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J. Awrejcewicz · Yu. Pyryev

Dynamic damper of vibrations with thermo-elastic contact

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Abstract Dynamics of a damper with two degrees of freedom (2-dof) and friction with reference to heating processes is discussed in the paper. A method to solve a nonlinear problem of thermo-elastic contacting bodies is proposed and a numerical analysis of the system kinetics is carried out.

Keywords Damper · Friction · Heating

1 Introduction

Nonlinear oscillations of mechanical systems are described extensively in a series of classic monographs [1–7], and some asymptotic methods for solution of equations of nonlinear oscillations are presented in well-known books [3,4,6,7]. They mainly address classical approaches to the study of vibrations exhibited by various engineering systems.

In many cases in engineering, the harmful effects of vibrations are suppressed by inclusion of so-called dynamic vibration dampers [8]. They have rather a wide spectrum of application and can be used to damp various longitudinal, torsional and transversal vibrations of both machines and civil engineering constructions [9]. Now, it is well known in engineering that, in order to avoid harmful effects of resonances, the majority of the externally driven mechanical systems should be damped.

A drawback of the current designs of vibration dampers is associated with heat transfer to the contacting bodies induced by frictional processes. This causes extension of the contacting bodies, and a change of contacting pressure and friction, often resulting in harmful damper wedging effects.

In this work a one-degree-of-freedom (1-dof) system driven by either a force or kinematic excitation is studied. An additional mass is added to the mentioned mechanical system via a special pressing device initiating dry friction on the contacting surfaces. Our proposed mathematical model of the described system includes thermal effects that appear on the contacting bodies [10,11]. Note that the damper geometrical properties, heat transfer between the bodies and a surrounding medium yield a change of friction on the contacting surface. We focus on a solution to the nonlinear problem of thermal stresses and strongly nonlinear equations governing the dynamics of this system. Based on the analysis, directions for the proper construction of mechanical vibration dampers are given.

J. Awrejcewicz (✉) · Y. Pyryev
Department of Automatics and Biomechanics, Technical University of Lodz,
1/15 Stefanowski St., 90-924 Lodz, Poland
E-mail: awrejcew@ p.lodz.pl
Tel: +48-42-6312225
Fax: +48-42-6312225
E-mail: jupyrjew@p.lodz.pl

2 Mathematical modeling

Below, we consider a dynamic model system with dry friction (Fig. 1). It may, for instance, model the Lanchester system with dry friction. Description of the Lanchester damper can be found in known text books, e.g., books by Den Hartog [8] and Giergiel [9]. It should be emphasized that we do not study the real Lanchester damper, but only its simplified model. A body with mass m_1 models a fundamental part of the system (shaft and bushes [8]). System vibrations are generated by harmonic force $F_1 = F_0 \sin \chi_0 t$. The following notation is used: m_1, mass; k_1, elasticity coefficient of the main system constraints; F_0, χ_0, amplitude and frequency of driving force, respectively. A damper (two coupled discs and screws [8]) of mass m_2 is added. It is coupled with mass m_1 by a pressing system and hence dry friction force F_{fr} occurs. Assume that the tested damper has the shape of a parallelepiped $(2L \times L_1 \times L_2)$ moving in direction Z_2 along the walls of the main system. The initial value of the distance between the walls, i.e. between the discs and the bush in the Lanchester system, is equal to the plate thickness $2L$ (the thickness of the friction washers [8]). Then, this distance is decreased according to the formula $2U_0 h_U(t)$, which is realized via power tightening of the screws [8], where U_0 is a constant larger than zero, and $h_U(t)$ is a known dimensionless function of time ($h_U(t) \to 1, t \to \infty$). As a result of this process, dry friction occurs on the parallelepiped surfaces $X = \pm L$. It is defined by the function $F_{\text{fr}}(V_r,\ Z_1)$, where V_r is the relative velocity of the plate and walls, i.e. $V_r = Z_1' - Z_2' (\mathrm{d}Z_i / \mathrm{d}t \equiv Z_i',\ i = 1,\ 2)$. According to Amonton's assumption, the friction force $F_{\text{fr}} = 2 f(V_r) P$ is equal to the product of the normal reaction component and the friction coefficient, $f(V_r)$ denotes the kinetic friction coefficient $[f(-V_r) = -f(V_r)]$, and we take $f(V_r) = f_s \operatorname{sgn}(V_r)$ for $V_r \neq 0$. The action of friction on the contact surface $X = \pm L$ generates heat. We follow the generally accepted assumption [12] that friction work is transformed into heat energy. Furthermore, we assume that walls ideally transform heat and that between the plate and walls the heat transfer is governed by Newton's law, and that the surrounding medium temperature is equal T_0. Plate surfaces not in contact with movable walls are thermally isolated. It is assumed that the contact areas have dimensions for which $L/L_1 \ll 1$, $L/L_2 \ll 1$. These assumptions allow us to introduce a one-dimensional model. Both thermal and stress–strain states of the plate are considered using the rectangular coordinates $0XYZ$. The governing equation of motion of uncoupled thermoelastic problem have the following form [13]:

$$\mu_2 \nabla^2 \mathbf{u} + (\lambda_2 + \mu_2) \operatorname{grad} \operatorname{div} \mathbf{u} = (3\lambda_2 + 2\mu_2)\alpha_2 \operatorname{grad} T + \rho_2 \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad \nabla^2 T = \frac{1}{a_2} \frac{\partial T}{\partial t},$$

where ∇^2 is the Laplace operator, $\mathbf{u} = U\mathbf{e}_X + V\mathbf{e}_Y + W\mathbf{e}_Z$ is the vector of relative displacement plate, U is the displacement in the X direction, T is the plate temperature, λ_2 and μ_2 are the Lamè coefficients, ρ_2 is the

