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# Theory of plates and shells: new trends and applications

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## Abstract

Numerous links of the theory of plates and shells with mathematics and physics are taken into consideration. Some new ideas and applications of plates and shells theory are analyzed along with the trends of development in applied science in the postindustrial information epoch.

**Keywords**: plates, shells, thin-walled structures, resonance, perturbations

## Introduction

A considerable number of popular books and reviews (see references [1-6]) have been dedicated to the theory of shells and plates. However only technical applications of the theory are mainly addressed. Although such a view is interesting, nevertheless it remains unilateral. The role of plates and shells theory in the modern natural history is much broader than that. Observe that, first of all in physics it is connected with the development of thin films and liquid crystal theory and, at the same time, the shells theory delivers not only ready results, but much more important general methodology of transition from 3D to 2D models.

On the other hand, in many important cases the original object cannot be described by the equations of 3D theory of elasticity. We may point out biological membranes, thin polymeric films, and thin-walled objects from materials with shape memory and with nanostructure. Besides the manufacturing technology of many types of shells essentially changes material properties. In particular, it is necessary to take into account the properties of the surface stratums being a considerable part of the shell.

The application of the latest physical concepts leads to exhibition of solitons and strange attractors to the theory of plates and shells, which is extremely interesting. The science of mathematics owes a lot to the theory of plates and shells, because the development of mathematical physics was simulated to a large extent by the problems of plates and shells theory. Many conceptions and methods originated from the plates shells theory or they were created in order to solve the problems from plates and shells theory. It is worth noting that a large number of concrete results of catastrophe theory were obtained in the theory of plates and shells before modern general rules were formulated. On the last years new problems in plates and shells theory connected with the appearance of modern materials and constructions stimulated the development of various asymptotical methods, like for example a homogenization method.

The paper includes two parts. In the first one, various applications of plates and shells theories are reported and their links with mathematics and physics are exhibited. In the second part, a relation between theories of plates and shells and computational shell theory is illustrated and discussed.

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## Part 1. Something about the theory itself

A simple experiment with thin sheet of paper will remind us that it is much easier to bend it than to switch it. The point is that the resistance of such a sheet to tension-compression $B$ is much greater than its resistance to bending $D$. It has been proved in the theory of shells and plates that $B$ is in proportion to Young's modulus $E$ and sheet's thickness $h$ and $D$ is in proportion to $Eh^3$. This is why if h is small the discussed resistance to bending is much smaller than the resistance to tension-compression. It results from this that the aim of the designer using the thin walled elements consists of making them to work via tension and not bending. Unfortunately, as a rule it is not possible to fully exclude momentous stresses. A very important feature of the shell ability to localize such states stresses comes here to help (see Figure 1).

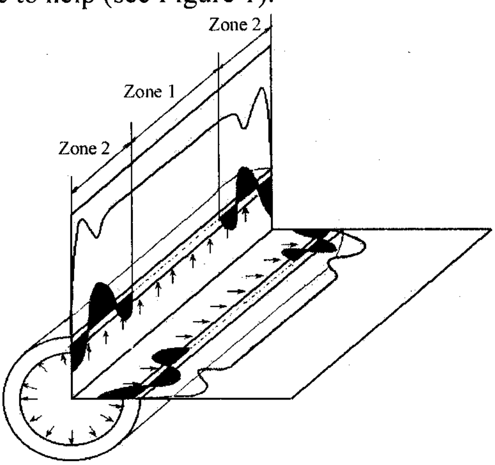


Figure 1.

High strength of shells is defined as the ability accepts edge and surface loads by tension deformation. It is the area of the membrane state which is described by the input equation with the limits h = 0. In a shell well designed the zones of big flexure appear on a small area. The stress state of edge effect is appeared here. Their definition is considerably simplified because of localization and fast changeability. Diminishing of the bending stress is usually achieved by local hardening of the edge of construction.

