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Tribological periodic processes exhibited by acceleration or braking of a shaft–pad system

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Abstract

A new one-dimensional thermoelastic frictional pair contact model of a shaft–bush system is proposed. The assumed model includes a study of vibration processes and contact characteristics exhibited by a relative velocity between the two mentioned bodies, contact temperature, pressure and wear. Some important conclusions are formulated.

Keywords: Friction; Wear; Frictional heat generation; Stick–slip vibrations

1. Introduction

Theoretical studies of one-dimensional frictional contact model of a shaft–bush system are mainly devoted to analysis of periodic vibrations caused by friction [1,4,5,10,14]. The classical problem of vibrations of a pad lying on a rotating shaft and linked to a base through the massless springs (a so-called Pronny's clamp) has been analyzed in [1]. In [9,14] a dry friction is investigated and its computational models are proposed. On the other hand, a thermoelastic contact between two bodies is often analyzed without an account of dynamical behavior (see, for example, a review given in [7]). For instance, a study of thermal conduction and wear of a cantilever beam being in frictional contact with a moving with a constant velocity rigid part is carried out in [12].

It is worth noticing that problems associated with friction, heat transfer and wear processes are less investigated. On the other hand, a deep analysis of friction generated vibrations and

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variations of the associated frictional characteristics (relative velocity, contact temperature, pressure and wear) may essentially contribute for explanation of various engineering phenomena occurred in machine tools with high treatment, torsion couplings, frictional dampers and other mechanisms with frictional pairs. A thermoelastic contact occurring between a rotating shaft and non-movable pad assuming that a shaft is non-inertial is investigated in [18,19]. In [6] the axially symmetric problem of thermoelastic contact of a shaft rotating with constant velocity and with attached pad is reported. The pad is fixed to the housing via massless springs, and self-excited vibrations caused by friction (taking into account wear and frictional heat generation) are analyzed. It is shown [6,18], among other results, that when the relative velocity achieves its critical value, then the so-called frictional thermoelastic instability occurs [7].

In this paper we analyze more complicated axially symmetric problem of self-excited vibrations of thermoelastic contact between rotating shaft and a pad elastically supported taking into account wear processes.

Since a frictional process is usually non-stationary one, all 'frictional' parameters are mutually coupled and depend on each other. Our investigation are mainly focused on the following question: if and to which extend both wear and frictional heat generation processes may influence a motion (and associated parameters) during braking and acceleration regimes of the shaft–pad system.

2. Mechanical model and formulation

An elastic and heat transferring shaft with a radius R_1 is inserted into a full braking pad (bush)—see Fig. 1. The pad with the internal R_1 and external R_2 radii is attached to the housing by the springs with stiffness coefficients k_2 and the damper with viscous coefficient c. The applied moment $M_0 h_M(t)$ is related to the shaft length unit, where M_0 is a constant with units of a moment and $h_M(t)$ is the known dimensionless time varying function ($h_M(t) \to 1, t \to \infty$). We assume that a pad transfers heat perfectly and that at the initial time moment pad temperature is governed by the formula $T_0 h_T(t)$, where T_0 is a constant measured in units of temperature, $h_T(t)$ is the known dimensionless time varying function ($h_T(0) = 0$, and $h_T(t) \to 2, t \to \infty$), and that between pad and shaft the so-called Newton's heat exchange takes place. The shaft begins to expand and a contact between two bodies occurs (note, that earlier both bodies were not in a contact). We assume that

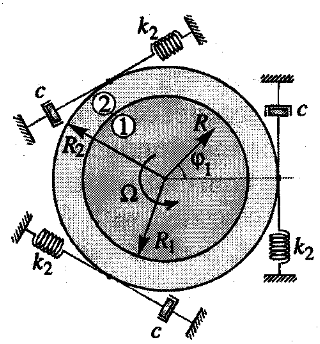

Fig. 1. The analyzed system (1—shaft; 2—braking pad).

between the pad and shaft, dry friction governed by the function $F_t(V_w)$ occurs, where $V_w = \dot{\varphi}_1 R_1 - \dot{\varphi}_2 R_1$ is the relative velocity between the bodies, i.e. pad and shaft. B_1 and B_2 are the inertial moments of shaft and pad (bush) measured in relation to their length units, respectively. We assume that the friction force is governed by the formula $F_t = f(V_w)N(t)$, where $N(t)$ is the normal reaction and $f(V_w)$ is the coefficient of kinetic friction. Note that in the contact surface between two bodies and for $R = R_1$ heat is generated by friction and the pad wear. $T_1(R,t)$ denotes shaft temperature, which is equal to zero at the initial time instant.