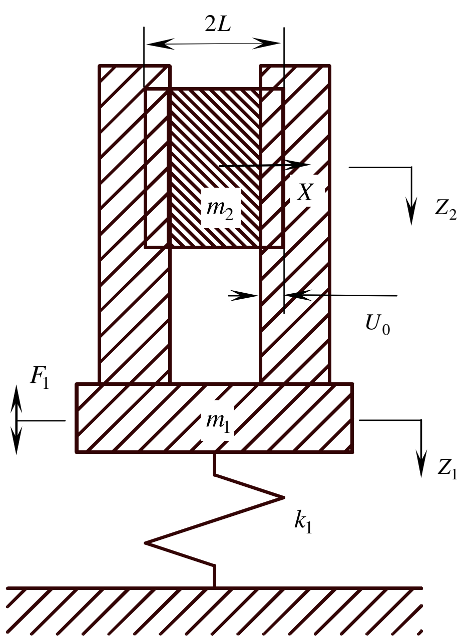

Fig. 1 Dynamical model of the system with dry friction

plate density, α_2 is the coefficient of thermal linear expansion of the plate, and a_2 is thermal diffusivity of the plate material.

We consider a one-dimensional model taking into account the following assumptions: (i) the external excitation of the system allows for neglecting of the term $\rho_2 \partial^2 \mathbf{u} / \partial t^2$ in the Lamè equation; (ii) the vector components related to displacements as well as the plate temperature do not depend on Y, Z, and the unequal zero components $U(X, t)$, $W(X, t)$ and $T(X, t)$ depend only on the X coordinate and time; and (iii) the heat flows q_1 and q_2 are generated on the contact surface $X = \pm L$ according to the Ling rule [12] and they are governed by the equation $q_1 + q_2 = f(V_r)V_r P(t)$. Both flows q_1 and q_2 go into the plate and wall, respectively: $q_1 = \pm\lambda_2 \partial T(\pm 1, t) / \partial X$, $q_2 = -\alpha^T (T_0 - T(\pm 1, t))$, where λ_2 is the thermal conductivity of the plate material, α^T is the heat transfer coefficient between the plate and the wall.

The governing equations of the system presented in Fig. 1 have the following form (see [8,10])

$$m_1 Z_1'' + k_1 Z_1 + F_{\mathrm{fr}}(Z_1' - Z_2',\ Z_1) = F_1, \quad m_2 Z_2'' - F_{\mathrm{fr}}(Z_1' - Z_2',\ Z_1) = 0, \tag{1}$$

with the friction model

$$F_{\mathrm{fr}}(V_r,\ Z_1) = \begin{cases} 2 f_s \operatorname{sgn}(V_r) P(t), & V_r \neq 0,\ \text{slip}, \\ \min\left(\frac{m_2}{m_1 + m_2}|F_1 - k_1 Z_1|,\ 2 f_s P(t)\right) \operatorname{sgn}(F_1 - k_1 Z_1), & V_r = 0,\ \text{stick}. \end{cases} \tag{2}$$

In order to solve Eqs. (1) and (2) knowledge of the contact pressure $P(t)$ is required. For this purpose we introduce some assumptions, and the following equation governing the theory of thermal stresses for an isotropic body [13] is solved first:

$$\frac{\partial}{\partial X}\left[\frac{\partial U(X, t)}{\partial X} - \alpha_2 \frac{1 + \nu_2}{1 - \nu_2} T(X, t)\right] = 0, \tag{3}$$

$$\frac{\partial^2 T(X, t)}{\partial X^2} = \frac{1}{a_2} \frac{\partial T(X, t)}{\partial t}, \quad X \in (-L, L), \tag{4}$$

with the associated mechanical

$$U(\mp L, t) = \pm U_0 h_U(t), \tag{5}$$

and heat boundary conditions

$$\mp\lambda_2 \frac{\partial T(\mp L, t)}{\partial X} + \alpha^T (T(\mp L, t) - T_0) = f(V_r) V_r P(t), \tag{6}$$

as well as zero initial conditions and $T(X, 0) = T_0$, $X \in (-L, L)$.

Normal stress occurring in the plate can be found knowing both the displacement $U(X,\ t)$ and temperature $T(X,\ t)$ in the plate [13], since

$$\sigma_{XX}(X, t) = \frac{E_2}{1 - 2\nu_2}\left[\frac{1 - \nu_2}{1 + \nu_2} \frac{\partial U(X, t)}{\partial X} - \alpha_2 (T(X, t) - T_0)\right]. \tag{7}$$

In the above, the following notation is taken: E_2, Young's modulus of the plate; ν_2, Poisson's ratio of the plate; while $P(t) = -\sigma_{XX}(\pm L, t)$ denotes the contact pressure. Quantities m_1, m_2, k_1, P, F_1 are measured per unit of the contact surface $S = L_1 \times L_2$ of the moving rigid plate and the wall. Note that, when the damper is neglected, the considered system is reduced to that with 1-dof with a natural frequency of $\omega_{01} = \sqrt{k_1 / m_1}$.

Integration of Eq. (3) with Eq. (7) and the boundary conditions (5) gives the contact pressure $P(t) = -\sigma_{XX}(\pm L, t)$ in the form

$$P(t) = \frac{E_2 (1 - \nu_2) U_0 h_U(t)}{(1 + \nu_2)(1 - 2\nu_2) L} + \frac{E_2 \alpha_2}{(1 - 2\nu_2) L} \frac{1}{2} \int_{-L}^{L} (T(\xi, t) - T_0) \mathrm{d}\xi. \tag{8}$$

The motion of the investigated system depends on both the ratios F_{fr} / F_0 and χ_0 / ω_{01}. For various values of the F_{fr} / F_0 ratio the system moves with one or more sticks within half of the period of motion. It should be emphasized that the exact solution of this problem without tribological processes and the case with one stop and without stops has been already reported by Den Hartog [8].

