D = Eh^3/(12(1-\nu^2))$; $P(x)$ is the normal pressure, R the shell radius, h the skin thickness, E, μ the Young and Poisson coefficients, respectively.

We assume an ideal contact between the shells and rings along lines. Hence, the compatibility conditions associated with the ith ring are as follows

$$\begin{aligned} & w^- = w^+ \\ & \left(\frac{dw}{dx}\right)^- = \left(\frac{dw}{dx}\right)^+ \\ & \left(\frac{d^2 w}{dx^2}\right)^- = \left(\frac{d^2 w}{dx^2}\right)^+; \quad \left(\frac{d^3 w}{dx^3}\right)^- - \left(\frac{d^3 w}{dx^3}\right)^+ = B_i w_{x=is} \end{aligned} \tag{2}$$

In the above notation $(\cdots)^\pm = \lim\limits_{x\to is\pm 0}(\cdots)$, s is the distance between the successive rings; $B_i = E_i F_i/(R^2 D)$; E_i, F_i – Young modulus of ring materials and the transversal cross-section of the ith ring, respectively, $i = 1, \ldots, N$.

Next, we introduce the following smooth function $\tilde{k}(x)$ such that $\tilde{k}(is) = B_i$, $\tilde{k}(0) = \tilde{k}(L) = 0$. We use the so far defined function while solving the inverse problems and it is defined by the optimality condition. The simply supported shell will be further studied, and hence the boundary conditions on the shell edges $x = 0, L$ are as follows

$$w = 0, \ \frac{d^2 w}{dx^2} = 0 \tag{3}$$

If the number of rings is large ($s/L = \varepsilon \ll 1$), then one may apply an asymptotic procedure to solve the problem governed by equations (1) to (3) [**13**]. We introduce the following variable

$$\xi = x/\varepsilon \tag{4}$$

The differential operator applied further has the following form

$$\frac{dw}{dx} = \frac{\partial w}{\partial x} + \varepsilon^{-1}\frac{\partial w}{\partial \xi} \tag{5}$$

Deflection w is represented by the following expansions [**13**]

$$\begin{aligned} w = {} & w_0(x) + \varepsilon w_{01}(x,\xi) + \varepsilon^2 w_{02}(x,\xi) + \varepsilon^3 w_{03}(x,\xi) \\ & + \varepsilon^4 w_1(x,\xi) + \varepsilon^5 w_2(x,\xi) + \cdots, \end{aligned} \tag{6}$$

where $w_{0j}(x,\xi)\,(j = 1,2,3)$, $w_i(i = 1,2,\ldots)$ are the periodic functions with respect to ξ having the period L.

Substitution of relations (5), (6) into (1) to (4) and applying asymptotic splitting regarding powers of ε yields the following relations (periodicity of functions w_i regarding ξ has been applied)

$$w_{01}(x,\xi) = w_{02}(x,\xi) = w_{03}(x,\xi) = 0 \tag{7}$$

$$\frac{\partial^4 w_1}{\partial \xi^4} + \frac{d^4 w_0}{dx^4} + \beta w_0 = q \tag{8}$$

$$\left(w_1; \frac{\partial w_1}{\partial \xi}; \frac{\partial^2 w_1}{\partial \xi^2}\right)_{\xi=0} = \left(w_1; \frac{\partial w_1}{\partial \xi}; \frac{\partial^2 w_1}{\partial \xi^2}\right)_{\xi=L} \tag{9}$$

$$\left(\frac{\partial^3 w_1}{\partial \xi^3}\right)_{\xi=L} - \left(\frac{\partial^3 w_1}{\partial \xi^3}\right)_{\xi=0} = k(x) w_0 \tag{10}$$

$$(w_0)_{x=0,\,L} = 0, \ \left(\frac{d^2 w_0}{dx^2}\right)_{x=0,\,L} = 0 \tag{11}$$

where $k(x) = L\tilde{k}(x)/s$. Observe that when deriving formula (10), the ratio of rings and shell stiffness satisfies the condition $R^3 Lk(x)/s \sim 1$.

Integration of equation (8) regarding ξ yields

$$w_1 = \frac{k(x)}{24L} w_0(x)\xi^4 + C_1\xi^3 + C_2\xi^2 + C_3\xi + C_4 \tag{12}$$

where $C_i = C_i(x)$.