The interest in the theory of deformed surfaces goes back to the beginning of the 19th century in connection with Chladni's experiments. He examined the shapes formed by the sand on the glass plate, if the span was made on the plate-edge (in other words, space forms of the plate vibrations are visualized). Considering the plate as the system of crossed beams to explain qualitatively the experiments of Chladni, but not allowing for mutual interaction between the beams while twisting. Then there is the next natural step connected with taking this factor into account: the plate is treated as a 2D deformable continuum having an inner structure. Historically this direct approach to establish the theory of shells has appeared before the 3D theory of elasticity was created. The publications of Euler and J.Bernoulli Jr. (1789), and finally Germain-Lagrange bending equation (1813) were based on the direct approach [7]. It is phenomenological approach to the formulation of the theory of plates and besides the rigidities of the system; it should be based on the special experiments. This is why many scientists tried to derive suitable formulas from the first principles, i.e. from basic relations and at the same time the problem of definition of rigidities properties would be solved. As those suitable formulas Poisson and Navier chose the relations of molecular theory, and they were based on the conviction deriving from Newton and stating that the elasticity feature can be explained from the point of view of attractive and repulsion forces acting between the smallest portions of bodies. But physics of those times as a science was not ready to examine the phenomena thoroughly on such level. It is interesting, however, that also in our times the similar altitude - obviously on a higher level - was developed in the theory if thin membranes consisting of one or many layers. As a matter of fact they are physical objects the descriptions of mechanism of which by the use of 3D continuous mechanics methods are radically impossible.

On the other hand Cauchy (1828) and Poisson (1829) tried to create the theory of plates starting from the theory of elasticity which was formulated for the first time not long ago, based on the arrangement of hypothesis.

They reduced 3D equations of elasticity theory to 2D ones splitting the unknown stress components according to the increasing degrees of thickness. However, not physical, but mathematical difficulties were created there, and this method was long forgotten having been the subject of fair criticism by Saint-Venant and Kirchhoff. The next mathematical deduction of plates and shells theory from the elasticity theory was possible only in the 60's-70's of the 20'th century, nearly a 100 years after they were phenomenological created.

Success was brought by phenomenological approach within the 3D elasticity theory. Kirchhoff (1850) created the first satisfactory theory of bending planes basing on the following arrangement of hypothesis: rectilinear fibers, perpendicular to the middle surface of the plate before the deformation, remain rectilinear and perpendicular to bent surface after the deformation, keeping their length; the lack of mutual interaction of the layers appears parallel to the middle surface in the direction normal to the layers.

Later on A. Love (1888, 1906) generalized this hypothesis for bent surfaces, and created the basic dependencies of shells theory.

## Traditional and new area of applications

Techniques. The thin shell combining the high durability with small weight, simplicity and technology of the product, has become one of the most widely spread constructions in numerous branches of contemporary technique, above all in the rocket, air-craft, ship building industries, industrial and civil engineering, and in construction of chemical devices. It is an extremely interesting subject (for instance oil extraction from the sea bottom and connected with it construction of enormous drilling platforms put new interesting problems before the theory), but as it is explicitly explained in scientific popular literature [1–6], this is why we shall discuss new applications of shells and plates theory.

Geomechanics. The model of lying in slimy liquid and compressed in its surface plate turned out to be quite effective in order to examine the structure of earth's crust. Such a model allows for making sure that elastic folding of the earth's crust is not possible.

The problem touching the subject is solved while crashing the ice by the ice-breaker. The ice field is regarded in this case as elastic plate lying on the liquid surface, and the ice-breaker is treated as an elastic beam.

It is interesting that the useful geophysical results can be obtained by the simplest model of the terrestrial globe, in which homogeneous spherical nucleus is surrounded by the spherical shell with constant density, different from nucleus' density mantle.

In accordance with modern geophysical date terrestrial shell is presented as a thin (the ratio of the thickness to the radius is about 0.005), solid deformed shell (lithosphere) situated on a viscous foundation (ozonosphere). The tidal waves are created on the Earth's surface because of the Moon's gravitation. The peaks of the waves are situated approximately on the axis the Earth - the Moon and their size is of 30-50 cm. The similar movements are used by some scientists to explain the continents drift and the calculation within the model can be based on. The theory of the plates and shells is also used by scientists who propose to treat the Earth as the detector of gravitation waves. The presence of these waves' results from general relativity theory of Einstein, however the experimental check of the some such forecasts remains of present beyond the reach of contemporary physics because its effects are very small.