3 Dimensionless differential and integral equations

Let us introduce the following similarity coefficients

$$t_* = 1\big/\omega_{01}\ [\mathrm{s}],\quad L_* = \frac{F_0}{k_1}\ [\mathrm{m}],\quad N_* = \frac{E_2(1-\nu_2)U_0 S}{(1+\nu_2)(1-2\nu_2)L}\ [\mathrm{N}],\quad T_* = \frac{(1-\nu_2)U_0}{\alpha_2(1+\nu_2)L}[^\circ\mathrm{C}], \tag{9}$$

and the following dimensionless parameters

$$x = \frac{X}{L},\quad \tau = \frac{t}{t_*},\quad \theta = \frac{T-T_0}{T_*},\quad p = \frac{P\,S}{N_*},\quad z_n = \frac{Z_n}{L_*},\quad n=1,2,\ \ \varepsilon = \frac{2N_* f_s}{F_0 S},$$
$$Bi = \frac{\alpha^T L}{\lambda_2},\quad \tau_T = \frac{t_*}{t_T},\quad l = \frac{L_*}{L},\quad \gamma = \frac{E_2\alpha_2 a_2}{(1-2\nu_2)\lambda_2},\quad \Omega = \frac{\Omega_1}{\tau_T},\quad \Omega_1 = \gamma l,$$
$$\omega_0 = \frac{\chi_0}{\omega_{01}},\quad \kappa = \frac{C}{2m_2\omega_{01}},\quad \mu = \frac{m_2}{m_1},\quad f(L_* t_*^{-1}\mathrm{v}_r) = f_s F(\mathrm{v}_r),$$

$$\mathrm{v}_r = \dot z_1 - \dot z_2,\quad \frac{dz_i}{d\tau} \equiv \dot z_i,\quad i=1,2\ \ b_1 = \frac{B_1}{L_*},\quad b_2 = \frac{B_2}{L_*}, \tag{10}$$

where $F(\mathrm{v}_r) = \mathrm{sgn}(\mathrm{v}_r)$ for $\mathrm{v}_r \neq 0$, $t_T = L^2\big/a$ is the characteristic time of thermal inertia; Bi is the Biot number. The dimensionless parameter ε represents the friction force, γ governs the body heat extension, and parameter Ω is responsible for heat generation on the surface contact.

In the dimensionless form the mathematical model reads

$$\ddot z_1 + z_1 + f_{\mathrm{fr}}(\dot z_1 - \dot z_2,\ z_1) = \sin(\omega_0\tau),\quad \mu\ddot z_2 - f_{\mathrm{fr}}(\dot z_1 - \dot z_2,\ z_1) = 0, \tag{11}$$

$$\frac{\partial^2\theta(x,\tau)}{\partial x^2} = \frac{1}{\tau_T}\frac{\partial\theta}{\partial\tau},\quad x\in(-1,\ 1),\ \ \tau\in(0,\infty), \tag{12}$$

with the friction model

$$f_{\mathrm{fr}}(\mathrm{v}_r,\ z_1) = \begin{cases} \varepsilon F(\mathrm{v}_r)p(\tau), & \mathrm{v}_r \neq 0,\ \text{slip},\\ \min\left(\frac{\mu}{1+\mu}|\sin(\omega_0\tau) - z_1|,\ \ \varepsilon p(\tau)\right)\mathrm{sgn}\left(\sin(\omega_0\tau) - z_1\right), & \mathrm{v}_r = 0,\ \text{stick}, \end{cases} \tag{13}$$

with the following boundary

$$\left[\frac{\partial\theta(x,\tau)}{\partial x} \mp \mathrm{Bi}\theta(x,\tau)\right]_{x=\mp 1} = \mp q(\tau), \tag{14}$$

and initial conditions

$$\theta(x,0)=0,\quad z_1(0)=0,\quad \dot z_1(0)=0,\quad z_2(0)=0,\quad \dot z_2(0)=0, \tag{15}$$

where:

$$q(\tau) = \Omega F(\dot z_1 - \dot z_2)p(\tau)(\dot z_1 - \dot z_2), \tag{16}$$

$$p(\tau) = h_U(\tau) + \frac{1}{2}\int_{-1}^{1}\theta(\xi,\tau)\mathrm{d}\xi. \tag{17}$$

In order to solve problems (12) and (14), the Laplace transformation is applied with respect to time τ. The theorem on convolution is used [14] to find an inverse transform. Finally, we get

$$p(\tau) = h_U(\tau) + \Omega_1\int_0^\tau F(\dot z_1 - \dot z_2)p(\xi)(\dot z_1 - \dot z_2)\dot G_p(\tau-\xi)\mathrm{d}\xi, \tag{18}$$

$$\theta(x,\tau) = \Omega_1\int_0^\tau F(\dot z_1 - \dot z_2)p(\xi)(\dot z_1 - \dot z_2)\dot G_\theta(x,\tau-\xi)\mathrm{d}\xi, \tag{19}$$

where:

$$\left\{G_p(\tau), G_\theta(\pm 1, \tau)\right\} = \frac{1}{\tau_T Bi} - \sum_{m=1}^{\infty} \frac{\left\{2Bi,\ 2\mu_m^2\right\} \exp(-\tau_T \mu_m^2 \tau)}{\tau_T \mu_m^2 (Bi(Bi+1) + \mu_m^2)}. \tag{20}$$

$\mu_m (m = 1, 2, 3, \ldots)$ are the roots of the characteristic equation $\tan(\mu) = Bi/\mu$. The functions $G_p(\tau)$, $G_\theta(\pm 1, \tau)$ have the following asymptotic estimations

$$G_p(\tau) \approx \tau, \quad G_\theta(\pm 1, \tau) \approx 2\sqrt{\tau/\tau_T \pi},\ \ \tau \to 0, \tag{21}$$

$$\left\{G_p(\tau), G_\theta(\pm 1, \tau)\right\} \approx 1/(\tau_T Bi),\ \ \tau \to \infty. \tag{22}$$

Observe that the considered problem is reduced to the system of nonlinear differential equations (11) and (13), and the integral equation (18) describing to dimensionless velocities $\dot{z}_1(\tau)$ and $\dot{z}_2(\tau)$, and the dimensionless contact pressure $p(\tau)$. The dimensionless temperature $\theta(r, t)$ is governed by Eq. (19).

