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Dynamics of a two-degrees-of-freedom system with friction and heat generation

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Abstract

A novel thermomechanical model of frictional self-excited stick-slip vibrations is proposed. A mechanical system consisting of two masses which are coupled by an elastic spring and moving vertically between two walls is considered. It is assumed that between masses and walls a Coulomb friction occurs, and stick-slip motion of the system is studied. The applied friction force depends on a relative velocity of the sliding bodies. Stability of stationary solutions is considered. A computation of contact parameters during heating of the bodies is performed. The possibility of existence of frictional auto-vibrations is illustrated and discussed.

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1. Introduction

Stick-slip motion is intimately related to the nature of frictional phenomena and is often attributed to the difference between the static and kinematic friction coefficients. Even though the topic

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of friction is a relatively old one and plays an important role in many practical and engineering applications, surprisingly it is not as well understood as might be expected.

Research reported in this paper extends the authors' earlier results, where both regular and chaotic vibrations in a cylinder-bush system have been analyzed [2–6]. In addition, two-degrees-of-freedom system, where a friction force depends on a distance between two masses, has been studied in reference [14].

It is worth noting that the main conditions for occurrence of self-excited vibrations in the models discussed earlier are associated with a difference between static and kinematic frictions, and with existence of an elastic coupling in a tribo-mechanical system.

Analysis of various references [1,2,5,8,10,12] leads to a conclusion that velocity of one of the contacting bodies is always given. The system in the condition of self-excited vibrations takes energy from a body moving at constant velocity. The self-excited vibrations do not appear when inertia of the contacting bodies is taken into account. The latter case is considered in this work. It has been shown that owing to heat extension, a body can be periodically heated, braked, cooled and accelerated. In some conditions, stick-slip self-excited vibrations may also appear.

2. Statement of the problem

We consider two masses M_1 (body 1) and M_2 (body 2) which are coupled by an elastic spring as indicated in Fig. 1. We assume that the initial length of the spring is l_0 and that the spring has stiffness k, which represents the overall elastic properties of the system. We also assume that the masses are constrained by walls to move only in the vertical direction, and that Z_1 and Z_2 denote positions of masses M_1 and M_2, respectively, as indicated in Fig. 1. Let us consider a one-dimensional model of the thermo-elastic contact of body 1 with a surrounding medium. Assume that this body 1 is represented by a rectangular plate ($l_1 \times l_2 \times 2L$) (Fig. 1). Both bodies are subjected to an action of the forces $F_n = F_*^n h_F(t)$, $n = 1,2$, ($h_F(t) \to 1, t \to \infty$). At the initial time

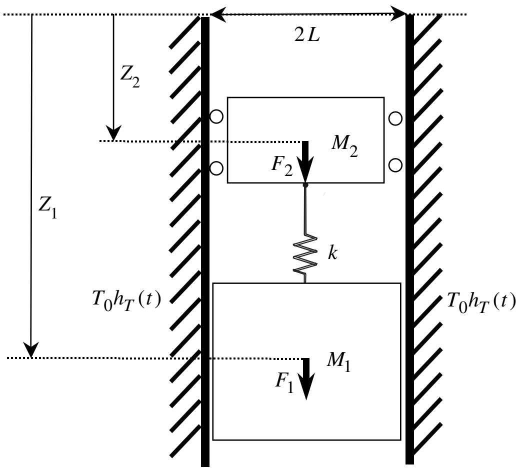

Fig. 1. Two coupled masses system.

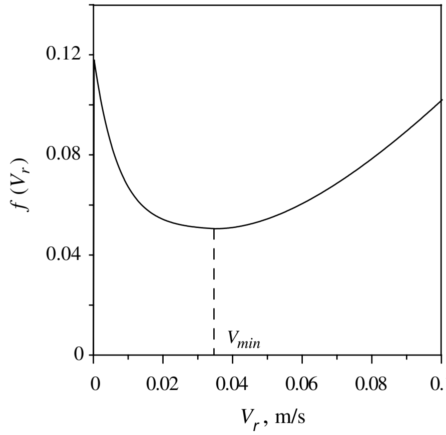

Fig. 2. The Stribeck friction-speed curve.

instant, body 1 (2) is situated at distance Z_1^0 (Z_2^0) and its velocity reads $\dot{Z}_1^0$ ($\dot{Z}_2^0$). The distance between walls is always equal to the initial plate thickness $2L$.