Thin films. There are some mechanical objects the description of which by the use of 3D continuous mechanics is impossible or little efficient because of the lack of the continuity of structure in one direction. In other words, the thickness of such objects is formed by 2-3 molecular grains; this is why it is impossible to use averaging. Thin membrane consisting of one or two layers of macromolecules is most widely spread and the revolution in microelectronics is above all connected with such thin-walled materials.

The test to check the physical process in two dimension results from the fact, that these materials can be created as 2D ones. Liquid crystals are one of the forms of 2D shape substances. Other forms of this type are thin membranes of superfluid gel, superconductors, liquid layers on the water surface (for instance the membranes of surfactants), and also the system of atoms adsorbed in the foundation. The complex examination is really indispensable. In order to understand the essence of the processes occurring in 2D systems, mechanical properties of thin membranes attract a particular interest. Let's consider as the example the atoms adsorbed on the surface, representing the ordered structure. In normal conditions the surface of the bodies and liquids are covered with the membranes of atoms or molecules adsorbed from the atmosphere or diffused from the some material. Similar structures can be observed in a large number of the systems: the adsorbent - the foundation. It turns out that in the most obvious case the 2D crystal on the foundation can be considered as a membrane on the elastic nonlinear base, which allows for taking into consideration all basic properties of mechanical behavior of 2D crystal.

Biology. Biological membranes are one of the most interesting objects of the nature. They are complex phenomena, enabling the transformation of chemical energy into mechanical energy, supervising the mass exchange, etc. Mechanical phenomena not always play decisive role in the biological processes, but their examination is essential in order to get the full image. The model of elastic anisotropy ((i.e. having different rigidity properties in different directions) plate on elastic base is appropriate from the mechanical point of view [8, 9].

The blood vessels are considered as multilayered deformed shells with active muscle layer through which a viscous fluid flows. Sudden heart catastrophes or heart disturbances can be explained on the basis of such model, and vessel spasm can be treated as the loss of stability. Such models should help in defining the effects of drugs taking and their requirements.

The definition of mechanical properties of red blood cells within theory of plates and shells helps to define the blood viscosity, the ability of erythrocytes to pass through the capillary vessels, the acceptable operation time with artificial blood transfusion devices.

A suitable model of eye deformation is of great importance when checkup the nerve, because, as it was already discovered in the 19$^{th}$ century, the strain of eye nerve is the most harmful factor influencing the eye during its work in short distances. In recent works an eyeball is modeled as elastic homogeneous spherical shell of fixed thickness and the eye nerve as the elastic bar of the fixed cross-section.

Many inner human organs (for instance stomach) can be considered as soft shells. Such models help to define suitable materials construct the artificial organs. For example, the prosthetic appliance of mitral valve can be calculated as the quarter of a spherical shell and the artificial important vessels - as cylindrical shell.

Physical phenomena in plates and shells. In September 1993 American Mathematician Martin Kruskal received National Science Medal - the lightest scientific reward in the United States. He reward for the series of research in the field of the nonlinear sciences.

It is worth noting, that for a very long time the whole science in reality has been based on the concept of linear processes for which the superposition rule is right: if the influence is n times bigger, then the results are also n times greater. Hooke's law, Ohm's law, isochronisms of linear oscillations of pendulum (the period of vibrations does not depend on the amplitude) can serve as examples. But actually all systems in the world are nonlinear - the superposition rule does come to be true. Obviously some corrections can be introduced if the oscillations from the linearity assumptions are accepted as small, but along with it the system is accepted as nearly linear, i.e. it is difficult to take into account qualitative changes of system behavior due nonlinearity. Mathematical difficulties have not allowed to get a solution of really nonlinear systems for a long time and it has came to relying upon some probable (as it scanned) hypothesis. Consequently, Debye stated quite obvious hypotheses for nonlinear discrete systems. At first sight small perturbations quickly leads to energy spreading into degrees of freedom of this system. It's analytical or numerical check has not been possible for a longtime, and only in 50$^{th}$ Fermi, Pasta and Ulam decided to display numerically 1D nonlinear chain of oscillators. However the authors of the experiment noticed: "Some phenomena which initially surprised us appeared as a result of our calculations. The exchange of energy occurred only between some low model values instead of uninterrupted flow of energy from the first modal value to higher. The tendency of the system to divide the energy evenly to the degrees of system freedom in a given time was hardly noticed. On other words, it was clear that in the system there was no mix" [10].