4 Analysis

4.1 Solution without heating

We take $\Omega_1 = 0$. Since $P(t) = N_*/S = P_*$, one gets

$$\ddot{z}_1 + z_1 + \varepsilon F(\dot{z}_1 - \dot{z}_2) = \sin(\omega_0 \tau), \tag{23}$$

$$\ddot{z}_2 - \varepsilon \mu^{-1} F(\dot{z}_1 - \dot{z}_2) = 0. \tag{24}$$

In order to solve the problem the method of equivalent linearization is applied. Assuming that the system motion is close to harmonic, one may apply linearization and use an equivalent viscous damping instead of dry friction. In other words, the equivalent damping is found by comparing the energy loss in a real A_T and in an equivalent A_C viscous system over the period $T_0 = 2\pi/\chi_0$.

Assuming the harmonic motion

$$z_1(\tau) = b_{11} \sin(\omega_0 \tau) + b_{12} \cos(\omega_0 \tau) = b_1 \sin(\omega_0 \tau + \varphi_1),$$

$$z_2(\tau) = b_{21} \sin(\omega_0 \tau) + b_{22} \cos(\omega_0 \tau) = b_2 \sin(\omega_0 \tau + \varphi_2), \tag{25}$$

the relative displacement is

$$z_1 - z_2 = d \sin(\omega_0 \tau + \psi)$$

where:

$$d = \sqrt{(b_{11} - b_{21})^2 + (b_{12} - b_{22})^2}. \tag{26}$$

Comparing the work done by the two dampers one gets

$$A_T = \int_{t_1}^{t_1+T_0} 2f(\dot{Z}_1 - \dot{Z}_2) P_* (\dot{Z}_1 - \dot{Z}_2) \mathrm{d}t, \quad A_C = \int_{t_1}^{t_1+T_0} C(\dot{Z}_1 - \dot{Z}_2)^2 \mathrm{d}t, \tag{27}$$

whereas comparing the works $A_T = 8P_* f_s L_* d$ and $A_C = C\pi \chi_0 L_*^2 d^2$ one gets the following equivalent dimensional

$$C = \frac{8P_* f_s}{\pi \chi_0 L_* d} \tag{28}$$

or non-dimensional damping coefficient of the form

$$\kappa = \frac{2\varepsilon}{\pi \mu \omega_0 d}. \tag{29}$$

Taking into account the equivalent damping coefficient, the governing equations (23) and (24) assume the form

$$\ddot{z}_1 + z_1 + 2\mu\kappa(\dot{z}_1 - \dot{z}_2) = \sin(\omega_0\tau), \tag{30}$$

$$\ddot{z}_2 - 2\kappa(\dot{z}_1 - \dot{z}_2) = 0. \tag{31}$$

A solution to Eqs. (30) and (31) is given by (25), where the corresponding dimensionless amplitudes are

$$b_{11} = \frac{(1-\omega_0^2(1+\mu))4\kappa^2 + \omega_0^2(1-\omega_0^2)}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}, \tag{32}$$

$$b_{12} = -\frac{2\mu\kappa\omega_0^3}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}, \tag{33}$$

$$b_{21} = \frac{(1-\omega_0^2(1+\mu))4\kappa^2}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}, \tag{34}$$

$$b_{22} = -\frac{2\omega_0\kappa(1-\omega_0^2)}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}, \tag{35}$$

$$b_1 = \sqrt{\frac{\omega_0^2 + 4\kappa^2}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}}, \tag{36}$$

$$b_2 = \sqrt{\frac{4\kappa^2}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}}. \tag{37}$$

According to (26) the dimensionless amplitude d is

$$d = \sqrt{\frac{\omega_0^2}{(1-\omega_0^2(1+\mu))^2 4\kappa^2 + \omega_0^2(1-\omega_0^2)^2}}. \tag{38}$$

According to formula (38), d depends on κ, and the equivalent damping depends on d [see (29)]. Solving (29), (38) one gets

$$\kappa = \frac{\omega_0\left|1-\omega_0^2\right|}{2}\sqrt{\frac{\omega_+\omega_-}{(\omega_0^2-\omega_+)(\omega_- - \omega_0^2)}}. \tag{39}$$

The amplitudes are

$$b_1 = \frac{1}{\omega_0\left|1-\omega_0^2\right|}\sqrt{\omega_0^2\left(1-\left(1+\frac{2}{\mu}\right)\left(\frac{4\varepsilon}{\pi}\right)^2\right)+\frac{2}{\mu}\left(\frac{4\varepsilon}{\pi}\right)^2}, \quad b_2 = \frac{4\varepsilon}{\pi\mu\omega_0^2} \tag{40}$$

where:

$$\omega_+ = \frac{\varepsilon}{(1+\mu)(\varepsilon+\varepsilon_0)}, \quad \omega_- = \frac{\varepsilon}{(1+\mu)(\varepsilon-\varepsilon_0)}, \quad \varepsilon_0 = \frac{\pi}{4}\left(1-\frac{1}{1+\mu}\right). \tag{41}$$

For $\varepsilon < \varepsilon_0$ and $\sqrt{\omega_+} < \omega_0$, dry friction does not limit the resonance amplitude, and for $\omega_0 \to 1$, $b_1 \to \infty$. In the case $\varepsilon_0 < \varepsilon < \pi/4$ and for $\sqrt{\omega_+} < \omega_0 < \sqrt{\omega_-}$ dry friction does not limit resonance amplitude either and for $\omega_0 \to 1$, $b_1 \to \infty$. When $\varepsilon > \pi/4$, we get the frequency interval $\sqrt{\omega_+} < \omega_0 < \sqrt{\omega_-}$, where the resonance frequency $\omega_0 = 1$ does not appear.