It is assumed that a heat conduction between the bodies and the walls obeys Newton's law. At an initial instant the temperature is governed by the formula $T_0 h_T(t)$ ($h_T(t) \to 1, t \to \infty$). It causes heat extension of the parallelepiped in the direction of $0X$, and body 1 starts to contact the walls. As a result of this process, a frictional contact on the parallelepiped sides $X = \pm L$ occurs. A simple frictional model is applied in the further considerations, i.e. friction force $F_{\rm fr}$ is a product of normal reaction force $N(t)$ and a friction coefficient. That means that $F_{\rm fr} = f(\dot{Z}_1)N(t)$ is the friction force defining resistance of the movement of two sliding bodies. Here, contrary to the assumption made in references [1,5,8], the kinematic friction coefficient $f(\dot{Z}_1)$ depends on the relative velocity $V_{\rm r} = \dot{Z}_1$ of the sliding bodies (Fig. 2).

The friction force $\sigma_{XZ}(X,t)$ per unit contact surface $X = -L$, $X = L$ generates heat. According to Ling's assumptions (cf. [9]), the work of the friction forces is transmitted into heat energy. Note, that the non-contacting plate surfaces are heat-isolated and have the dimensions of $L/l_1 \ll 1$, $L/l_2 \ll 1$, which is in agreement with the assumption of our one-dimensional modeling for body 1. Quantities M_n, F_*^n, k are related to a unit contacting surface.

Below, the problem is reduced to determination of the mass plate (body 2) center displacement $Z_1(t)$ ($Z_2(t)$), plate (body 2) velocity $\dot{Z}_1(t)$ ($\dot{Z}_2(t)$), contact pressure $P(t) = N(t)/l_1 \ l_2 = -\sigma_{XX}(-L,t) = -\sigma_{XX}(L,t)$, plate temperature $T_1(X,t)$, and displacement $U(X,t)$ in the direction of X axis.

3. Mathematical problem formulation

In the considered case, the studied problem is governed by two equations of motion in the form

$$M_1\ddot{Z}_1(t) + k(Z_1(t) - Z_2(t) - l_0) = M_1 g + F_*^1 h_F(t) - 2f(\dot{Z}_1)P(t), \tag{1}$$

$$M_2\ddot{Z}_2(t) - k(Z_1(t) - Z_2(t) - l_0) = M_2 g + F_*^2 h_F(t), \tag{2}$$

where Z_1, Z_2 denote position of both masses as shown in Fig. 1; $\dot{Z}_1$, $\dot{Z}_2$ denote their respective velocities. Equations of the heat stress theory for an isotropic body 1 [11] follow

$$\frac{\partial}{\partial X}\left[\frac{\partial}{\partial X}U_1(X,t) - \alpha_1\frac{1+\nu_1}{1-\nu_1}T_1(X,t)\right] = 0, \tag{3}$$

$$\frac{\partial^2}{\partial X^2}T_1(X,t) = \frac{1}{a_1}\frac{\partial}{\partial t}T_1(X,t), \quad X \in (-L,L) \tag{4}$$

and mechanical

$$U_1(-L,t) = 0, \quad U_2(L,t) = 0, \tag{5}$$

heat

$$-\lambda_1\frac{\partial T_1(-L,t)}{\partial X} + \alpha_T(T_1(-L,t) - T_0 h_T(t)) = f(\dot{Z}_1)\dot{Z}_1(t)P(t), \tag{6}$$

$$\lambda_1\frac{\partial T_1(L,t)}{\partial X} + \alpha_T(T_1(L,t) - T_0 h_T(t)) = f(\dot{Z}_1)\dot{Z}_1(t)P(t) \tag{7}$$

and initial conditions

$$T_1(X,0) = 0, \quad X \in (-L,L), \quad Z_1(0) = Z_1^0, \quad Z_2(0) = Z_2^0, \quad \dot{Z}_1(0) = \dot{Z}_1^0, \quad \dot{Z}_2(0) = \dot{Z}_2^0 \tag{8}$$

are attached. Normal stresses that occur in the plate are defined via the relation

$$\sigma_{XX} = \frac{E_1}{1-2\nu_1}\left[\frac{1-\nu_1}{1+\nu_1}\frac{\partial U}{\partial X} - \alpha_1 T_1\right]. \tag{9}$$