In 1965 Kruskal together with Zabusky derived a continuous model from a discrete chain and analyzed it numerically. As a result they found nonlinear solitary waves, which always reconstruct its form. As some definite analogy with the particles behaviors was perceived, Kruskal and Zabusky introduced the term 'soliton'. Further intensive research supported by many scientists from many countries showed that we deal with much generalized situations and the use of solitons allows in same sense to extend superposition principle for nonlinear systems [11, 12]. Naturally, this mathematical technique has found application while analyzing nonlinear problems of shells and plates theory. Moreover the examination of solitons in the shells and plates theory is not only a matter of theoretical problem - similar waves can appear, for instance during the transport of liquid in the pipeline (especially during the blood flow in the blood vessels). One can mention consequence that the theory of blood pressure measurement (Korotkov's sound definition) is not fully elaborated. One of the probable explanations is as follows: the wave which is running through the artery generates high frequencies vibrations, which are treated by therapeutist as a sound. There are also some forecasts that the pulse is constituted by located nonlinear waves.

Strange attractors. Attractors, i.e. attracting areas of phase space (the space of system position and its speed), where the system gets stuck when it gets there always played important role during investigation of motion.

The system with energy dissipation is the simplest example of such behavior (in this case equilibrium regime or the regime of periodic oscillations). It would seem understandable by itself that similar sets include exponentially stable trajectories. It turned out, however, that the presence of attracting sets of unstable trajectories is possible (strange attractors) [13]. A possibility of the chaotic behavior of deformed systems is conditioned by the lack of stability of the motion trajectory. At present the theory of strange attractors is spread over the turbulence research in hydromechanics. For elastic system the chaotic behaviors was discovered for the vibrations of compressed bars and plates in postbuckling state [13]. At the same time the ideology of strange attractors can turn out to be important in the theory of shells stability. The following analogy can prove it. The so called Benard's cells are one of the basic objects of the considered domain. They are hexahedron prismatic; appear in liquid layer under beating. But during buckling of spherical shell under external pressure, the shell surfaces turns to hexagon structure resembling Bernard's structure. This is why I.Vorovich notices: "It is interesting to use the turbulence theory methods into the examination of the buckling of very thin shells for which shell's dividing into small patterns is achieved with the buckling".

Cavitation. Lavrent'ev [14] carried out a famous experiment which brought unexpected results. In a thick-walled container filled with water with a hole covered with a thin metal plate the explosions were carried out in the liquid opposite the plate. The plate's deflection increased along with the power of explosion and the plate reached deflection inside the container contrary to expectation.

Theoretical explanation of these interesting phenomena is as follows: the increase of explosion power leads to cavitation, i.e., intensive occurrence of cavitation bubbles and unloading of the plate occurs. The time of loading the plate does not exceed the time of plate's displacement to reach the maximum displacement.

It is interesting to recall one more paradoxical result obtained by Hertz in connection with the discussed problems. While examining the plate's deformation under the influence of concentrated force on the elastic base he reached the conclusion that the displacement surface contains convexities and concavities, i.e., the displacement become considerable and a shell is obtained instead of a plate. Because of such transformation the plate mode of weight heavier than water can be made float by imposing in the middle of it the concentrated force large enough.