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Below, we define $C_1 - C_3$ from conditions (9) and we take $C_4 = 0$, and since this term refers to the averaged equation, one gets

$$w_1 = \frac{k(x)}{L} w_0(x)\xi^2(\xi - L)^2 \tag{13}$$

Substitution of (13) into (10) results in getting the following homogenized equation defining w_0

$$\frac{\mathrm{d}^4 w_0}{\mathrm{d}x^4} + \left(\frac{k(x)}{L} + \beta\right) w_0 = q \tag{14}$$

Equation (14) governs the axially symmetric deformation of structurally orthotropic shell having rings located continuously over the shell's length. Correction term (13) takes into account the discreteness of the location of rings.

3 INVERSE PROBLEM

In order to apply optimization, we use the shell compliance to formulate the minimum functional of the form

$$J = \int_0^L qw\,\mathrm{d}x \to \min_k \tag{15}$$

Here, the right-hand term of (15) means that we study the problem of minimizing energy of the elastic shell deformation through an appropriate choice of $k(x)$, i.e., through the appropriate choice of the stringers stiffness distribution along the shell length.

Further, we apply a condition of the summed ring stiffness of the following form

$$\sum_{i=1}^{N} k(is) = c \equiv const \tag{16}$$

Manevitch et al. [**13**] showed that the structurally orthotropic approximation made it possible to define accurately the deflection of reinforced plates and shells. For this reason for problem (15), we only take zero-order approximation for deflection (14). It is convenient to present constraint (16) in the following averaged form

$$\frac{1}{L}\int_0^L k(x)\mathrm{d}x = c \tag{17}$$

Since function $k(x)$ defines the stringers stiffness then owing to its physical interpretation, it should satisfy the following inequality $k(x) \geq 0$. Observe that the given inequality is satisfied if in optimization performances (15) and (16), we take function $\varphi(x)$ as the control one, which satisfies the following condition

$$k(x) = \varphi^2(x) \tag{18}$$

Owing to the known methods of optimal control [**15**], we formulate an optimality condition of the problem governed by (3), (14), (15), and (17) regarding the chosen control function $\varphi(x)$ displayed by (18). For this purpose, we apply the following variations of integrals (15), (17), and equation (14)

$$\delta J = \int_0^L q\delta w_0\,\mathrm{d}x, \quad \delta J_1 = 2\int_0^L \varphi\,\delta\varphi\,\mathrm{d}x \tag{19}$$

$$\delta\frac{\mathrm{d}^4 w_0}{\mathrm{d}x^4} + \left(\frac{\varphi^2}{L} + \beta\right)\delta w_0 + 2\frac{\varphi\, w_0}{L}\delta\varphi = 0 \tag{20}$$

Note that equation (20) is obtained by substituting w_0 and φ by $w_0 + \delta w_0$ and $\varphi + \delta\varphi$, respectively, then equation (14) is used, and only linear terms are kept.

Next, we take $\delta\varphi$ as the minimum function. We introduce the conjugated variable $v(x)$ defined through the condition that the expressions for minimizing functional should not contain variation δw_0. Multiplying equation (20) by $v(x)$, and then integrating in the interval from 0 to L, we get

$$\int_0^L v\left[\delta\frac{\mathrm{d}^4 w_0}{\mathrm{d}x^4} + \left(\frac{\varphi^2}{L} + \beta\right)\delta w_0 + 2\frac{\varphi\, w_0}{L}\delta\varphi\right]\mathrm{d}x = 0 \tag{21}$$

Now, the first term of (21) is integrated four times by parts with inclusion of the boundary conditions for w_0 (3) and equation (14), and in the next step integral (21) is cast to the following form

$$\int_0^L \left[\left(\frac{d^4 v}{d x^4} + \left(\frac{\varphi^2}{L} + \beta\right) v\right)\delta w_0 + 2\frac{\varphi\, w_0\, v}{L}\delta\varphi\right]\mathrm{d}x \tag{22}$$

where the following boundary conditions are applied

$$(v)_{x=0,L} = 0, \left(\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}\right)_{x=0,L} = 0 \tag{23}$$

We add variation δJ_1 to the variation of minimizing functional δJ through the Lagrange multiplier, and hence formula (22) takes the following form

$$\delta J = \int_0^L \left[\left(\frac{d^4 v}{d x^4} + \left(\frac{\varphi^2}{L} + \beta\right) v + q\right)\delta w_0 + 2\frac{\varphi}{L}(\lambda + v\, w_0)\delta\varphi\right]. \\ \mathrm{d}x = 0 \tag{24}$$

In order to keep variation δJ independent of δw_0 of the conjugated variable v, the following equation should be satisfied

$$\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + \left(\frac{\varphi^2}{L} + \beta\right) v = -q \tag{25}$$

