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RESEARCH PAPER

Modified Muravskii model for elastic foundations

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Abstract A frequency equation for the vibration of an engine seating and an equation for pressure under the bottom of the engine are obtained. The present approach extends the so called Muravskii model possessing high practical accuracy of the ground modeling with its simultaneous simplicity.

Keywords Elastic foundation · Vibration · Boundary value problems

1 Introduction

In general, the problem of suppressing engine seating vibrations is reduced to a 3D contact dynamical problem [1]. Although in general one may apply numerical approaches to solve a 3D problem, say e.g. Finite Element Method (FEM), but even using high-tech computers, this approach causes various difficulties in getting a reliable solution in an economical way.

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From the viewpoint of an analytical treatment, the so far applied methods may be divided into mathematically rigorous and approximate analytical techniques. Methods of the first group are associated with the application of integral equations [2], and therefore it is necessary to prove the existence and uniqueness theorems. Next, the discussed equations are usually reduced to coupled integral equations which can be transformed to the second order Fredholm type equations by using various approaches (for instance, applying the method of orthogonal polynomials) to get finally a system of linear algebraic equations. The latter ones are tested according to their regularity and then a truncated system is solved numerically.

Practically oriented computational methods include already earlier introduced various physically motivated simplifications [3]. For instance, the engine seating mass is assumed to be concentrated to a point, or a ground reaction is uniformly distributed, and so on. The latter approach is commonly accepted in engineering society. Models proposed by Winkler [4], Pasternak and Vlasov [5,6] do not allow to get sufficiently accurate results, whereas the model of an elastic half-plane (or an elastic half-space) is not adequately complex in comparison with simplifications introduced for the foundation. On the other hand, when inertial properties of foundation are neglected while computing vertical vibrations of a plate-type engine, the seating may cause serious errors, as both experimental and numerical investigations as shown in Refs. [6–11]. Owing to the critical state-of-the art review carried out, the model proposed by Muravskii [11] seems to be one of the most appropriate ones for engineering as it exhibits high practical accuracy of the ground modeling with its simultaneous simplicity. Here some natural modification of this model are proposed. The paper is organised in the following manner. In Sect. 2 the static plane problem of elasticity for a strip subjected to a rigid punch action is studied. In Sect. 3 the boundary value problem (BVP) of the analyzed system with dynamic loading is solved. Finally in Sect. 4 some concluding remarks are given.

2 Static loading

Next, consider the plane problem of elasticity for a strip subjected to a rigid punch action (Fig. 1).

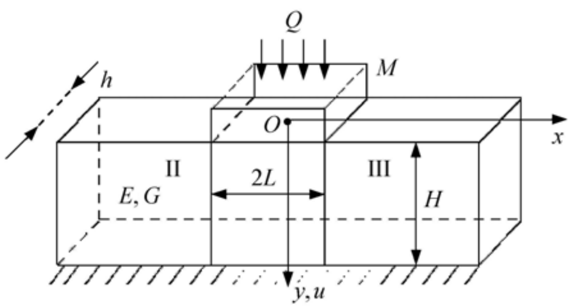

Fig. 1 Physical model of a rigid foundation on an elastic layer

The following computational scheme is applied (Fig. 2): on zone I strip is reduced to a rod with stiffness EF, where $F = 2lh$, and E denotes Young modulus; zones II and III are associated with the Muravskii model [11]. The problem defined so far is governed by the following equations

$$E[h + F\delta(x)]U_{yy} + GhU_{xx} = 0, \tag{1}$$

$$E[h + F\delta(x)]U_y = -p\delta(x), \quad \text{for } y = 0, \tag{2}$$

$$U = 0, \quad \text{for } y = H,$$

$$U \to 0, \quad \text{for } |x| \to \infty, \tag{3}$$

where $\delta(x)$ is the Dirac function; G is the shear modulus; $p = 2lhp + Mg$.