The edge resonance. It is possible to discover new effects even in such see mingle fully elaborated theory as the theory of plates and shells. In 1956 the edge resonance was discovered [15]. The experiments showed the existence of intensive movements located strongly near the edge of the disk. These forms of planar vibrations were called the edge resonance. Some time later similar phenomena were discovered theoretically and experimentally for planar vibrations of the plate and also for shells (in some conditions). As the edge resonance frequency really depends on Poisson's coefficient $\upsilon$, increasing when stress increases, the phenomenon can be used to define experimentally the value of $\upsilon$.

Localization of vibrations. About 50 years ago Andersen proved that it is possible to locate vibrations in some space domain in the framework of quantum mechanics (for this discovery the was honored the Nobel prize). It turned out later that a similar like behavior may appear in periodic constructions. This localization may lead to vibration forms located in the part of the system volume [16].

Mutual interaction with the physical fields. Constructions elements in modern technology (included shells) are subjected to a large influence of high temperature and pressure, neutron and X-rays radiation, diffusion and strong magnetic fields. The complexity of such problems requires the introduction of corrected theory of plates and shells, and also corrected theory of processes (i.e., irreversible processes in thermodynamics).

The development of physics also constantly requires the solution of shell theory problems. The appearance of high temperature superconductive materials brought up to date the problem of magnetic elasticity of superconductive thin shells [17]. It comes to considering the shells both front surfaces covered with a thin layer of superconductive material and the shell is in a strong magnetic field. The practical application of discovered newly high temperature superconductors depends to a large extent on the solution of the shown problem.

The star war program (as well as peace problems of method forming) decides about the unusual actuality problem of plates and shells deformation subjected to thermal shock from a concentrated energy source. In this case the problem of heat exchange should be solved first, and then thermal elasticity problem should be followed. The problem may be simplified since the dynamical thermo-elastic stresses are quickly running processes in a thin (boundary) layer adherent to the body surface.

We can also mention the problems of piezoelectric plates and shells [18].

## Mathematical models in the shell's theory

Asymptotic methods belong to a set of mathematical ways of problem solutions neglecting of a "small" (in some sense) components [19, 20]. The neglecting of these components increases the symmetry of problems and allows for their solution in the limiting cases. The meaningful simplification of the process of solution finding is obtained, whereas the introduction of corrections permits to show the real image of process of the analyzed phenomenon. Indeed, asymptotic methods are something more than the set of technical methods - in natural sciences they represent the whole system of views or even ideology. Asymptotic methods found broad applications in the shells and plates theory because of the number of reasons.

Singular perturbations. The theory of shells and plates can be treated as the elasticity theory for those bodies, the two dimensions of which are considerably larger then third one [21, 22]. As a result a small essential parameter appears in the equation, for instance ratio of shell's thickness to its radius. This is why asymptotic methods turn out to be most adequate in their nature from both mathematical and physical points of view. Besides, the theory of plates and shells belongs to the science with clearly defined applied character, where the problem creation of approximate solution methods is the most important, especially the problem of elimination in input equations of those quantities, which can noticeable influence final result and which cause complications not represented by the nature of the problem. Since the input equations of the shell theory has a small parameter, in this connection (from the point of view of asymptotic methods) the so-called singular perturbation takes place. It means that the deviation of input solution from the limiting one is focused in narrow located range of the edge effect. A singular phenomenon plays an important role in hydromechanics, where it is called the boundary layer: the viscous properties may be neglected everywhere, but not in the narrow near body surface, where the viscosity of the liquid should be taken into consideration. The study of the boundary layers is simplified because of its narrowness. Observe that the edge effect was created and understood in the shells theory much earlier than in hydromechanics [6, 23].

The homogenization method. The basic cost of thin-walls is the danger of the buckling, which leads to the strong bending stresses. Proper buckling loads define the construction durability, because strong displacements lead to destruction or irreversible displacements.