For $\mu \to \infty (m_2 \to \infty, \varepsilon_0 = \pi/4)$ we obtain the case of 1-dof vibration with friction considered by Den Hartog [8], where:

$$b_1 = \frac{1}{\left|1-\omega_0^2\right|}\sqrt{1-\left(\frac{4\varepsilon}{\pi}\right)^2}. \tag{42}$$

Observe that for $\varepsilon < \pi/4$ (b_1 is a real number) one gets: $\omega_0 \to 1, b_1 \to \infty$.

Now we consider the problem of optimal damping of vibrations. We assume that the body of mass m_1 has velocity $\dot{z}_1(\tau) = \omega_0 b_1 \cos(\omega_0\tau + \varphi_1)$. Note that the velocity of body m_2 is governed by

$$\dot{z}_2(\tau) = \frac{\varepsilon}{\mu}\tau + C_+, \quad \text{if } \dot{z}_1 - \dot{z}_2 > 0, \tag{43}$$

$$\dot{z}_2(\tau) = -\frac{\varepsilon}{\mu}\tau + C_-, \quad \text{if } \dot{z}_1 - \dot{z}_2 < 0. \tag{44}$$

The constants $C_+,\ C_-$ are different for all intervals of motion. Assuming that only sliding occurs, the work done within one period by the damping force $2f_s P_* \operatorname{sgn}(\dot{Z}_1 - \dot{Z}_2)$ is [8]:

$$A_T = 4F_0 L_* b_1 \varepsilon \sqrt{1 - \frac{\pi^2}{4}\left(\frac{\varepsilon}{\mu\omega_0^2 b_1}\right)^2}. \tag{45}$$

An optimal value of friction is obtained from the following equation $\frac{\partial A_T}{\partial \varepsilon} = 0$, if $\left(\partial^2 A_T/\partial\varepsilon^2\big|_{\varepsilon=\varepsilon_{\text{opt}}} < 0\right)$.

The optimal dimensionless friction is

$$\varepsilon_{\text{opt}} = \frac{\sqrt{2}}{\pi}\mu\omega_0^2 b_1, \tag{46}$$

and

$$A_{T\,\text{opt}} = \frac{4}{\pi} F_0 L_* b_1^2 \mu\omega_0^2. \tag{47}$$

The obtained value corresponds to the maximally damped fundamental system. However, in this case the amplitude b_1 should be known (say, from an experiment).

4.2 Numerical analysis

Numerical analysis of the considered problem [differential equations (11), (13) and integral equation (18)] is carried out using the Runge–Kutta method and the method of quadrature with estimations (21). Estimation (21) is required to to compute kernels of the integral equations at zero. The temperature on the contact surface is given by formula (19). Observe that during temperature computation the kernel defined by formula (21) possesses the singularity $\sim 1/\sqrt{\tau}$, which can be integrated. The function $\operatorname{sgn}(\mathrm{v}_r)$ has been approximated in the following way

$$\operatorname{sgn}(\mathrm{v}_r) = \begin{cases} 1, & \mathrm{v}_r > \delta, \\ \left(2 - \dfrac{|\mathrm{v}_r|}{\delta}\right)\dfrac{\mathrm{v}_r}{\delta}, & |\mathrm{v}_r| < \delta, \\ -1, & \mathrm{v}_r < -\delta \end{cases}$$

where: $\delta = 0.0001$.

If heat is not generated by friction ($\gamma = 0$), then the contact pressure $p(\tau) = h_U(\tau)$. Let us assume that $h_U(\tau) = H(\tau)$, where $H(\cdot)$ is the Heaviside step function ($H(\tau) = 1,\ \tau \geq 0, H(\tau) = 0,\ \tau < 0$). The previous analysis (Sect. 4.1) indicates that resonance occurs in the system. Taking $\mu = 0.5$, $\varepsilon = 0.5$ and using (41) one finds $\varepsilon_0 = 0.26$, $\omega_+ = 0.44$, $\omega_- = 1.4$. We have $\varepsilon > \varepsilon_0$, $\sqrt{\omega_+} < \omega_0 < \sqrt{\omega_-}$ and for $\omega_0 = 1$ the system is in resonance. In Fig. 2a for a lack of heat extension ($\Omega_1 = 0$), time histories of both dimensionless velocities $\dot{z}_1$ and $\dot{z}_2$, and dimensionless relative velocity (Fig 2b) $\mathrm{v}_r = \dot{z}_1 - \dot{z}_2$ are reported.

For the general case, numerical computations were carried out for various values of the parameters μ, Ω_1. Figures 3, 4, 5 and 6 show results of the numerical analysis for $\Omega_1 = 0.1$ and $Bi = 1$, $\tau_T = 0.1$. In Fig. 3a the dimensionless displacements z_1 of the body with mass m_1 and z_2 of the damper with mass m_2 versus time τ are shown (the same is done for the velocities in Fig. 3b). Figure 4 illustrates the dimensionless relative velocities v_r of two bodies versus dimensionless time τ. The system oscillations are out of resonance and