Both thermal and stress–strain state of the shaft are considered using the cylinder coordinates R, ϕ, Z rotating with the angular velocity Ω with its origin situated in the center of the rotating elastic body. The shaft center is rigidly fixed. The governing equations of motion of uncoupled thermoelastic problem along Z-axis have the following form [16,20]:

$$\mu_1 \nabla^2 \mathbf{u} + (\lambda_1 + \mu_1)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \rho_1 \Omega^2 R \mathbf{e}_R = (3\lambda_1 + 2\mu_1)\alpha_1\,\mathrm{grad}\,T_1 + \rho_1 \frac{\partial^2 \mathbf{u}}{\partial t^2}, \tag{1}$$

$$\nabla^2 T_1 = \frac{1}{a_1}\frac{\partial T_1}{\partial t}, \tag{2}$$

where ∇^2 is the Laplace operator, $\mathbf{u} = U\mathbf{e}_R + V\mathbf{e}_\phi + W\mathbf{e}_Z$ is the vector of relative displacement shaft, U is the radial displacement, T_1 is the shaft temperature, λ_1, μ_1 are the Lamè coefficients, ρ_1 is the shaft density, α_1 is the coefficient of thermal linear expansion of the shaft, and a_1 is the thermal diffusivity of the shaft material.

We consider a one-dimensional model of thermal friction and wear during a stick–slip motion taking into account the following assumptions:

1. We assume that the pad is a perfectly rigid body.
2. The external excitation of the system allows for neglecting of the term $\rho_1 \partial^2\mathbf{u}/\partial t^2$ in the Lamè equation (1). Since the shaft rotates with a small angular velocity $\Omega(t) = \dot{\varphi}_1(t)$, the centrifugal forces $\rho_1 \Omega^2 R$ can be neglected [20].
3. The vector components related to displacements as well as the shaft temperature do not depend on ϕ, Z, and the unequal zero components $U(R,t)$, $V(R,t)$ and $T_1(R,t)$ depend only on the radial coordinate and time.
4. The heat flows q_1 and q_2 are generated on the contact surface due to the Ling rule [13,15] and they are governed by the equation $q_1 + q_2 = f(V_w)V_w P(t)$. Both flows q_1 and q_2 go into shaft and bush, respectively:

$$q_1 = \lambda_1 \frac{\partial T_1(R_1,t)}{\partial R}, \quad q_2 = -\alpha_T(T_0 h_T(t) - T_1(R_1,t)), \tag{3}$$

where λ_1 is the thermal conductivity of the shaft material, and α_T is the heat transfer coefficient between shaft and bush.
5. One of the most popular wear model is governed by the equation [11]

$$\frac{\mathrm{d}U_z(t)}{\mathrm{d}t} = K_z |V_w(t)|^m P(t)^n, \tag{4}$$

where m, n are exponents, $\mathrm{d}U_z/\mathrm{d}t$ is the pad wear rate, K_z is the wear constant determined from an experiment [9,14]. Owing to the second assumption, K_z does not depend on temperature.

However, in some Refs. [9,17] a temperature dependence of K_z is also considered. $P(t)$ is the time depended contact pressure. We assume Archard's law of wear [2,3] in the form of (4), where $m = n = 1$. The taken rule is typical for an abrasive wear.

Owing to the introduced assumptions, a balance of the moments in relation to shaft axis [4] is carried out and the equations governing thermal stresses for the isotropic body [16,20] in the cylinder coordinates are derived.

The equations and initial conditions governing the rotational motion of the bush have the following form:

$$B_2\frac{\mathrm{d}^2\varphi_2}{\mathrm{d}t^2} = M_t - cR_2^2\frac{\mathrm{d}\varphi_2}{\mathrm{d}t} - M_s, \quad \varphi_2(0) = 0, \quad \frac{\mathrm{d}\varphi_2(0)}{\mathrm{d}t} = 0, \tag{5}$$

whereas the equations and initial conditions governing the rotational motion of the shaft can be cast in the form

$$B_1\frac{\mathrm{d}^2\varphi_1}{\mathrm{d}t^2} = M - M_t, \quad \varphi_1(0) = \varphi_1^\circ, \quad \frac{\mathrm{d}\varphi_1(0)}{\mathrm{d}t} = \omega_1^\circ. \tag{6}$$

In order to determine the amount of wear at the bush during sliding process, Archard's law of wear is used

$$\frac{\mathrm{d}U_z(t)}{\mathrm{d}t} = K_z|V_w(t)|P(t), \tag{7}$$

and the quasi-static thermoelasticity equations (1) and (2) for the shaft have the following form [16,20]:

$$\frac{\partial^2 U(R,t)}{\partial R^2} + \frac{1}{R}\frac{\partial U(R,t)}{\partial R} - \frac{1}{R^2}U(R,t) = \alpha_1\frac{1+\nu}{1-\nu}\frac{\partial T_1(R,t)}{\partial R}, \tag{8}$$

$$\frac{\partial^2 T_1(R,t)}{\partial R^2} + \frac{1}{R}\frac{\partial T_1(R,t)}{\partial R} = \frac{1}{a_1}\frac{\partial T_1(R,t)}{\partial t}, \quad 0 < R < R_1, \ 0 < t < \infty, \tag{9}$$

with the attached mechanical conditions

$$U(0,t) = 0, \quad U(R_1,t) = U_z(t), \qquad 0 < t < \infty. \tag{10}$$

The thermal boundary conditions have the following form (see assumption 4):