"The commonest way to guard buckling is to stiffen the skin of a thin-walled structure by attaching extra members, such as ribs or stringers, to it. Stiffeners which run circumferentially are generally called 'ribs', while those which run lengthwise are called 'stringers'. The shell-platting of ships is traditionally stiffened by means of ribs and bulkheads, though, recently, large tankers have been built on the 'Isherwood' system, which largely depends upon longitudinal stringers. A sophisticated shell structure, such as an aircraft fuselage, is usually stiffened by both ribs and stringers. The hollow stems of grasses and bamboos, which tend to flatten when they are bent, are very elegantly stiffened by means of 'nodes' or partitions or bulkheads, spaced at intervals along the stem.

Wood is a cellular material, and so are most other plant issues, notably the stem-walls of grasses and bamboos. Furthermore, in the competitive struggle for existence, many plants depend critically upon the structural efficiency of their leaves, because they must try to expose the maximum area to sunlight, for photosynthesis, as the minimum metabolic cost. Leaves are therefore important panel structures, and they seem to make use of most of the known structural devices to increase their stiffness in bending. Nearly all leaves are provided with an elaborate rib structure; the membranes between the ribs are stiffened by being of cellular construction, and in some cases they are further stiffened by corrugations" [2, pp.294, 295].

The introduction of stiffening ribs may lead to big heterogeneity of stress deformation state of construction, and it may worsen operating conditions. These all requests precise enough analysis. In practice, one often passes to the structurally-orthotropic theory. Rigidity and inertial features of supporting elements uniformly spreading on a surface of the shell, which is now considered to be homogeneous, but endowed with some new features in accordance with the construction properties of the object (structural orthotropy). The introduction of structural orthotropy gives the possibility of leaving the peculiarities of force cooperation between ribs and the skin, and the problem can be simplified radically. Meanwhile, the structurally orthotropic scheme allows to define precisely enough only global, not local components of the construction. This difficulty can be excluded by the introduction of the homogenization method based on separation of fast and slow components of the solution (Figure 2). The solution of structurally-orthotropic theory problems comes out via homogenization and the corrected solution allows for the definition of all stress deformation state components, which is not more complicated than the one received as result of structurally-orthotropic theory.

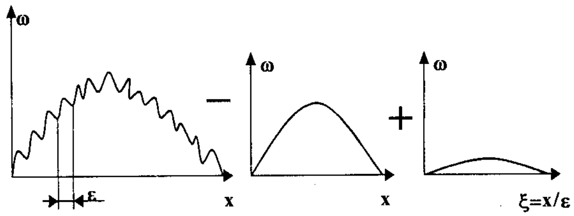


Figure 2.

The modern homogenisation method can be described as follows: the solution is presented as the global (slow) and local (fast) components. The last one is solution on the periodically repeated cell, considered in new “stretched” coordinates). As a result we get two slowly changing systems instead of one initial fast changing system.

An interesting peculiarity of asymptotic methods is briefly addressed now. The rib shell is really more complicated for calculations than the smooth one. But the presence of new parameters and new structures (the arrangement of ribs) leads to new broader possibilities, than in the isotropic case, an especially for an asymptotic integration.

The second example is connected with the perforated plates [24]. The investigation of similar system is complicated because of occurrence of the multi-connected domains (Figure 3,a).

Obviously, the homogenization method can be applied here as well, but what about getting the solutions to the problem on a shell (i.e. periodically separated continuous sector with one hole, Figure 3,b). Two separated cases can be examined. If the hole is small, the outer boundary influence may not be taken into consideration, and the plane with one hole may be considered. However, if the hole is large, then the round one is replaced by the square one, the initial plate by the grid (Figure 3,c). Then the limiting solution can be matched, and one gets the solution for any value of whole radius.

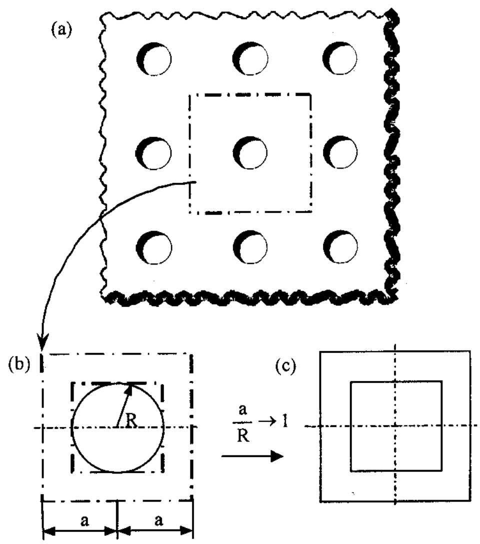


Figure 3.