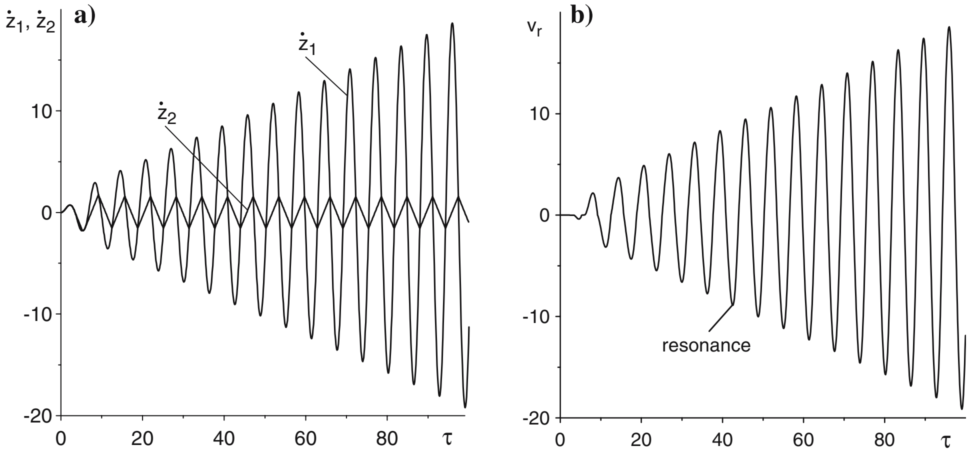

Fig. 2 The dimensionless velocity $\dot{z}_1(\tau)$ of the fundamental body and velocity $\dot{z}_2(\tau)$ of the damper **(a)** versus dimensionless time τ during resonance ($\omega_0 = 1$); dimensionless dependence of sliding velocity $v_r(\tau) = \dot{z}_1(\tau) - \dot{z}_2(\tau)$ versus time τ **(b)** ($\Omega_1 = 0, \mu = 0.5, \varepsilon = 0.5, \omega_0 = 1, \Omega_1 = 0$)

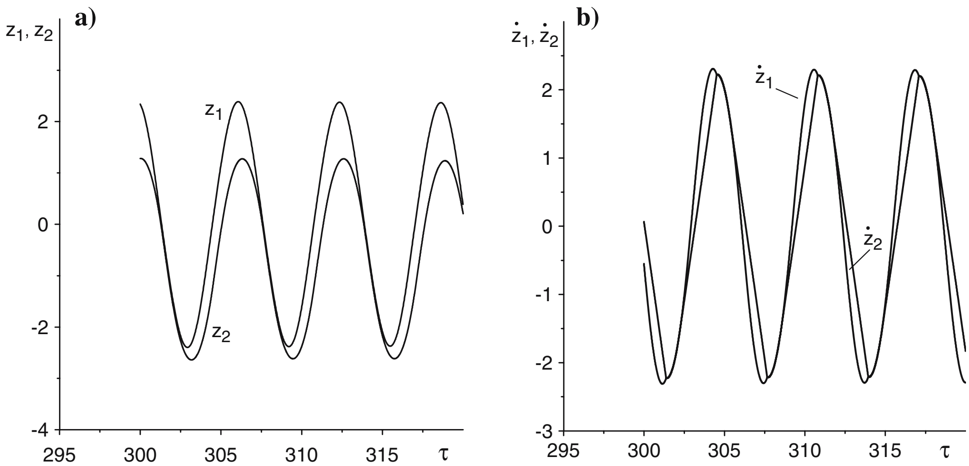

Fig. 3 The dependence of the dimensionless displacements z_1 of the fundamental body and the displacements z_2 of the damper **(a)** the dimensionless velocity $\dot{z}_1(\tau)$ of the fundamental body and the velocity $\dot{z}_2(\tau)$ of the damper **(b)** versus dimensionless time τ for $\tau \in (300,\ 320)$, taking into account heat generation ($\Omega_1 = 0.1, \mu = 0.5, \varepsilon = 0.5, Bi = 1, \tau_T = 0.1, \omega_0 = 1$)

they reach the periodic attractor (Figs 3, 4b) with period $T_1 = 2\pi/\omega_0 = 2\pi$, whereas the damper undergoes stick–slip oscillations (Fig. 4b). Evolutions of the dimensionless contact pressure $p(\tau)$ and the temperature $\theta(\tau)$ on the contacting surface are shown in Fig 5. Both of these characteristics change periodically with the dimensionless period of $T_2 = \pi$. The same period is seen for the variable $|v_r|$, occurring in (16), which governs heat generation on the sliding surface.

In order to investigate how the damper mass influences the system motion we also investigated our system for $\mu = 2(m_2 = 2m_1)$. In this case one gets $\varepsilon_0 = 0.52, \omega_+ = 0.16$. We have $\varepsilon < \varepsilon_0, \sqrt{\omega_+} < \omega_0$ and for $\omega_0 = 1$ the system is in resonance assuming that the heat extension is omitted. In Figs. 6 and 7 time evolutions of the contact characteristics are reported. In Fig. 6a the dependence of dimensionless displacement z_1 and z_2 versus dimensionless time is shown, whereas Fig. 6b illustrates the corresponding relative velocity. Figure 7 shows dimensionless the contact pressure and temperature versus dimensionless time. Note that an increase of the parameter μ causes a decrease of the contact time (see Figs. 4b, 6b) and the vibration amplitude decreases (see Figs. 3a, 6a), but the temperature amplitude in the periodic state increases (see Figs. 5b, 7b).

A numerical analysis of the results for $\Omega_1 = 0.2, \mu = 0.5, \varepsilon = 0.5, Bi = 1, \tau_T = 0.1$ is illustrated in Figs. 8 and 9. An increase of the coefficient Ω_1 causes an increase of time τ_r (the so-called time of passive regulation), when the trajectory achieves periodic motion. For $\Omega_1 = 0.1$ time $\tau_r \approx 250$, whereas for $\Omega_1 = 0.2$