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Finally, we obtain the formula being sought for the variation of the optimized functional expressed through variation $\delta\varphi$ as

$$\delta J = \frac{2}{L}\int_0^L \varphi(\lambda + v\, w_0)\delta\varphi\, \mathrm{d}x = 0 \tag{26}$$

Therefore, the following optimality condition is obtained

$$\varphi(\lambda + v\, w_0) = 0 \tag{27}$$

Observe that a comparison of the boundary value problems for w_0 (cf. (3), (14)) and for v (cf. (23), (25)) yields $v = -w_0$, and finally condition (27) takes the form

$$\varphi\,(\lambda - w_0^2) = 0 \tag{28}$$

Observe that a trivial solution $\varphi = 0$ of this equation does not have a physical interpretation for the problem considered by us, since for that case, the shell support does not exist. On the other hand, the solution $w_0 = \sqrt{\lambda} - const$ does not satisfy boundary conditions (3).

4 SOLUTION OF THE OPTIMIZATION SINGULAR PROBLEM

One of the typical properties of the optimization of numerous elements of structures is the occurrence of singular points [**15**]. In these points, the highest order derivatives of the differential equations are equal to zero. In this case, in order to get a closed system of stationary conditions, one should take into account the Weierstrass–Erdmann relations in the singular points. While optimizing bending variable stiffness, Masur [**18**] reported that the Weierstrass–Erdmann conditions were reduced to the continuity conditions of the derivative of a bending function in singular points. We observe the same behaviour in our case. Since problems (3), (14), (17), and (28) have no solution in the class of continuous functions, solution $\varphi(x)$ is sought in the class of piece-wise functions having first-order discontinuities. The so far defined problem will have a solution if in intervals $(0, x_1)$ and (x_2, l), we have $\varphi = 0$, whereas in interval (x_1, x_2), one gets $w_0 = \sqrt{\lambda}$. In this case, equilibrium equation (13) yields the following result being valid in interval (x_1, x_2)

$$\varphi^2 = \pm\frac{q}{\sqrt{\lambda}} - \beta \tag{29}$$

The discontinuity points x_1, x_2 are deduced from conditions of both deflection function w as well as their derivatives $\frac{\mathrm{d}w}{\mathrm{d}x}$, $\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$ continuity

$$w^{(-)} = w^{(+)}, \quad \left(\frac{\mathrm{d}\,w}{\mathrm{d}\,x}\right)^{(-)} = \left(\frac{\mathrm{d}\,w}{\mathrm{d}\,x}\right)^{(+)},$$
$$\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)^{(-)} = \left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)^{(+)} \tag{30}$$

In the above notation $(\ldots)^{(\pm)} = \lim_{x\to x_j\pm 0}(\ldots)$, $j = 1, 2$. The choice of conditions (30) is motivated by physical observations, since they keep smooth shell deformations. However, derivative $\frac{\mathrm{d}^3 w}{\mathrm{d}x^3}$ may have a jump in singular points; therefore, those points may be reinforced by stringers. Note that relations (30) also ensure satisfaction of the Weierstrass–Erdmann conditions [**18**].

In the considered case, conditions (30) may be written as follows

$$\left(w^{(1)}\right)_{x=x_1} = \left(w^{(2)}\right)_{x=x_2} = \sqrt{\lambda},$$
$$\left(\frac{dw^{(1)}}{dx}\right)_{x=x_1} = \left(\frac{dw^{(2)}}{dx}\right)_{x=x_2} = 0,$$
$$\left(\frac{d^2 w^{(1)}}{dx^2}\right)_{x=x_1} = \left(\frac{d^2 w^{(2)}}{d\,x^2}\right)_{x=x_2} = 0. \tag{31}$$

Expressions for $w^{(1)}$ and $w^{(2)}$ are derived from solutions of the corresponding boundary value problems in intervals $(0, x_1)$ and (x_2, L) for equation (14) for $\varphi^2 \equiv 0$ and for boundary conditions (3), and by taking into account the adjunct condition (31). The ten mentioned conditions (3) and (31) allow us to find eight constants of integration (four for each interval), and then describe coordinates of discontinuity points x_1, x_2 by λ.

The Lagrange constant λ is defined *via* isoperimetric condition (17), which in this case takes the following form

$$\frac{1}{L}\int_{x_1}^{L-x_2} \varphi^2\, \mathrm{d}x = c \tag{32}$$

Coordinates x_1, x_2 correspond to the first and last ring positions, respectively.