The periodically perforated plate can be replaced by homogenous one with reduced parameters.

The analysis of both quoted examples, as well as many engineering calculation methods shows that all sensible reductions are asymptotic in nature. What would be the role of pure asymptotic approach (would be, because unfortunately, the cases of direct use of asymptotic methods in engineering practice are not too numerous)? First of all, it is an estimation of applicability engineering theories. In other way, a judgment of their application would have to be introduced by way of expensive experiments or time-consuming computer calculations. Second, asymptotic discovers subtle effects influencing the operating construction, e.g. stress concentrations.

The problem of optimal designing, the most important for technology will be considered as the last one. This problem is in inverse ratio to the calculations of the input system. If we were to use exact algorithms, enormous amount of machine time is lost with every step of optimization and the steps may count in thousands. A simple engineering scheme may be taken, but there is fear that we shall get results, where principally it is not applied, as we shall miss important effects of stresses concentrations represented by the boundary layers not considered in simple schemes. So, it means that asymptotic approaches are highly useful owing to their simplicity and sufficient precision, and with precise estimation of the domain of application.

Catastrophe theory describes jump changes of systems or processes. The elastic compressed ruler can be the simplest model showing such behavior - at first it keeps its straight shape but when the load is increased, it buckles [25-27].

Theoretical calculations of bars and plates buckling were well verified by experimental investigations; however this has not been the case for shells. Observe that theoretical value of buckling load of cylindrical shell P may be 2-3 times larger than the experimental value Q. The whole sequence of factors is quoted in order to explain this phenomenon: the influence of boundary conditions, material plasticity, etc. Particular precise experiments conducted on this situation which gave the values Q = 0.9-0.95 P, helped to understand the situation. However, in practice there are always deviations from an idealized system: eccentricity of the applied load, deviations of the middle surface from the ideal one, etc. Their occurrence causes a sudden decrease of critical forces and of shells buckling (Figure 4, 5).

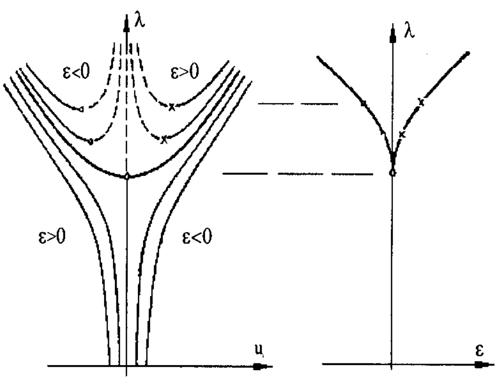


Figure 4 [25].

The behaviour of compressed bar according to the occurrence of initial imperfections (e.g. deviation of the bar shape from the ideal form). The dotted line signifies unstable solution branches. Dependence from ε is soft, imperfections does not lead to qualitative changes, only to small quantitative ones.

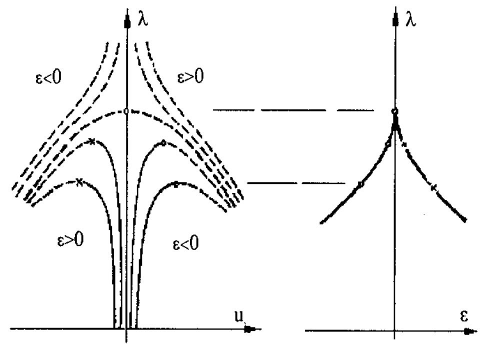


Figure 5 [25].

The diagram of dependence of load on displacement, and the diagram of sensitivity to imperfections of cylindrical shell behaviour under axial compression is present or spherical shell at the external pressure. Stiff dependence on imperfections is obvious.