$$\lambda_1\frac{\partial T_1(R_1,t)}{\partial R} + \alpha_T(T_1(R_1,t) - T_0h_T(t)) = f(V_w)V_wP(t), \tag{11}$$

$$R\frac{\partial T_1(R,t)}{\partial R}\bigg|_{R\to 0} = 0, \quad 0 < t < \infty, \tag{12}$$

and initial conditions read

$$T_1(R,0) = 0, \quad 0 < R < R_1, \tag{13}$$

where $M_t = f(R_1(\dot\varphi_1 - \dot\varphi_2))2\pi R_1^2P(t)$ is the moment of friction force, M is the moment acting on the shaft, the elastic forces moment is defined by $M_s = k_2R_2^2\varphi_2$, and $U_z(t)$ denotes wear. Radial stresses in shaft $\sigma_R(R,t)$ can be found knowing both radial displacement $U(R,t)$ and temperature $T_1(R,t)$ in the shaft [16,20], since

$$\sigma_R(R,t) = \frac{E_1}{1-2v}\left[\frac{1-v}{1+v}\frac{\partial U(R,t)}{\partial R} + \frac{v}{1+v}\frac{U(R,t)}{R} - \alpha_1 T_1(R,t)\right]. \qquad (14)$$

The following notation is used: $P(t) = N(t)/2\pi R_1 = -\sigma_R(R_1,t)$ is the contact pressure, E_1 is Young's modulus of the shaft material, v is Poisson's ratio of the shaft, α_1 is the coefficient of thermal linear expansion of the shaft, $\varphi_1(t)$ and $\varphi_2(t)$ are angles of bush and pad rotation.

Let us introduce the following dimensionless parameters:

$$r = \frac{R}{R_1}, \quad \tau = \frac{t}{t_*}, \quad p = \frac{P}{P_*}, \quad u_z = \frac{U_z}{U_*}, \quad \theta = \frac{T_1}{T_0}, \quad \phi_2 = \varphi_2(t_*\tau), \quad \phi_1 = \varphi_1(t_*\tau), \quad \phi_1^\circ = \varphi_1^\circ,$$

$$\dot\phi_1^\circ = t_*\omega_1^\circ, \quad \varepsilon = \frac{P_* t_*^2 2\pi R_1^2}{B_2}, \quad Bi = \frac{\alpha_T R_1}{\lambda_1}, \quad \gamma = \frac{2E_1\alpha_1 R_1^2}{\lambda_1(1-2v)t_*}, \quad \tilde\omega = \frac{t_* a_1}{R_1^2}, \quad a_M = \frac{P_* t_*^2 2\pi R_1^2}{B_1},$$

$$m_0 = \frac{M_0}{P_* 2\pi R_1^2}, \quad k_z = \frac{K_z R_1 P_*}{U_*}, \quad h = \frac{cR_2^2}{2B_2 t_*}, \quad h_M(\tau) = h_M(t_*\tau), \quad h_T(\tau) = h_T(t_*\tau),$$

$$F(\dot\phi_1 - \dot\phi_2) = f(R_1 t_*^{-1}(\dot\phi_1 - \dot\phi_2)),$$

where $t_* = \sqrt{B_2/k_2}/R_2$, $U_* = 2\alpha_1 T_0(1+v)R_1$, $P_* = 2E_1\alpha_1 T_0/(1-2v)$.

Applying the Laplace transformations [6,8] to Eqs. (8)–(13), the following non-dimensional equations are obtained:

$$\ddot\phi_2(\tau) + 2h\dot\phi_2(\tau) + \varphi(\tau) = \varepsilon F(\dot\phi_1 - \dot\phi_2)p(\tau), \; 0 < \tau < \infty, \quad \phi_2(0) = 0, \quad \dot\phi_2(0) = 0, \qquad (15)$$

$$\ddot\phi_1(\tau) = a_M(m_0 h_M(\tau) - F(\dot\phi_1 - \dot\phi_2)p(\tau)), \; 0 < \tau < \infty, \quad \phi_1(0) = \phi_1^\circ, \quad \dot\phi_1(0) = \dot\phi_1^\circ, \qquad (16)$$

$$p(\tau) = Bi\tilde\omega \int_0^\tau G_p(\tau-\xi)h_T(\xi)\,d\xi - u_z(\tau) + \gamma\tilde\omega \int_0^\tau G_p(\tau-\xi)F(\dot\phi_1 - \dot\phi_2)p(\xi)(\dot\phi_1 - \dot\phi_2)\,d\xi, \qquad (17)$$

$$u_z(\tau) = k_z \int_0^\tau |\dot\phi_1(\xi) - \dot\phi_2(\xi)| p(\xi)\,d\xi, \qquad (18)$$

$$\theta(r,\tau) = Bi\tilde\omega \int_0^\tau G_\theta(r,\tau-\xi)h_T(\xi)\,d\xi + \gamma\tilde\omega \int_0^\tau G_\theta(r,\tau-\xi)F(\dot\phi_1 - \dot\phi_2)p(\xi)(\dot\phi_1 - \dot\phi_2)\,d\xi, \qquad (19)$$