5 THE INFLUENCE OF RING POSITIONS

First of all, let us emphasize that taking into account discreteness of the ring positions (13) yields an essential problem in getting a solution to the stated problem. In this case, taking into account (4) and (6), a function governing the shell deflection has the form $w = (1 + \varphi^2 p)\, w_0$, where $p = x^2(x - L)^2 s/L^2$. Then, the minimizing functional (15) takes the following

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form

$$J = \int_0^L (1+\varphi^2 p)\, w_0 q\, \mathrm{d}x \to \min_{\varphi} \tag{33}$$

and the corresponding variation of functional (33) is as follows

$$\delta J = \int_0^L [(1+\varphi^2 p)\delta w_0 + 2\varphi p w_0 \delta\varphi] q\, \mathrm{d}x \tag{34}$$

Equations of equilibrium (20) and integral for the conjugated variable v defined by (22) do not change. Therefore, the expression for extended variation δJ governed by (24) takes the following form

$$\delta J = \int_0^L \left[\left(\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + \left(\frac{\varphi^2}{L} + \beta\right) v + (1+\varphi^2 p) q \right) \right.$$

$$\left. \delta w_0 + 2\,\frac{\varphi}{L}\,(\lambda + (v+p)\, w_0)\,\delta\varphi \right] \mathrm{d}x = 0 \tag{35}$$

As a result, we get the following equation for the conjugated variable v

$$\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + \left(\frac{\varphi^2}{L} + \beta\right) v + (1+\varphi^2 p) q = 0 \tag{36}$$

and the optimality conditions take the form

$$\varphi\,(\lambda + (v+p)\, w_0) = 0 \tag{37}$$

Optimality condition (37) together with equilibrium equation (14), and with the equation for conjugated variable (36) and boundary conditions (3) and (23) creates a closed boundary value problem for the determination of an optimal distribution of ring stiffness $k(x)$ (cf. (18)) and deflection $w(x)$ taking into account discreteness of the ring positions. The Lagrange multiplier λ is defined *via* constraint condition (32).

6 NUMERICAL EXAMPLE

Let us consider a particular case of shell loading of $q = const$. Below, we take into account only the structural orthotropic properties. In this case due to symmetry $x = L/2$ and so $x_1 = x_2$, and for their determination, one gets a boundary value problem for equilibrium equation (14) for $k=0$ with boundary conditions (3) and adjunct conditions (31) in the interval of $(0, x_1)$. A solution to equilibrium equation (14) is defined *via* the Krylov functions K_i, $i=1,2,3,4$ in the following way [**19**]

$$w_1 = D_1 K_1 + D_2 K_2 + D_3 K_3 + D_4 K_4 + \mathrm{q}/\beta \tag{38}$$

where $D_1 - D_4$ are the arbitrary constants, and

$K_1 = \cosh\eta\cos\eta$, $K_2 = (\cosh\eta\sin\eta + \sinh\eta\cos\eta)/2$

$K_3(x) = (\sinh\eta\sin\eta)/2$, $K_4(x) = (\cosh\eta\sin\eta - \sinh\eta\cos\eta)/4$, $\eta = \sqrt[4]{\frac{\beta}{4}}x_1$

Boundary conditions (3) yield

$$D_1 = -q/\beta, \qquad D_3 = 0 \tag{39}$$

Substituting (38) into adjunct condition (31), and taking into account (39) yields a system of equations to determine D_2, D_4, and x_1. Next, taking expressions for D_2, D_4 from the second and third equations of (31) and substituting them into the first equation, one obtains the following equation to determine x_1

$$\frac{\begin{pmatrix} \sinh(\eta)\cosh(\eta) + \cos(\eta)\sin(\eta) - \cosh(\eta)\sin(\eta) \\ -\sinh(\eta)\cos(\eta) \end{pmatrix}}{\sinh(\eta)\cosh(\eta) + \cos(\eta)\sin(\eta)} = \frac{\beta\sqrt{\lambda}}{\mathrm{q}} \tag{40}$$

Developing the left-hand side of equation (40) into a series regarding η yields

$$0.167\eta^4 - 0.021\eta^8 + \mathrm{O}(\eta^9) = \frac{\beta\sqrt{\lambda}}{q} \tag{41}$$

Now, taking into account that $0 \le x_1 \le L/2$, we keep in equation (41) only the first term and we get $x_1^4 = 24\sqrt{\lambda}/q$. Then, taking into account (29) and condition (31), the following equation is obtained to determine x_1

$$2\beta x_1^5 - (\beta L + c) x_1^4 - 48 x_1 + 24L = 0 \tag{42}$$

Numerical solutions to equation (42) are found by Maple, for the following fixed parameters: $L=100$, $\beta=0.1$, $c=0.1, \ldots, 1$, and they are shown in Fig. 1. Observe that for all considered values of c, the problem of determination of x_1 has been solved uniquely, since for all five roots of (42), only one was less than $L/2$.