In the above, the following notation is applied: E_1—elasticity modulus, ν_1, λ_1, a_1, α_1, α_{T} are Poisson's ratio, thermal conductivity, thermal diffusivity, thermal expansion and heat transfer coefficients, respectively.

Integration of Eq. (3), owing to (9) and boundary conditions (5), yields the contact pressure $P(t) = -\sigma_{XX}(-L,t) = -\sigma_{XX}(L,t)$ cast in the form

$$P(t) = \frac{E_1\alpha_1 T_0}{1-2\gamma_1}\frac{1}{2L}\int_{-L}^{L} T_1(\xi,t)\,\mathrm{d}\xi. \tag{10}$$

Let us introduce the following similarity coefficients

$$t_* = L^2/a_1\ [\mathrm{s}], \quad v_* = a_1/L\ [\mathrm{m/s}], \quad P_* = T_0E_1\alpha_1/(1-2\nu_1)\ [\mathrm{N/m^2}] \tag{11}$$

and the following non-dimensional parameters

$$x = \frac{X}{L}, \quad \tau = \frac{t}{t_*}, \quad z_n = \frac{Z_n}{L}, \quad p = \frac{P}{P_*}, \quad \theta = \frac{T_1}{T_0}, \quad z_n^0 = \frac{Z_n^0}{L}, \quad \dot{z}_n^0 = \frac{\dot{Z}_n^0}{v_*}, \quad l = \frac{l_0}{L},$$

$$\tau_{n\mathrm{D}} = t_*/t_{n\mathrm{D}}, \quad n = 1,2, \quad m_{n0} = (M_n g + F_n)/2P_*, \quad \varepsilon_n = 2P_* t_*^2/M_n L, \quad n = 1,2, \tag{12}$$

$$\gamma = \frac{E_1\alpha_1 a_1}{(1-2\nu_1)\lambda_1}, \quad Bi = \frac{L\alpha_T}{\lambda_1}, \quad F(\dot{z}) = f(v_*\dot{z}),$$

where $t_{n\mathrm{D}} = \sqrt{M_n/k}$, $n = 1,2$.

The examined problem is governed by the following non-dimensional equations

$$\frac{\partial^2\theta(x,\tau)}{\partial x^2} = \frac{\partial\theta(x,\tau)}{\partial\tau}, \quad x \in (-1,1), \quad \tau \in (0,\infty), \tag{13}$$

$$\ddot{z}_1(\tau) + (z_1(\tau) - z_2(\tau) - l)\tau_{1\mathrm{D}}^2 = \varepsilon_1(m_{10} - F(\dot{z}_1)p(\tau)), \tag{14}$$

$$\ddot{z}_2(\tau) - (z_1(\tau) - z_2(\tau) - l)\tau_{2\mathrm{D}}^2 = \varepsilon_2 m_{20} \tag{15}$$

with both boundaries

$$\left[\frac{\partial\theta(x,\tau)}{\partial x} - Bi\theta(x,\tau)\right]_{x=-1} = -q(\tau), \quad \left[\frac{\partial\theta(x,\tau)}{\partial x} + Bi\theta(x,\tau)\right]_{x=1} = q(\tau) \tag{16}$$

and initial conditions,

$$\theta(x,0) = 0, \quad z_1(0) = z_1^0, \quad z_2(0) = z_2^0, \quad \dot{z}_1(0) = \dot{z}_1^0, \quad \dot{z}_2(0) = \dot{z}_2^0, \tag{17}$$

where

$$q(\tau) = Bih_T(\tau) + \gamma F(\dot{z}_1)\dot{z}_1(\tau)p(\tau), \quad p(\tau) = \frac{1}{2}\int_{-1}^{1}\theta(\xi,\tau)\,\mathrm{d}\xi. \tag{18}$$