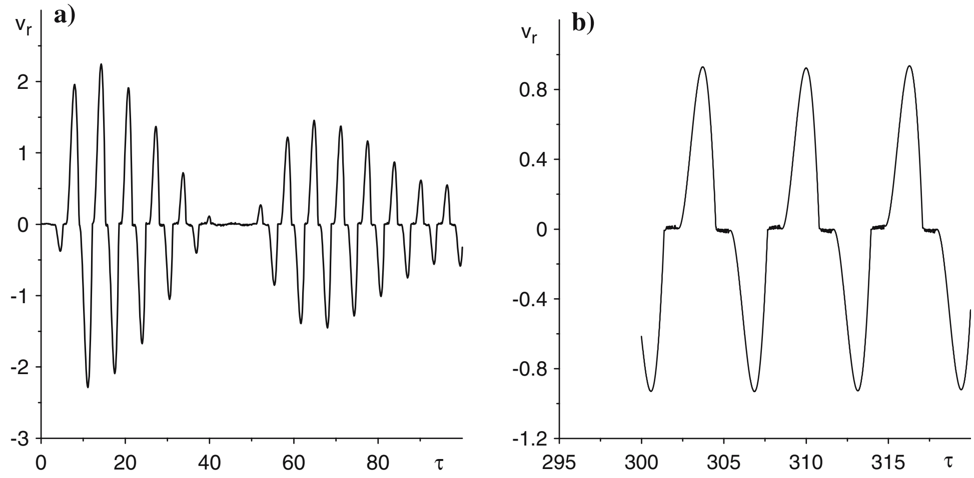

Fig. 4 The dimensionless sliding velocity $v_r(\tau) = \dot{z}_1(\tau) - \dot{z}_2(\tau)$ versus time τ for $\tau \in (0,\ 100)$ **(a)** and for $\tau \in (300,\ 320)$ **(b)** taking into account heat generation ($\Omega_1 = 0.1$, $\mu = 0.5$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

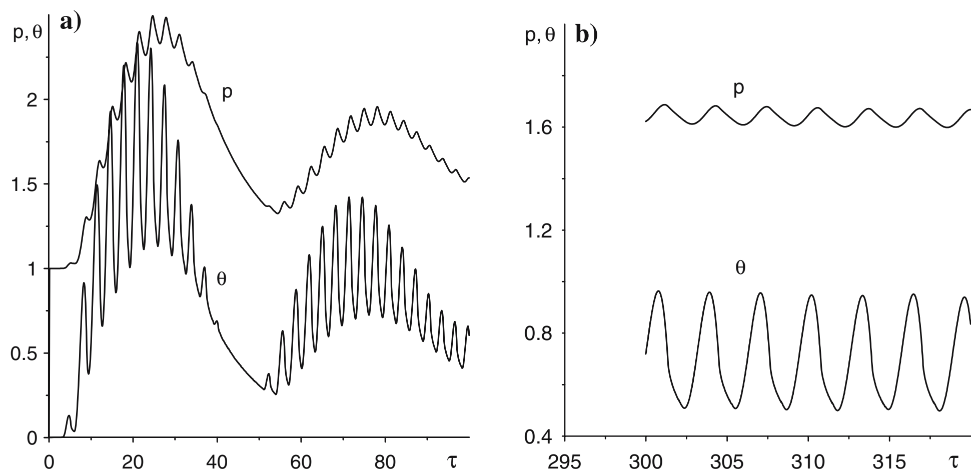

Fig. 5 Time histories of dimensionless contact pressure $p(\tau)$ and contact surface temperature $\theta(\tau)$ for $\tau \in (0,\ 100)$ **(a)** and for $\tau \in (300,\ 320)$ **(b)**, taking into account heat generation ($\Omega_1 = 0.1$, $\mu = 0.5$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

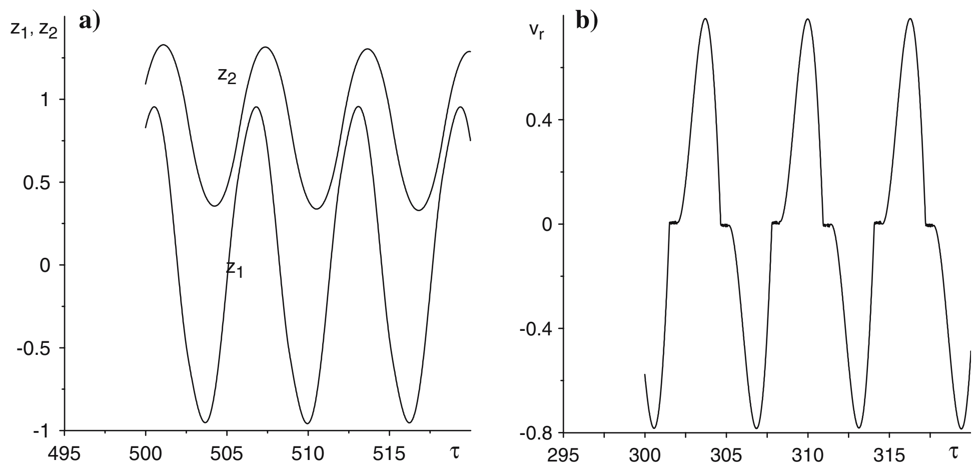

Fig. 6 The dependence of the dimensionless displacements z_1 of the fundamental body and displacements z_2 of the damper **(a)** the dimensionless dependence of sliding velocity $v_r(\tau) = \dot{z}_1(\tau) - \dot{z}_2(\tau)$ **(b)** versus dimensionless time τ for $\tau \in (300,\ 320)$, taking into account heat generation ($\Omega_1 = 0.1$, $\mu = 2$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