A solution to the BVP (1)–(3) is sought in the form satisfying the boundary conditions (3)

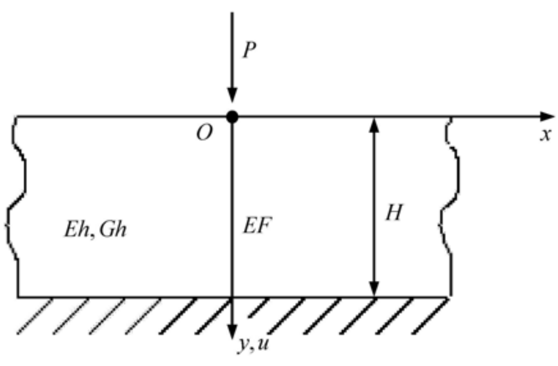

Fig. 2 Computational scheme of the static problem

$$U(x, y) = \sum_{k=1,3,5,\cdots} U_k(x) \cos\frac{k\pi y}{2H}. \tag{4}$$

Substituting Eq. (4) to Eq. (1) and after splitting regarding cosines one gets

$$E[h + F\delta(x)]\left(\frac{n\pi}{2H}\right)^2 U_n(x) - GhU_{nxx}(x) = p\delta(x),$$

$$n = 1, 3, 5, \cdots. \tag{5}$$

Applying the Fourier transformation

$$\overline{U}_n(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U_n(x) \mathrm{e}^{-\mathrm{i}xq} \mathrm{d}x \tag{6}$$

to Eq. (5), one gets

$$Eh\left(\frac{n\pi}{2H}\right)^2 \overline{U}_n(p) + Ghq^2 \overline{U}_n(p)$$

$$= \frac{1}{\sqrt{2\pi}}\left[p - EF\left(\frac{n\pi}{2H}\right)^2 U_n(x)\right]. \tag{7}$$

Using an inverse Fourier transformation governed by formula (6), one finds $U_n(q)$, and then $U_n(0)$ is defined.

Finally the function $U(x, y)$ may be written as follows

$$U(x, y) = \frac{8pH}{\pi^2 EF} \sum_{k=1,3,5,\cdots} \frac{\exp\left(-\frac{\pi\alpha k|x|}{2H}\right)}{k(k+d)} \cos\frac{k\pi y}{2H}, \tag{8}$$

where $\alpha = \sqrt{\frac{E}{G}}$, $d = 2hH(\pi\alpha F)$.

Displacement of the engine seating is given by formula (8) for $x = y = 0$ and after a summation procedure owing to a formula from Ref. [12] one obtains

$$U(0, 0) = \frac{P}{2\pi h\sqrt{EG}}\left[\psi(d+1) - \frac{1}{2}\psi\left(\frac{d+1}{2}\right)\right.$$

$$\left. + \frac{1}{2}\psi(d) + C + \ln 2\right], \tag{9}$$

where $\psi(z)$ is the so called psi-function; $\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$, Γ is the gamma-function; C is the Euler constant, $C = 0.577\,215\,664\,9\cdots$.

Define now pressure under the engine seating bottom

$$T = EFU_y\Big|_{x=0}. \tag{10}$$

Since differentiation of the series (8) term by term is not allowed, first Krylov's method is applied [13]. The identity is used

$$\frac{8pH}{\pi^2 EF} \sum_{k=1,3,5,\cdots} \frac{\cos\left(\frac{k\pi y}{2H}\right)}{k^2} = \frac{pH}{EF}\left(1 - \frac{y}{H}\right). \tag{11}$$

Finally, combine formula (8) for $x = 0$ with formula (11) to get

$$U(0,y) = \frac{pH}{EF}\left(1 - \frac{y}{H}\right) - \frac{8pHd}{\pi^2 EF} \sum_{k=1,3,5,\cdots} \frac{\cos\left(\frac{k\pi y}{2H}\right)}{k^2(k+d)}. \quad (12)$$

The series in expression (12) can be differentiated term by term and after a substitution of formula (12) into formula (10) one gets

$$T = -p + \frac{4pd}{\pi} \sum_{k=1,3,5,\cdots} \frac{\sin\left(\frac{k\pi y}{2H}\right)}{k(k+d)}. \quad (13)$$

Furthermore, shear stress for $x = 0$ is

$$\tau(y) = \frac{EF}{2} U_{yy}\Big|_{x=0} = \frac{1}{2} T_y. \quad (14)$$

In order to allow the term-by-term differentiation, the following expression are derived from the right hand side of Eq. (13)

$$\frac{4pd}{\pi} \sum_{k=1,3,5,\cdots} \frac{\sin\left(\frac{k\pi y}{2H}\right)}{k^2} = \frac{2pd}{\pi} \int_0^{\frac{\pi y}{2H}} \left[\ln\left(2\cos\frac{k\pi t}{4H}\right) - \ln\left(2\sin\frac{k\pi t}{4H}\right)\right] \mathrm{d}t. \quad (15)$$

For summation of the series, formula 5.4.2.12 in Ref. [13] is used.

