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STRING AND BEAM-LIKE MODELS AND THE REDUCTION PROBLEM

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Abstract. *In this short communication two types of belt vibrations are discussed and boundaries of their application are established.*

Key words: *string, reduction, vibration, boundaries.*

The investigation of moving objects approximated by one-dimensional equations (belts, tapes, and cables) is very important from the view of applications (see [1-6] and the references therein). One may expect that the equations governing the dynamics of the given objects are properly derived for both linear and non-linear cases. However, even for the linear case, the problem is reduced to a consideration of the infinite systems of ordinary differential or algebraic equations (see examples given in references [1-4]).

In this report only a linear case is considered, although the obtained results can be easily generalized into a non-linear case.

As it has been mentioned in reference [1], the belt vibrations can be classified into two types, i.e. that of a string-like type or of a beam-like type, depending on the bending stiffness of a belt.

We are going to establish boundaries of applications of two mentioned models. For a linear case elementary transformations are needed to carry out the study. We show that the obtained linear estimations hold also for a non-linear case.

In the computational scheme, a conveyor belt is modelled by a stretched beam of length L (Figure 1).

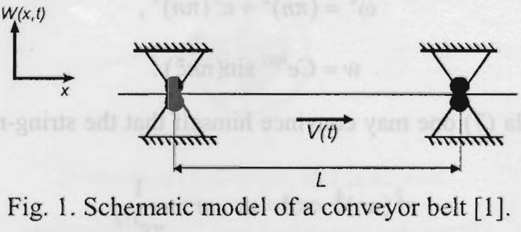

Fig. 1. Schematic model of a conveyor belt [1].

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The governing equations can be reduced to the following form [1, 2]

$$\rho F\frac{\partial^2 W}{\partial t^2}+2V\frac{\partial^2 W}{\partial x\partial t}+V^2\frac{\partial^2 W}{\partial x^2}-T\frac{\partial^2 W}{\partial x^2}+V_1\frac{\partial W}{\partial x}+EI\frac{\partial^4 W}{\partial x^4}=0, \quad (1)$$

where: $W(x,t)$ is the displacement of the belt in the vertical direction; V is the time-varying belt speed, $V_1=\partial V/\partial t$; ρ is the mass density of the belt; F and I are the area and first moment of the beam cross-section; t is time; x is spatial coordinate; T is constant tension.

The following boundary conditions are applied:

$$W=0 \quad \text{for} \quad x=0,L; \quad (2)$$

$$\frac{\partial^2 W}{\partial x^2}=0 \quad \text{for} \quad x=0,L. \quad (3)$$

In computations both equation (1) with boundary conditions (2), (3) (beam like approximation), and the so called string-like approximation governed by equation

$$\rho F\frac{\partial^2 W}{\partial t^2}+2V\frac{\partial^2 W}{\partial x\partial t}+V^2\frac{\partial^2 W}{\partial x\partial x}-T\frac{\partial^2 W}{\partial x\partial x}+V_1\frac{\partial W}{\partial x}=0 \quad (4)$$

with boundary condition (2) are applied. It should be emphasized that while solving equation (4), infinite systems appear which cannot be reduced to finite ones [1].

In what follows we show that the occurred difficulties are only of mathematical character and they do not possess any physical insight. We put $V=0$ and transform equation (1) to non-dimensional form

$$\frac{\partial^2 w}{\partial \tau^2}-\frac{\partial^2 w}{\partial \xi^2}+\varepsilon^2\frac{\partial^4 w}{\partial \xi^4}=0 \quad (5)$$

where: $w=W/h$; $\xi=x/L$; $\tau=tL\sqrt{T/\rho F}$; $\varepsilon^2=EI/(TL^2)$.

Recall that for physically motivated considerations the parameter ε is small, i.e. $\varepsilon \ll 1$.

The associated boundary conditions (2), (3) are also transformed into the equivalent non-dimensional form

$$w=\frac{\partial^2 w}{\partial \xi^2}=0 \quad \text{for} \quad \xi=1. \quad (6)$$

Eigenfrequencies and associated modes of systems (6), (7) vibrations read:

$$\omega^2=(\pi n)^2+\varepsilon^2(\pi n)^4, \quad (7)$$

$$w=Ce^{i\omega\tau}\sin(\pi n\xi). \quad (8)$$

Owing to formula (7) one may convince himself that the string-model (4) can be only applied either for

$$\varepsilon^2(\pi n)^4 \ll 1 \quad \text{or} \quad n>\frac{1}{\pi\varepsilon^{1/2}} \quad (9)$$

This observation yields a conclusion that the problem of occurrence of infinite systems associated with analysis of string-like model does not appear at all.

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MODELI TIPA STRUNE I GREDE U ZADACIMA REDUKCIJE

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U ovom kratkom radu dva tipa oscilacija trake su prikazana, kao i oblasti njihove primene.

Ključne reči: *struna, redukcija, vibracije, granice.*