4. Solution of the problem

Applying an inverse Laplace transformation [7], our non-linear problem governed by Eqs. (13), (16) and (17) is reduced to the following integral equation

$$p(\tau) = Bi\int_0^\tau \dot{h}_T(\xi)G_p(\tau-\xi)\,\mathrm{d}\xi + \gamma\int_0^\tau F(\dot{z}_1)\dot{z}_1(\xi)p(\xi)\dot{G}_p(\tau-\xi)\,\mathrm{d}\xi \tag{19}$$

which yields both non-dimensional pressure $p(\tau)$ and velocity $\dot{z}_1(\tau)$. The temperature is defined by the following formula

$$\theta(x,\tau) = Bi\int_0^\tau \dot{h}_T(\xi)G_\theta(x,\tau-\xi)\,\mathrm{d}\xi + \gamma\int_0^\tau F(\dot{z}_1)\dot{z}_1(\xi)p(\xi)\dot{G}_\theta(x,\tau-\xi)\,\mathrm{d}\xi, \tag{20}$$

where

$$\{G_p(\tau), G_\theta(1,\tau)\} = \frac{1}{Bi} - \sum_{m=1}^{\infty}\frac{\{2Bi, 2\mu_m^2\}}{\mu_m^2[Bi(Bi+1)+\mu_m^2]}e^{-\mu_m^2\tau} \tag{21}$$

and μ_m are the roots of the following characteristic equation

$$tg\mu_m = \frac{Bi}{\mu_m}, \quad m = 1,2,\ldots \tag{22}$$

5. Steady-state solution analysis

A stationary solution to the problem reads

$$p_{\mathrm{st}} = \theta_{\mathrm{st}} = \frac{1}{1-v}, \quad v = F(v_{\mathrm{st}})\frac{v_{\mathrm{st}}\gamma}{Bi}, \tag{23}$$

where v_{st} is the solution to the non-linear equation

$$F(v_{\mathrm{st}}) = \frac{m_0}{1+\gamma m_0 v_{\mathrm{st}}/Bi}, \quad m_0 = m_{10}^{\mathrm{st}} + m_{20}^{\mathrm{st}}, \quad m_{n0}^{\mathrm{st}} = \frac{M_n g + F_*^n}{2P_*}. \tag{24}$$

Graphical solution of Eq. (24) is presented in Fig. 3 for various parameters m_0 and Bi. Recall that for steel $\gamma = 1.87$.

The right-hand side of Eq. (24) is represented by solid curves for different values of the parameters m_0 and Bi. Solid curve 1 is associated with parameters $m_0 = 0.15$, $Bi = 50$; solid curve 2—$m_0 = 0.1$, $Bi = 50$; solid curve 3—$m_0 = 0.1$, $Bi = 0.5$; solid curve 4—$m_0 = 0.15$, $Bi = 0.5$. The dashed curve displays the function $F(v_{\mathrm{st}})$.

For different values of parameters, Eq. (24) can have a different number of solutions: for $m_0 = 0.15$, $Bi = 50$ (first case) it has one solution $v_{\mathrm{st}}^{(3)}$ $(F'(v_{\mathrm{st}}^{(3)}) > 0)$; for $m_0 = 0.1$, $Bi = 50$ (second case)—three solutions $v_{\mathrm{st}}^{(1)}$, $v_{\mathrm{st}}^{(2)}$, $v_{\mathrm{st}}^{(3)}$ $(F'(v_{\mathrm{st}}^{(1)}) > 0,\ F'(v_{\mathrm{st}}^{(2)}) < 0,\ F'(v_{\mathrm{st}}^{(3)}) > 0)$; and for $m_0 = 0.1$, $Bi = 0.5$ (third case) one solution $v_{\mathrm{st}}^{(1)}$ $(F'(v_{\mathrm{st}}^{(1)}) > 0)$. Owing to approximation (37) we have $v_{\mathrm{st}}^{(1)} \approx \varepsilon_0 m_0/2F_0$ and $F'(v_{\mathrm{st}}^{(1)}) \approx 2F_0/\varepsilon_0$. For $m_0 = 0.15$, $Bi = 0.5$ (fourth case), again one solution exists $v_{\mathrm{st}}^{(2)}$ $(F'(v_{\mathrm{st}}^{(2)}) < 0)$.