Note that the relation illustrated in Fig. 1 completely represents a physical meaning of the problem. Increase of the rings total stiffness causes a decrease of the shell deflection within interval (x_1, x_2). This uniform deflection is achieved faster on the boundary shell parts free from rings, that is in intervals $(0, x_1)$, (x_2, L).

The uniform deflection of the optimally supported shell in interval (x_1, x_2) is defined by formula $w = \sqrt{\lambda} = q x_1^4/24$. We compare this deflection with shell deflection $\bar{w}$, uniformly supported by rings with the same stiffness. In this case in the frame of the structurally orthotropic theory, we get $\varphi^2/L = c$. As it is known, in the case of a sufficiently long shell, its middle surface deflection can be defined *via* the solution to equation (1) of the form $\bar{w} = q/(\varphi^2/L + \beta)$. The computational results are given in Fig. 2.

Next, we consider a second (often met in applications) shell loaded by a hydrostatic pressure $q = px$, $p = const$. In the case of boundary conditions keeping the upper shell edge free, while its bottom edge is clamped, the following relations hold

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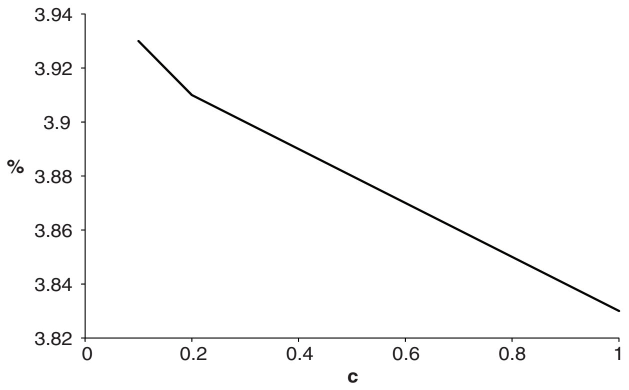

Fig. 1 Free shell parts $(0, x_1) = (x_2, L)$ (in % of shell length L) versus sum of the stiffness c of the rings for $q = const$

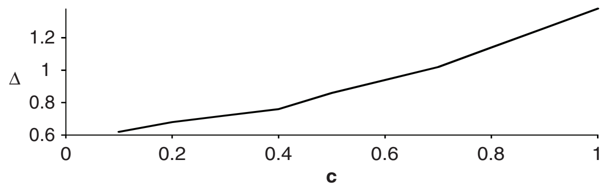

Fig. 2 Dependence of the shell deflection decrease in its middle surface for optimal support $(\Delta = (\bar{w} - w)/\bar{w}\%)$ *vs.* the sum of ring stiffness for $q = const$

$$\left(\frac{d^2 w}{dx^2}\right)_{x=0} = 0, \left(\frac{d^3 w}{dx^3}\right)_{x=0} = 0,$$
$$(w)_{x=L} = 0, \left(\frac{dw}{dx}\right)_{x=L} = 0. \quad (43)$$

This approach makes it possible to validate the efficiency of both optimal support of the shell being non-uniformly loaded as well as the chosen boundary conditions, which may differ from these considered so far. Needless to say, a proper solution to the so far formulated problem plays a key role while analysing numerous engineering problems, because shells considered by us may be used to carry large volumes of fluids.

In the considered case, formula (29) is used to conclude that stringers stiffness on the shell part (x_1, x_2) is governed by the following equation

$$\varphi^2 = \left(\frac{px}{\sqrt{\lambda}} - \beta\right) L \quad (44)$$

in order to keep the stringer-type shell support optimal.

Graphs displaying dependencies of dimensions of free and supported shell parts *versus* entire stringers stiffness c for the upper $(0, x_1)$ and bottom (x_2, L) shell edges are shown in Figs 3 and 4, respectively.

Our computations show that length of the upper non-stretched shell part is practically equal to zero.