$$\{G_p(\tau), G_\theta(1,\tau)\} = \sum_{m=1}^{\infty} \frac{\{2Bi, 2\mu_m^2\}}{Bi^2 + \mu_m^2} e^{-\mu_m^2\tilde\omega\tau}, \qquad (20)$$

where $d\phi_n/d\tau \equiv \dot\phi_n$, $d^2\phi_n/d\tau^2 \equiv \ddot\phi_n$, $n = 1,2$, μ_m are the roots of the characteristic equation $(m = 1,2,3,\ldots)$

$$BiJ_0(\mu) - \mu J_1(\mu) = 0, \qquad (21)$$

$J_n(\mu)$ is a Bessel function of the first kind of order n $(n = 0,1)$, $F(\dot\phi_1 - \dot\phi_2)$ is the dimensionless coefficient of kinetic friction, k_z is the dimensionless wear constant, Bi is the Biot number, m_0 is the dimensionless applied moment, γ is the dimensionless thermomechanical parameter proportional to α_1, and $\tilde{\omega}$ is the coupling parameter.

Observe that the considered problem is reduced to the system of non-linear differential equations (15) and (16), and the integral equation (17) describing the angular velocities of pad $\dot{\phi}_2(\tau)$ and shaft $\dot{\phi}_1(\tau)$, and the dimensionless contact pressure $p(\tau)$. Dimensionless temperature $\theta(r,\tau)$ and dimensionless wear $u_z(\tau)$ are governed by Eqs. (19) and (18), respectively.

Observe also that the derived governing equations include known in the literature particular cases of frictional contact analysis of two bodies.

For example, if both thermal shaft extension ($\gamma = 0$) and wear ($k_z = 0$) do not appear, than after some time a contact pressure $p(\tau) \to 1$. Our general investigations can be also reduced to the particular cases considered in the following references: [1] $p(\tau) = 1$ $(\gamma = 0, k_z = 0)$, $h = 0$, $\dot{\phi}_1 = \text{const}$; [6] $h = 0$, $k_z = 0$, $\dot{\phi}_1 = \text{const}$; [18] $\phi_2 = 0$ $(k_2 \to \infty)$, $F(\dot{\phi}_1 - \dot{\phi}_2) = \text{const}$; [19] $\phi_2 = 0$, $F(\dot{\phi}_1 - \dot{\phi}_2) = \text{const}$, $\dot{\phi}_1 = \text{const}$.

3. Results

A dependence of the friction coefficient on the non-dimensional relative velocity (the so-called Stribeck's curve [10,14]) is approximated by the following function [4]:

$$F(y) = F_0\,\mathrm{Sgn}(y) - \alpha y + \beta y^3, \quad \mathrm{Sgn}(y) = \begin{cases} \{y/|y|\} & \text{for } y \neq 0, \\ [-1,1] & \text{for } y = 0, \end{cases} \tag{22}$$

where the following parameters are fixed: $F_0 = 0.3$, $\alpha = 0.3$, $\beta = 0.3$. Note that for $y_{\min} = \sqrt{\alpha/3\beta}$ the function $F(y)$, $y \in (0,\infty)$, reaches its minimum $F_{\min} = F(y_{\min})$. Observe also that the $F'(y) < 0$ for $y \in (0, y_{\min})$, and $F'(y) > 0$ for $y \in (y_{\min}, \infty)$. The function $\mathrm{Sgn}(x)$ has been approximated by [6]

$$\mathrm{Sgn}_{\varepsilon_0}(y) = \begin{cases} y/|y| & \text{for } |y| > \varepsilon_0, \\ (2 - |y|/\varepsilon_0)y/\varepsilon_0 & \text{for } |y| < \varepsilon_0, \end{cases} \quad \text{where } \varepsilon_0 = 0.0001. \tag{23}$$

Numerical analysis is carried out using Runge–Kutta method for Eqs. (15) and (16), and the quadratures method is applied to Eqs. (17)–(19).

Assume that in the initial time instant the force moment $h_M(\tau) = 1 - \exp(-\delta\tau^2)$ acts on the shaft and dimensionless temperature of the pad is governed by the formula $h_T(\tau) = 2(1 - \exp(-\delta\tau^2))$. This moment forces the shaft to rotate with an acceleration. Since a heat transfer occurs, the rotating cylinder starts to expand and then a contact with the pad occurs.

First, the case of wear absence, when the shaft after a transitional state achieves the constant rotational speed $\dot{\phi}_1(\tau) = \omega_{st} = \text{const}$ is studied and the bush is non-movable ($\phi_2 = 0$). In what follows a stationary solution (in Eqs. (5), (6) and (9) the derivaties with respect to time are omitted) has the form

$$p_{st} = \frac{1}{1-v}, \quad \theta_{st} = \frac{2}{1-v}, \quad \varphi_{st} = \varepsilon m_0, \quad v = \frac{\gamma\omega_{st}F(\omega_{st})}{2Bi}, \tag{24}$$

where ω_{st} is a solution of the following non-linear equation:

$$F(\omega_{st}) = \frac{m_0}{1 + \gamma m_0 \omega_{st}/2Bi}. \tag{25}$$

Graphical analysis of Eq. (25) yields four different cases depending on the following parameters: m_0 (non-dimensional torque), γ (thermomechanical parameter associated with shaft) and Bi (Biot number).