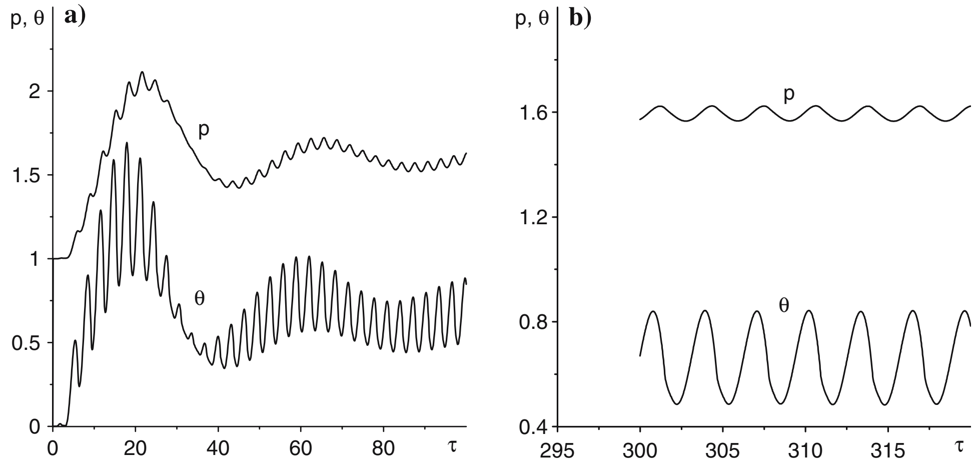

Fig. 7 Time histories of dimensionless contact pressure $p(\tau)$ and contact surface temperature $\theta(\tau)$ for $\tau \in (0,\ 100)$ **(a)** and for $\tau \in (300,\ 320)$ **(b)**, taking into account heat generation ($\Omega_1 = 0.1$, $\mu = 2$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

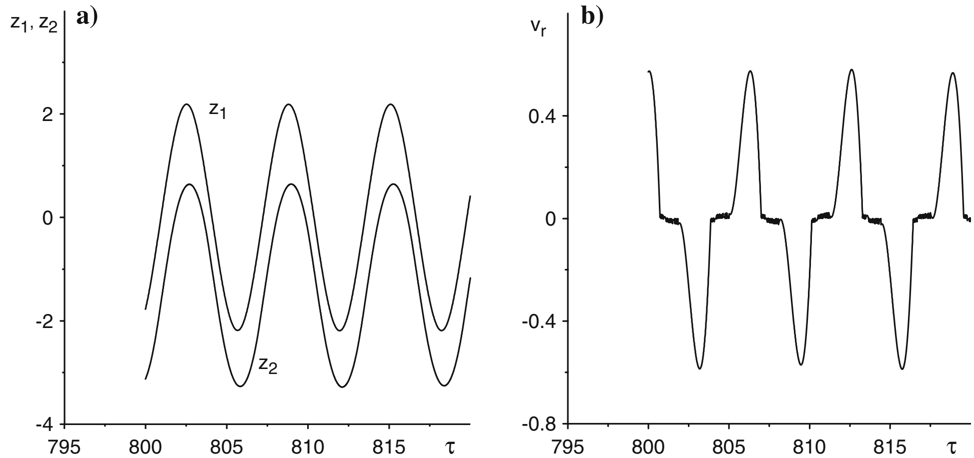

Fig. 8 The dependence of the dimensionless displacements z_1 of the fundamental body and the displacements z_2 of the damper **(a)** the dimensionless dependence of sliding velocity $\mathrm{v}_r(\tau) = \dot{z}_1(\tau) - \dot{z}_2(\tau)$ **(b)** versus dimensionless time τ for $\tau \in (800,\ 820)$, taking into account heat generation ($\Omega_1 = 0.2$, $\mu = 0.5$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

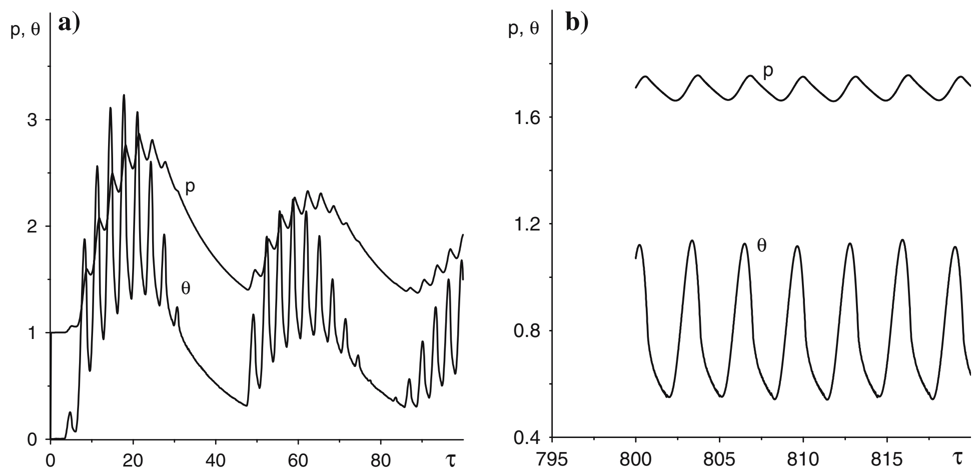

Fig. 9 Time histories of the dimensionless contact pressure $p(\tau)$ and the contact surface temperature $\theta(\tau)$ for $\tau \in (0,\ 100)$ **(a)** and for $\tau \in (800,\ 820)$ **(b)**, taking into account heat generation ($\Omega_1 = 0.2$, $\mu = 0.5$, $\varepsilon = 0.5$, $Bi = 1$, $\tau_T = 0.1$, $\omega_0 = 1$)

time $\tau_r \approx 500$. Additionally, the contact time of both bodies also increases (see Figs. 4b, 8b). An increase of Ω_1 causes a decrease of the relative velocity amplitude (see Figs. 4b, 8b).

5 Conclusions

A dynamic 2-dof damper with dry friction and heat generation has been modeled mathematically. The proposed method of solution may also be applied to model any other nonlinear problem of dynamics of thermo-elastic contacting bodies. A series of practical results regarding the kinetics of the main system and of the dynamic damper are formulated as a result of the analysis of various contact characteristics (contact pressure, temperature on the contacting surface).

It should be emphasized that the dynamic damper with dry friction may not achieve the expected properties. As we have shown, heat generation on the contacting surface between the damper and the oscillating body as well as heat expansion eliminate, for certain parameters, resonance phenomena. The real system, in certain conditions, behaves as a self-regulating one, i.e. it controls achievement of an optimal contacting pressure. The thermo-elastic parallelepiped extends itself according to the conditions of both sliding velocity and heat transfer.

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