Now, applying formulas (13)–(15), one gets

$$\tau(y) = \frac{pd}{H}\left[\ln\left(2\cos\frac{k\pi y}{4H}\right) - \ln\left(2\sin\frac{k\pi y}{4H}\right)\right] - 2d \sum_{k=1,3,5,\cdots} \frac{\cos\left(\frac{k\pi y}{2H}\right)}{k(k+d)}. \quad (16)$$

3 Dynamic loading

Next, consider a BVP governed by the following equations (Fig. 3)

$$E[h + F\delta(x)]U_{yy} + GhU_{xx} - \rho h U_{tt} = 0, \quad (17)$$

$$E[h + F\delta(x)]U_y = -M\delta(x)U_{tt}, \quad \text{for } y = 0, \quad (18)$$

$$U = 0, \quad \text{for } y = H, \quad (19)$$

$$U \to 0, \quad \text{for } |x| \to \infty.$$

Aimed at natural vibrations assuming

$$U = U(x,y)\exp(\mathrm{i}\omega t). \quad (20)$$

Substituting Eq. (20) into the BVP Eqs. (17)–(19) one gets

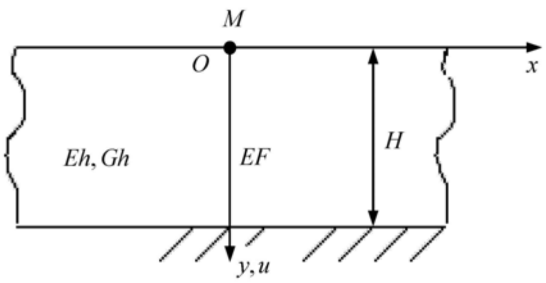

Fig. 3 Computational model of dynamical problem

$$E[h + F\delta(x)]U_{yy} + GhU_{xx} + \rho h\omega^2 U = 0, \quad (21)$$

$$E[h + F\delta(x)]U_y = -M\delta(x)\omega^2 U, \quad \text{for } y = 0, \quad (22)$$

$$U = 0, \quad \text{for } y = H, \quad (23)$$

$$U \to 0, \quad \text{for } |x| \to \infty. \quad (24)$$

Substituting the solution to the BVP equations (21)–(24) in the form of Eq. (4) one obtains

$$E[h + F\delta(x)]\left(\frac{n\pi}{2H}\right)^2 U_n(x) - GhU_n(x) + \rho h\omega^2 U_n = M\omega^2\delta(x)U_n(x). \quad (25)$$

Now, applying the Fourier transformation (6), one gets

$$E\left(\frac{n\pi}{2H}\right)^2 \overline{U}_n(q) + Ghp^2\overline{U}_n(q) - \rho h\omega^2 U = \frac{1}{\sqrt{2\pi}}\left[M\omega^2 - EF\left(\frac{n\pi}{2H}\right)^2\right]U_n(0). \quad (26)$$

Application of the inverse Fourier transformation defined by formula (6) yields

$$U_n(x) = \frac{\left[M\omega^2 - EF\left(\frac{n\pi}{2H}\right)^2\right]U_n(0)}{4Gh} \times \sqrt{\frac{E}{G}\left(\frac{n\pi}{2}\right)^2 - \frac{\rho H^2}{G}\omega^2} \times \exp\left(-\sqrt{\frac{E}{G}\left(\frac{n\pi}{2}\right)^2 - \frac{\rho H^2}{G}\omega^2}|x|\right).$$

Taking $x = 0$ the following formula for a frequency of natural vibration is finally obtained

$$\frac{M\omega^2 - EF\left(\frac{n\pi}{2H}\right)^2}{Gh}\sqrt{\frac{E}{G}\left(\frac{n\pi}{2}\right)^2 - \frac{\rho H^2\omega^2}{G}} = 1.$$

4 Conclusions

In spite of many proposals, the problem regarding modeling of engine seating vibrations is still not adequately solved. Our contribution to the challenging topic relies on extension and modification of the Muravskii foundation model.

Modification of the Muravskii foundation model allows to obtain a simple analytical solution of the static and dynamic problem for engine seating on elastic foundations.

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