Let us introduce a perturbation of the stationary solution (23) by means of the following formulas

$$z_n = v_{\mathrm{st}}\tau + z_n^*, \quad \dot{z}_n = v_{\mathrm{st}} + \dot{z}_n^*, \quad n = 1,2, \quad p = p_{\mathrm{st}} + p^*,$$
$$\theta = \theta_{\mathrm{st}} + \theta^*, \quad h_T = 1 + h_T^*, \quad h_F = 1 + h_F^*, \quad |h_F^*| \ll 1, \quad |h_T^*| \ll 1. \tag{25}$$

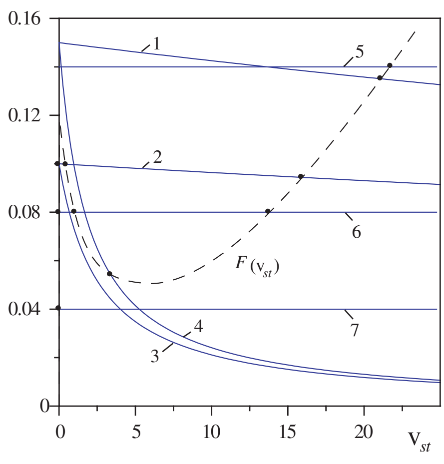

Fig. 3. Graphical solution of Eq. (24) (solid curves: (1) $m_0 = 0.15$, $Bi = 50$; (2) $m_0 = 0.1$, $Bi = 50$; (3) $m_0 = 0.1$, $Bi = 0.5$; (4) $m_0 = 0.15$, $Bi = 0.5$; dashed curve corresponds to $F(v_{\mathrm{st}})$).

Owing to linearization of the right-hand sides of (14) and with boundary condition (16) the following linear problem is obtained

$$\frac{\partial^2\theta^*(x,\tau)}{\partial\tau^2}=\frac{\partial\theta^*(x,\tau)}{\partial\tau}, \tag{26}$$

$$\ddot{z}_1^*(\tau)+(z_1^*-z_2^*)\tau_{1D}^2=\varepsilon_1[m_{10}^*(\tau)-F(v_{st})p^*(\tau)-F'(v_{st})p_{st}\dot{z}_1^*], \tag{27}$$

$$\ddot{z}_2^*(\tau)-(z_1^*-z_2^*)\tau_{2D}^2=\varepsilon_2 m_{20}^*(\tau), \tag{28}$$

$$\left[\frac{d\theta^*(x,\tau)}{dx}-Bi\theta^*(x,\tau)\right]_{x=-1}=-q^*(\tau),\quad \left[\frac{d\theta^*(x,\tau)}{dx}+Bi\theta^*(x,\tau)\right]_{x=1}=q^*(\tau), \tag{29}$$

where

$$q^*(\tau)=Bih_T^*(\tau)+\gamma(v_{st}p_{st}(\beta_1+\beta_2)\dot{z}_1^*(\tau)+v_{st}F(v_{st})p^*(\tau)),$$

$$m_{n0}^*(\tau)=F_*^n h_F^*(\tau)/(2P_*),\quad p^*(\tau)=\frac{1}{2}\int_{-1}^{1}\theta^*(\xi,\tau)\,d\xi,\quad \beta_1=\frac{F(v_{st})}{v_{st}},\quad \beta_2=F'(v_{st}). \tag{30}$$

Further, applying the Laplace transformation, a solution of problem (26)–(29) in the transform domain is found. For example, the Laplace transformation of the velocity perturbation of body 1 reads

$$\begin{aligned}s\bar{z}_1^*(s)=&-\Delta^{-1}(s)\{\varepsilon_1\beta_1 v_{st}Bi(s^2+\tau_{2D}^2)\overline{S}(s)\bar{h}_T^*(s)+\left(Biv\overline{S}(s)-\Delta_1(s)\right)\\&\times\left[\varepsilon_1(s^2+\tau_{2D}^2)\bar{m}_{10}^*(s)+\varepsilon_2\tau_{1D}^2\bar{m}_{20}^*(s)\right]\},\end{aligned} \tag{31}$$

where

$$\{\bar{z}_1^*(s),\bar{h}_T^*(s),\bar{h}_F^*(s),\bar{m}_{n0}^*(s)\}=\int_0^\infty\{z_1^*(\tau),h_T^*(\tau),h_F^*(\tau),m_{n0}^*(\tau)\}e^{-s\tau}\,d\tau.$$