One may expect one solution ω_{st}^3 ($F'(\omega_{st}^3) > 0$) (first case), three solutions ω_{st}^1, ω_{st}^2, ω_{st}^3 ($F'(\omega_{st}^1) > 0$, $F'(\omega_{st}^2) < 0$, $F'(\omega_{st}^3) > 0$) (second case), one solution $\omega_{st}^1 = 0$ (using approximation (23) $\omega_{st}^1 \approx \varepsilon_0 m_0/2F_0$, $F'(\omega_{st}^1) \approx 2F_0/\varepsilon_0$) (third case).

For small γ ($\gamma \ll 1$) and: (i) $m_0 \in [0, F_{min})$, Eq. (5) can possess one solution $\omega_{st}^1 = 0$; (ii) for $m_0 \in (F_{min}, F_0)$, three solutions ω_{st}^1, ω_{st}^2, ω_{st}^3 may exist; whereas (iii) for $m_0 \in (F_0, \infty)$, one solution ω_{st}^3 is possible.

For larger γ one solution ω_{st}^2 satisfying the inequality $F'(\omega_{st}^2) < 0$ is expected (case four).

In order to trace stability of the stationary solutions (24), their perturbations are analyzed. The characteristic equation of a linearized problem has the following form:

$$s(\Delta_1(s)\Omega_2(s) - 2Bi\,v\Delta_2(s)\Omega_1(s)) + a_M p_{st}(s^2 + 2hs + 1)(\beta_2\Delta_1(s) + 2Bi\,v\beta_1\Delta_2(s)) = 0, \tag{26}$$

$$\Omega_1(s) = s^2 + (2h - p_{st}\varepsilon\beta_1)s + 1, \quad \Omega_2(s) = s^2 + (2h + p_{st}\varepsilon\beta_2)s + 1, \quad \beta_2 = F'(\omega_{st}),$$

$$\beta_1 = F(\omega_{st})/\omega_{st}, \quad \Delta_1(s) = (s/\tilde{\omega})\Delta_2(s) + Bi I_0(\sqrt{s/\tilde{\omega}}), \quad \Delta_2(s) = I_1(\sqrt{s/\tilde{\omega}})/\sqrt{s/\tilde{\omega}},$$

where $I_n(x) = \mathrm{i}^{-n}J_n(\mathrm{i}x)$ is the modified first order Bessel function with the argument x.

Roots s_m ($\mathrm{Re}s_1 > \mathrm{Re}s_2 > \cdots > \mathrm{Re}s_m > \cdots$, $m = 1, 2, 3, \ldots$) of the characteristic equation (26) may be situated either in the left hand side $\mathrm{Re}s < 0$ (a stationary solution is stable) or in the right hand side $\mathrm{Re}s > 0$ (a stationary solution is unstable) of the complex variable s.

The characteristic equation (26) is transformed into the following one:

$$\sum_{m=0}^{\infty}(s/\tilde{\omega})^m b_m = 0, \tag{27}$$

$$b_0 = a_M p_{st} c_0, \quad c_0 = Bi\,p_{st}(\beta_2 + v\beta_1), \quad b_1 = \tilde{\omega}d_0 + a_M p_{st} c_1, \quad d_0 = Bi(1 - v),$$

$$c_1 = (2(2 + Bi)\beta_2 + Bi\,v\beta_1)/8 + 2h\tilde{\omega}Bi(\beta_2 + v\beta_1),$$

$$d_1 = 0.5 + Bi(0.25 - 0.125v + \tilde{\omega}p_{st}\varepsilon(\beta_2 + v\beta_1) + 2h\tilde{\omega}(1 - v)),$$

$$c_m = (d_m^{(1)} + 2h\tilde{\omega}d_{m-1}^{(1)} + \tilde{\omega}^2 d_{m-2}^{(1)})\beta_2 + 2Bi\,v(d_m^{(2)} + 2h\tilde{\omega}d_{m-1}^{(2)} + \tilde{\omega}^2 d_{m-2}^{(2)})\beta_1,$$

$$b_m = \tilde{\omega}d_{m-1} + a_M p_{st} c_m,$$

$$d_m = d_m^{(1)} - 2Bi\,vd_m^{(2)} + \tilde{\omega}\varepsilon p_{st}(\beta_2 d_{m-1}^{(1)} + 2Bi\,v\beta_1 d_{m-1}^{(2)}) + \tilde{\omega}^2(d_{m-2}^{(1)} - 2Bi\,vd_{m-2}^{(2)})$$
$$+ 2h\tilde{\omega}(d_{m-1}^{(1)} - 2Bi\,vd_{m-1}^{(2)}), \quad m = 2, 3, \ldots$$

$$d_m^{(1)} = \frac{Bi + 2m}{2^{2m}(m!)^2}, \quad d_m^{(2)} = \frac{1}{2^{2m+1}m!(1 + m)!}, \qquad m = 0, 1, \ldots$$