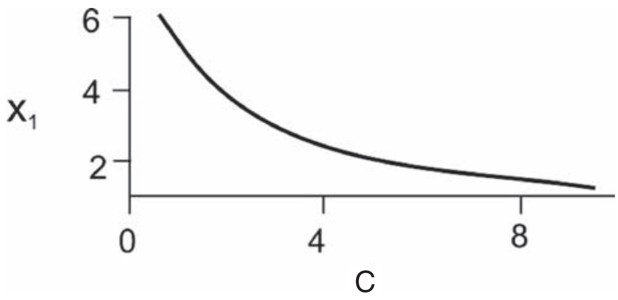

Fig. 3 Function of x_1 $\left(x_1 = \frac{x_1}{L} \cdot 10^{-4}\right)$ versus c

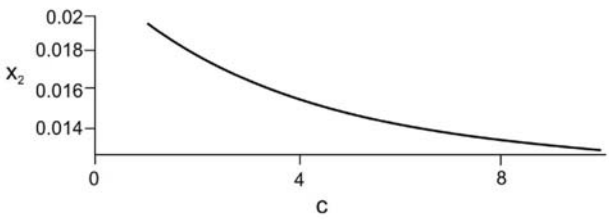

Fig. 4 Function of x_2 $\left(x_2 = \frac{x_2}{L}\right)$ versus c

Length of the lower shell part decreases with an increase of summed up stiffness c of all stringers. The latter observation is in full agreement with a physical meaning of the problem. Namely, the upper part is free and not loaded, whereas the lower part is forced by clamping and therefore an increase of the entire stringers stiffness yields a decrease of uniform deflection $\sqrt{\lambda}$, and this value is achieved on the free (not reinforced) shell part.

It is evident that the uniformly and optimally supported shell deflection $w = \sqrt{\lambda}$ is achieved in the interval (x_1, x_2). Then, we compare this deflection with shell deflection $\bar{w}$. The latter one corresponds to the shell being uniformly supported by stringers of the same stiffness (the orthotropic design oriented theory yields $\varphi^2 = c \cdot L$ in this case).

As it is well known, in the case of a sufficiently long shell, its middle surface part deflection can be estimated by a particular solution of equation (1), which in this case yields $\bar{w} = px/(\varphi^2/L + \beta)$. Results of computations are presented in the form of function $\Delta(c)$, where $\Delta = (\bar{w} - w)/\bar{w}$ per cent. They refer to the point $x = 1 - x_2$ and are shown in Fig. 5. In other words, this function presents the dependence of the deflection decrease of the optimally supported shell *versus* the summed up stiffness of all applied stringers.

Let us emphasize that the shell deflection in its point $x = 1 - x_2$ is computed through the design-oriented anisotropic theory and becomes close to the maximal value achieved by the shell uniformly supported by stringers of the same stiffness. As i.e., shown in Fig. 5, the decrease of the maximum deflection achieves more than 50 per cent and practically does not depend on the entire stiffness c of the applied stringers.

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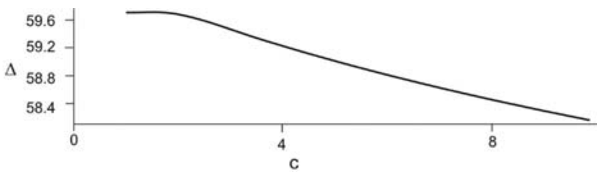

Fig. 5 Function $\Delta(c)$ *vs* c

7 CONCLUSIONS

We propose an optimization procedure of a cylindrical shell reinforced by rings carried out in two steps, that is through isolation of free (without rings) shell parts on its edges and then through reorganization of the stiffness of rings with a rule corresponding to the changes of acting load. We applied only the first step for the case of $q = const$, since $k = const$ is already optimal. For other shapes of loading distribution two steps were applied, and the obtained results improved essentially the carrying shell load ability. It should be emphasized that for an optimal support the shell deflection will be uniform regardless the applied loading and boundary conditions. Finally, the effects of loading, boundary conditions and the ratio of shell and rings stiffness are expressed by the rings stiffness variations and by dimensions of edge shell parts (free of rings) as well as by the magnitude of uniform middle shell surface deflection.

The so far proposed method can be applied also to solve direct and inverse problems of other functional gradient constructions, that is those having characteristics smoothly changed along either one or two coordinates. The mentioned structural members may include shells reinforced by ribs of non-constant stiffness, gopher-like plates and shells with variable heights of the gopher wave, and others. Another field of application of the introduced method includes the computation of physical fields of the functionally gradient composites reinforced by threads made from various materials and having variable thickness and stiffness.

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