The characteristic equation of the linearized problem reads

$$\begin{aligned}\Delta(s)=&(\varepsilon_1 p_{st}\beta_2(s^2+\tau_{2D}^2)+s(s^2+\tau_{1D}^2+\tau_{2D}^2))\Delta_1(s)+Biv(\varepsilon_1 p_{st}\beta_1(s^2+\tau_{2D}^2)\\&-s(s^2+\tau_{1D}^2+\tau_{2D}^2))\overline{S}(s)=0,\end{aligned} \tag{32}$$

where $\Delta_1(s)=s\overline{S}(s)+Bi\overline{C}(s)$, $\overline{S}(s)=\sinh(\sqrt{s})/\sqrt{s}$, $\overline{C}(s)=\cosh(\sqrt{s})$.

The characteristic function $\Delta(s)$, in the form of an infinite order polynomial takes the form

$$\Delta(s)=\sum_{m=0}^{\infty}s^m a_m, \tag{33}$$

where

$$a_0=\tau_{2D}^2 b_0,\quad a_1=\tau_{2D}^2 b_1+\tau_{1D}^2(d_0^{(1)}-Biv),\quad a_m=b_{m-2}+\tau_{2D}^2 b_m+\tau_{1D}^2(d_{m-1}^{(1)}-Bivd_{m-1}^{(2)}),$$

$$m=2,3,\ldots,\quad b_0=\varepsilon_1 p_{st}c_0,\quad b_m=\varepsilon_1 p_{st}c_m+d_{m-1},\quad m=1,2,\ldots,$$

$$c_0 = Bi(\beta_2 + v\beta_1), \quad d_m = d_m^{(1)} - Bivd_m^{(2)}, \quad c_m = \beta_2 d_m^{(1)} + Biv\beta_1 d_m^{(2)},$$

$$d_m^{(1)} = \frac{2m + Bi}{(2m)!}, \quad d_m^{(2)} = \frac{1}{(2m+1)!}.$$

Observe that owing to analysis of the roots of characteristic Eq. (33), the parameter $v_{\rm st}$ represents a solution to non-linear Eq. (24).

If the frictional heat generation is not taken into account ($\gamma = 0$), the characteristic equations are governed by the following cubic equation: $s^3 + \varepsilon_1\beta_2 s^2 + (\tau_{1D}^2 + \tau_{2D}^2)s + \varepsilon_1\beta_2\tau_{2D}^2 = 0$. Its roots lie in the right-hand part of the complex plane if $\beta_2 < 0$.

In the case of a perfectly stiff spring ($k \to \infty$), we have $\tau_{1D} \to \infty$, $\tau_{2D} \to \infty$. The Laplace transformation of the velocity perturbation of body 1 reads

$$s\bar{z}_1^*(s) = -\Delta^{-1}(s)\left\{\varepsilon_1\beta_1 v_{\rm st} Bi\bar{S}(s)\bar{h}_T^*(s) + \left(Biv\bar{S}(s) - \Delta_1(s)\right)(\varepsilon_1\bar{m}_{10}^*(s) + \varepsilon_2\bar{m}_{20}^*(s))\right\}, \tag{34}$$

where

$$\Delta(s) = (\varepsilon_1 p_{\rm st}\beta_2 + s)\Delta_1(s) + Biv(\varepsilon_1 p_{\rm st}\beta_1 - s)\bar{S}(s) = 0 \tag{35}$$

is the characteristic equation of the linearized problem, and velocity $v_{\rm st}$ is a solution to Eq. (24). Note that Bern has carried out a detailed analysis of roots of Eq. (35) for $\beta_2 = 0$ in reference [13].