A general analysis of roots of Eq. (27) yields a conclusion that the following four cases should be considered. In the first case one stable stationary solution is found. In the second case, three

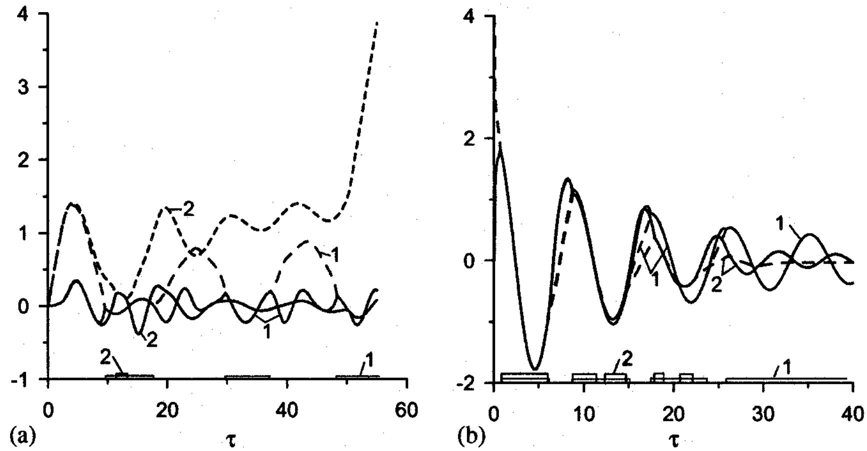

Fig. 2. Time histories of the angular velocities of braking pad $\dot{\phi}_2$ (solid curves) and shaft $\dot{\phi}_1$ (dashed curves) during acceleration (a) and braking (b) for different values of the parameter k_z (curve 1: $k_z = 0.01$; curve 2: $k_z = 0.1$).

different solutions exist, but one of them is unstable. In the third case, one stable solution exists. Finally, in the fourth case, one may expect an occurrence of one unstable stationary solution. However, in the latter case the phase space trajectories are attracted by stable self-excited stick–slip vibrations. Our further analysis is focused on this case.

The following parameters are fixed during computational process: $\varepsilon = 1$, $a_M = 1$, $Bi = 1$, $\gamma = 20$, $h = 0.05$, $\tilde{\omega} = 0.1$, $\delta = 10$. These parameters correspond to the following stationary solution: $p_{st} = 2.1$, $\theta_{st} = 4.2$, $\omega_{st} = 0.2$. However, this solution is unstable, since the roots of Eq. (27) $s_{1,2} = 0.23 \pm i0.26$, $s_{3,4} = 0.21 \pm i0.89$ have positive real parts. One may also expect a contribution of the largest period of amount of $2\pi/\mathrm{Im} s_1 = 24.2$ in the occurred self-excited vibrations.

The dependencies of non-dimensional angular speed of the shaft $\dot{\phi}_1$ (dashed curves) and the pad $\dot{\phi}_2$ (solid curves) versus the non-dimensional time during acceleration ($m_0 = 0.5$, $\phi_1^\circ = 0$, $\dot{\phi}_1^\circ = 0$) and braking ($m_0 = 0$, $\phi_1^\circ = 0$, $\dot{\phi}_1^\circ = 4$) are reported in Fig. 2a and b, respectively, for a few values of the parameter k_z characterizing pad's wear (curves 1 and 2 corresponding to $k_z = 0.01$ and $k_z = 0.1$). The dependencies of non-dimensional contact pressure $p(\tau)$ versus the non-dimensional time during acceleration ($m_0 = 0.5$) and braking ($m_0 = 0$) are reported in Fig. 3a and b, respectively, for a few values of the parameter k_z characterizing pad's wear (curves 1 and 2 corresponding to $k_z = 0.01$ end $k_z = 0.1$). The dependencies of non-dimensional wear $u_z(\tau)$ versus the non-dimensional time during acceleration ($m_0 = 0.5$) and braking ($m_0 = 0$) are reported in Fig. 4a and b, respectively (for a few values of the parameter k_z characterizing pad's wear (curves 1 and 2 corresponding to $k_z = 0.01$ end $k_z = 0.1$).

3.1. Dynamics of shaft and bush during acceleration process

Consider first dynamics for small value of the wear coefficient $k_z = 0.01$ (curve 1). In response to a driven moment action, the shaft starts to rotate (solid curves). Owing to thermal shaft radial expansion, the contact pressure p increases. To conclude, both dimensionless contact pressure $p(\tau)$

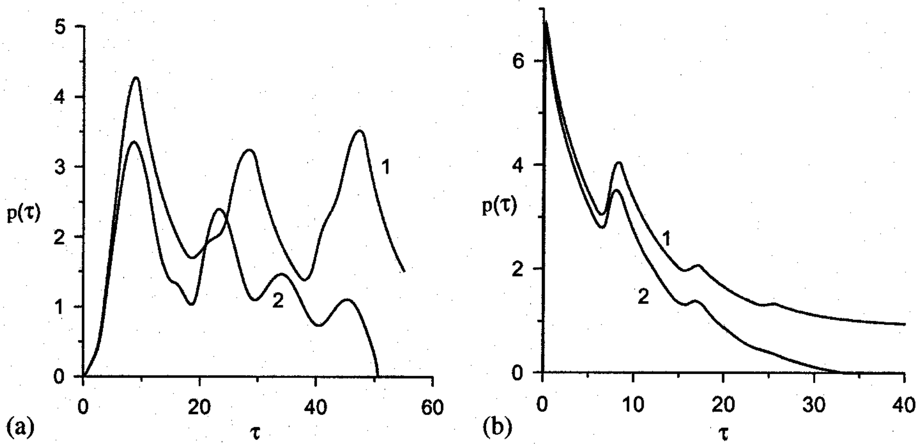

Fig. 3. Behavior of dimensionless contact pressure p versus dimensionless time τ during acceleration (a) and braking (b) for different values of k_z (curve 1: $k_z = 0.01$; curve 2: $k_z = 0.1$).

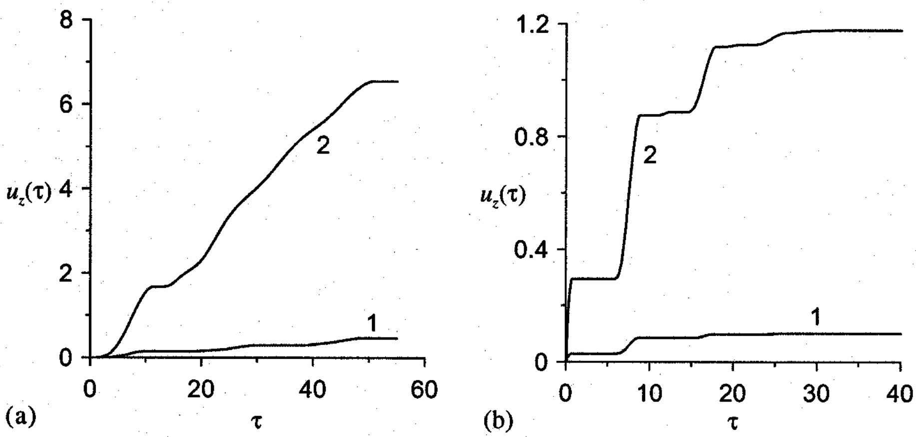

Fig. 4. Behavior of dimensionless wear u_z versus dimensionless time τ during acceleration (a) and braking (b) for different values of k_z (curve 1: $k_z = 0.01$; curve 2: $k_z = 0.1$).

(Fig. 3a) and dimensionless friction force $F(\dot{\phi}_1 - \dot{\phi}_2)p(\tau)$ (right hand side of Eq. (15)) increase yielding an increase of bush velocity and contact temperature $\theta(\tau)$ and wear bush $u_z(\tau)$. For example, in time instant $\tau = 4.27$ ($\tau = 4.76$) the shaft (bush) velocity starts to decrease (Fig. 2a). The maximal values of contact pressure are achieved for non-dimensional time units $\tau = 8.95$ (see curve 1 in Fig. 3a). For time instant $\tau_1 = 9.54$ the relative sliding velocity of both bodies is equal to zero, and a stick phase begins, which ends for $\tau_2 = 17.5$. In the stick state for $\tau \in (\tau_1, \tau_2)$ the shaft temperature decreases owing to heat exchange, and therefore both contact pressure (see curve 1 in Fig. 3a) and friction decrease, but wear does not undergo any changes. Beginning from $\tau_2 = 17.5$, a sliding phase appears within interval of $\tau \in (\tau_2, \tau_3)$, where $\tau_3 = 29.7$. In this phase both increase and decrease of shaft velocity is observed, the bush vibrates, and also a contact temperature and pressure exhibit oscillating character (see curves 1 in Fig. 3a). Friction accompanied by vibrations also increases yielding an increase of bush wear (Fig. 4a). For $\tau_3 = 29.7$ the next stick phase occurs for $\tau \in (\tau_3, \tau_4)$, where $\tau_4 = 37.3$. It is worth noticing that during stick phases $\tau \in \tau_{st} = (\tau_1, \tau_2) \cup \cdots (\tau_{2i-1}, \tau_{2i}) \cup \cdots$ the system velocity oscillates periodically with the period $2\pi(1 + \varepsilon/a_M)/\sqrt{1 + \varepsilon/a_M - h^2} = 8.89$.

Consider now the system dynamics for larger value of the wear coefficient $k_z = 0.1$ (curve 2). In the beginning of the sliding phase for $\tau \in (0, \tau_1)$, where $\tau_1 = 11.7$, all earlier mentioned characteristics of two contact bodies are similar to the previous case associated with small wear (see curves 2 in Figs. 2a–4a). Only one sliding phase $\tau \in (\tau_1, \tau_2)$, where $\tau_2 = 13.3$, is exhibited. After $\tau_2 = 13.3$ the bush starts to vibrate and the shaft rotation velocity, as well as contact temperature and wear, are increased. The contact pressure approaches zero for $\tau_c = 50.6$ in an oscillatory manner (Fig. 3a). Beginning from this time instant the contact between two bodies is lost. The shaft starts to rotate with an acceleration, whereas the bush vibrates with the period $2\pi/\sqrt{1 - h^2}$. Zones of sticks are reported in Fig. 2a and are marked by horizontal intervals 1 and 2.