In the case, when $M_2 \to 0$, we have $\tau_{2D} \to \infty$, and for $F_*^2 = 0$ ($\bar{m}_{20}^*(s) = 0$) the Laplace transformation of the velocity perturbation of body 1 is defined by Eq. (34), whereas the characteristic equation is given by (35).

In this case, the examination concerns the steel made parallelepiped plate ($\alpha_2 = 14 \times 10^{-6}$ °C^{-1}, $\lambda_1 = 21$ W/(m °C^{-1}), $v_1 = 0.3$, $a_1 = 5.9 \times 10^{-6}$ m^2/s, $E_1 = 19 \times 10^{10}$ Pa) with $\varepsilon_1 = 100$, $\varepsilon_2 = 900$, $\tau_{1D} = 2$, $\tau_{2D} = 6$ and with a non-constant friction coefficient. The function $F(\dot{z}) = f(v_*\dot{z})$ is defined by the formula (cf. [5])

$$f(V_{\rm r}) = {\rm sgn}(V_{\rm r})\begin{cases} f_{\min} + (F_0 - f_{\min})\exp(-b_1|V_{\rm r}|), & \text{for } |V_{\rm r}| < V_{\min} \\ [-F_0; F_0], & \text{for } V_{\rm r} = 0 \\ f_{\min} + (F_0 - f_{\min})\exp(-b_1|V_{\min}|) & \\ +\dfrac{b_2 b_3(|V_{\rm r}| - V_{\min})^2}{1 + b_2(|V_{\rm r}| - V_{\min})}, & \text{for } |V_{\rm r}| > V_{\min}, \end{cases} \tag{36}$$

where $F_0 = 0.12$, $f_{\min} = 0.05$, $b_1 = 140$ sm^{-1}, $b_2 = 10$ sm^{-1}, $b_3 = 2$ sm^{-1}, $V_{\min} = 0.035$ mc^{-1}. The function sgn(x) is approximated by the formula (cf. [5,10])

$${\rm sgn}_{\varepsilon_0}(x) = \begin{cases} 1, & \text{if } x > \varepsilon_0, \\ \left(2 - \dfrac{|x|}{\varepsilon_0}\right)\dfrac{x}{\varepsilon_0}, & \text{if } |x| < \varepsilon_0, \\ -1, & \text{if } x < -\varepsilon_0, \end{cases} \tag{37}$$

where $\varepsilon_0 = 0.0001$.

In the first case (for $m_0 = 0.15$, $Bi = 50$), one solution $v_{\rm st}^{(3)} = 21$, $p_{\rm st}^{(3)} = \theta_{\rm st}^{(3)} = 1.12$ ($v = 0.105$) is found. It is always stable (the roots of equation (32) $s_{1,2} = -0.05 \pm 6.3i$ lie in the left-hand side of the complex variable s). The contact characteristics achieve their limiting values through a damped oscillation process with the expected 'period' $T = 0.99$.

In the second case ($m_0 = 0.1$, $Bi = 50$), three solutions appear. The solution $v_{st}^{(3)} = 15.94$, $p_{st}^{(3)} = \theta_{st}^{(3)} = 1.06$, ($v = 0.056$) is stable (the roots of equation (32) $s_{1,2} = -0.04 \pm 6.32i$ lie in the left-hand side of the complex plane s). The solution $v_{st}^{(2)} = 0.41$ ($v = 0.0015$) is unstable ($s_1 = 3.8$, $s_{2,3} = 0.15 \pm 6.23i$). The solution $v_{st}^{(1)} \approx 0$ corresponds to an equilibrium state. In the considered case, the contact characteristics, depending on initial conditions, tend to one of the two stable solutions.

In the third case ($m_0 = 0.1$, $Bi = 0.5$), there is only one solution, which is stable. Note that in this case, a braking process always occurs (the applied external force is smaller than the friction force).

In the fourth case ($m_0 = 0.15$, $Bi = 0.5$), the only solution that exists is of the form: $v_{st}^{(2)} = 3.01$, $p_{st}^{(2)} = \theta_{st}^{(2)} = 2.69$, ($v = 0.63$) and it is unstable ($s_{1,2} = 0.5 \pm 0.68i$, $s_{3,4} = 0.06 \pm 6.32i$). If a solution is unstable, then solving non-stationary problem this solution may approach a stable limit cycle or it can be expressed via oscillations increasing in time (its behavior depends on other non-linear terms).

6. Numerical analysis of transient solution

Let us consider the fourth case as an example. Fig. 4a shows the dependence of displacement of non-dimensional body 1 (body 2) $z_1(\tau)$ ($z_2(\tau)$) versus non-dimensional time τ, whereas Fig. 4b displays a dependence of velocity of non-dimensional body 1 (body 2) $\dot{z}_1(\tau)$ ($\dot{z}_2(\tau)$) versus non-dimensional time τ. Solid curves correspond to body 1, dashed curve corresponds to body 2. Note that body 1 is in a stick-slip state. Zones with stick ($\dot{z}_1 = 0$) are substituted by zones of slips.

Evolution of non-dimensional contact pressure in time (curve 1) and a temperature on the contact surface (curve 2) is shown in Fig. 5.

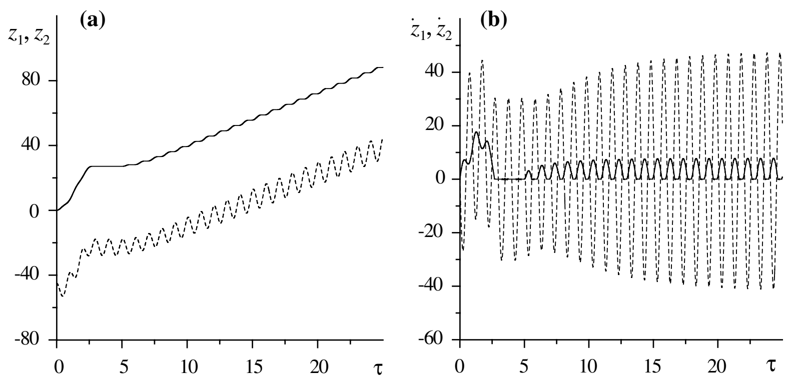

Fig. 4. Time history of non-dimensional body displacement (a) and velocity (b). Solid curves correspond to body 1, dashed curve corresponds to body 2.

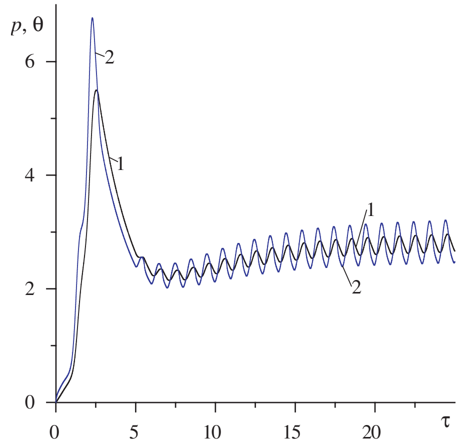

Fig. 5. Dimensional contact pressure p (curve 1) and dimensionless contact temperature θ (curve 2) versus dimensionless time τ.

7. Conclusions

In this work a new physical and mathematical model of a two-degrees-of-freedom system with an account of friction and heating processes is studied. It is assumed that the friction coefficient depends on sliding velocity.

It has been shown that when a heat transfer is not taken into account ($\gamma = 0$), the system cannot exhibit a stick-slip motion. This potential behavior of the studied system is displayed by a solid curve in Fig. 3. For $m_0 > F_0$ (solid curve 5) equation $F(v_{st}) = m_0$ has one stable static solution, which attracts a non-stationary one. For $f_{min} < m_0 < F_0$ (solid curve 6) the mentioned equation has three static solutions, and one of them is unstable. In this case, a non-stationary solution will be attracted by one of two static stable solutions. In the cases when $m_0 < f_{min}$ (solid curve 7) the discussed equation has only one stable solution.

In order to realize a stick-slip motion the parameter γ should be positive (see case 4). In this case, one deals with only one solution, which is unstable and a non-stationary solution can be attracted by a limiting cycle. The numerical analysis is in agreement with theoretical prediction of the occurrence of stick-slip dynamics of our investigated system with friction and heat generation.

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