3.2. Dynamics of shaft and bush during braking process

Consider first dynamics for small value of the wear coefficient $k_z = 0.01$ (curve 1 in Figs. 2b–4b). In result of temperature increase of a surrounding medium, a temperature of the shaft rotating with non-dimensional velocity $\dot{\phi}_1^\circ = 4$ also increases. The shaft and bush start to touch each other, a contact pressure increases and achieves its maximal value $p = 6.72$ for time instant $\tau = 0.26$, and both friction force and bush velocity increase. For $\tau_1 = 0.75$ the velocity of two bodies (sliding velocity) will be equal to zero, and a stick phase begins (it ends for $\tau_2 = 5.9$). Similarly to the previous case, due to heat exchange the shaft temperature starts to decrease in the stick phase $\tau \in (\tau_1, \tau_2)$, which causes a decrease of both contact pressure (curve 1 in Fig. 3b) and friction force. The latter one changes its sign rapidly for $\tau = 4.57$. Wear process is constant during stick phase (Fig. 4b), i.e. for $\tau \in (\tau_1, \tau_2)$. Zones of sticks are reported in Fig. 2b and marked by horizontal intervals 1. Beginning from $\tau_7 = 26.0$ the stick phase is exhibited, which is observed until the damped oscillations (with the period $2\pi(1 + \varepsilon/a_M)\sqrt{1 + \varepsilon a_M - h^2} = 8.89$) vanish.

Finally, let us consider the braking process for largest value of the wear coefficient $k_z = 0.1$ (curve 2). The corresponding stick phases are shown in Fig. 2b and denoted by horizontal intervals 2. For $\tau_c = 33.3$ the contact pressure is equal to zero (Fig. 3b). The shaft stops, whereas damped bush vibrations are observed with the period $2\pi/\sqrt{1 - h^2}$.

Note that when the shaft displacement achieves its extremal values ($\ddot{\phi} = 0$, see dashed curves), the friction force changes its sign. In the stick phases $\tau \in \tau_{st} = (\tau_1, \tau_2) \cup \cdots (\tau_{2i-1}, \tau_{2i}) \cup \cdots$ wear process is not observed.

4. Conclusions

One-dimensional mathematical model of a thermoelastic contact of moving bodies taking into account with an account their inertia, frictional heat generation and wear is developed. Assuming a friction dependence on the relative velocity of the contacting bodies, both acceleration and braking processes are studied. The constituted mathematical model consists of one integral equation and two non-linear differential equations. The introduced model includes both small time t_* (associated with the system stiffness) and large time R_1^2/a_1 associated with heat transfer process. Both of them are characterized by their ratio $\tilde{\omega} = t_* a_1/R_1^2$. Note that this model can be applied for a small value of the parameter $\tilde{\omega}$.

First, the stationary solutions (equilibria) are found, and further their stability is investigated following a standard approach. A stability investigation is reduced to a study of a transcendental characteristic equation. The characteristic function is presented in a power series form suitable for roots estimation. Among other results, it is shown that for some parameters one of the stationary solution is unstable. It is associated with an occurrence of stick–slip self-excited vibrations exhibited by the system. In the beginning a minor wear does not influence stick–slip dynamics essentially. However, increase of wear causes a contact loss between two bodies.

In the braking regime with large wear, a contact between two bodies is lost. In the case of small wear both of two bodies keep a contact after some time of independent motion, and a damped vibrational process is exhibited.

Let us emphasize that wear governed by the formula (18) depends on all contact parameters (relative velocity, contact pressure, and contact temperature). In addition, a total wear amount in both acceleration and braking regimes depends on initial conditions. It is demonstrated, among other results, how for some chosen parameters a total wear is increased in the acceleration regime owing to increase of time contact intervals between the bodies $\tau \in \tau_{sl} = (0, \tau_1) \cup (\tau_2, \tau_3) \cup \cdots$

It is worth noticing that for $\gamma = 0$, i.e. when a radial thermal expansion is neglected, the system does not exhibit self-oscillation behavior (one and only one unstable stationary solution does not exist in this case). Although in the second case there exist one unstable stationary solution, but there are two more stable ones. Hence, in the latter case one may observe self-excited vibration, but only if the shaft velocity ω_{st}^2 ($F'(\omega_{st}^2) < 0$) is a priori given and constant one (in fact, this case is reduced to the classical case considered in the monograph [1]).

Owing to the previous considerations, the following general observation can be formulated. If the radial thermal expansion is neglected, then self-excitation of two contacting bodies does not occur. However, it can be realized in the contacting pair for a given velocity ω_{st}^2 of one of the bodies, only if $F'(\omega_{st}^2) < 0$. Owing to radial thermal expansion of the contacting bodies, the self-excited vibrations may appear. The exhibited stick–slip vibrations are associated with a heat (slip) and cooling (stick) processes.

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