It is interesting that suitable works were done in the theory of shells and the catastrophe theory was formed as an independent branch of science. Arnold notices [26]: "Of course contemporary general theory allows checking more complicated properties more precisely and with the smaller loss of forces. But the examinations of simplest and next frequently occurring properties have the biggest practical value. Fundamental works of the predecessors of catastrophe theory keep their significance also now, when the mathematical structure is fully explained".

Along with it the catastrophe theory allows for explaining some important features of constructions operating with the buckling. For instance the integration of two subsystems, each of which has soft sensitivity to imperfections, may lead to the system stiffly depended on imperfections (i.e. the dangerous system can be created as a result of unification of two safe ones). The presented danger increases particularly, when the buckling loads of the subsystems are close (and it worth noting that the designers tend to this case during optimum design). For example, the strengthening of plates by stiffening ribs can lead to the system sensitive to defects, when the plate and ribs separately are deprived of this feature. If the pointed out is not taken into consideration, the drawn conclusions may be not correct. This is why it has been believed for a long time that the introducing of ribs decreases the sensitivity of elastic cylindrical shell to initial imperfections, but this argument turned out to be inaccurate in case of narrow and tall enough ribs.

## Part 2. Computers and science

It is evident that thousands of papers and books (for instance see [28-35] and the cited references therein) are devoted to this subject, and it is regularly discussed either on special scientific conferences or in a general public way. We would like to address one aspect of the problem, not clearly stated in the cited references. Because due to computer simulation a new virtual world is created, and the same rules hold for both real and virtual worlds, therefore (especially in the computer age) a particular attention must be paid to application of analytical and asymptotical methods.

Such phrases as "Nowadays computers expand very widely human being's possibilities", "Computers are needed but they cannot fully substitute human behaviour", and so on, are well known, many times repeated in numerous journals and books. However, this issue becomes more complicated if somebody tries to read more carefully papers by J. Guckenheimer [29, 30], M. Gromov [31], I. Elishakoff [32], and others [33-35]. We are going to present our point of view on the subject of computers versus the traditional analytical approach on the basis of the cited scientists' opinions as well as our own understanding.

As it known, social response to new event is characterized by three steps: unlimited optimism, a reaction showing strong criticism with an emphasis on disadvantages, and finally, an inclusion of a new event in a cultural paradigm.

It seems that an influence of computers on our life is still in the initial phase. Really, the number of books, papers, talks, TV programmes, etc. devoted to advertising the power of computers is extremely high. On the other hand, many scientists are fully confident that serious estimation of limits of computer oriented science is needed [28-35]. A belief in unlimited possibilities of computers leads to manipulation and control of the social behaviour of a human being. Real complex processes like a climate change caused by a thermal effect or a nuclear war, or a state of economics depend on extremely high numbers of parameters. Very often a priori model of the process is not known but it depends on the chosen parameters. For instance, according to one group of researchers, a thermal effect leads to a temperature decrease on the Earth, whereas another group predicts its increase. Simulation models also lead to prediction of the duration time of a "nuclear winter" either within tens (or hundreds) of years or within weeks (or months). Both groups accuse each other of a juggling with initial data or an improper choice of essential parameters, and so on.

A dangerous illusion is introduced, when one assumes that accuracy can be improved by increasing the number of parameters, which are sometimes not easy to define or the price of their estimation is high. Computer algorithms should be stable against unknown parameter changes, but one can observe a lack of the corresponding theories in this field [29, 30].

According to Hamming, a good theoretical researcher should estimate a result a priori and should behave rather in a sceptic way as far as the obtained results are concerned. The main problem occurs not due to potential wrong results introduced by the subroutines used, but due to a formulation of the problem and due to the fact that the expectations are different. This observation corresponds with the paradoxical phrase of Hamming "It is better to solve a problem properly formulated in an improper way than to solve an improperly formulated problem using a proper way" [28].

Furthermore, even if a problem is properly stated, one should recognise advantages as well as disadvantages of the numerical algorithms used. "When we bundle exacting algorithms into libraries and wrap them into packages to facilitate easy use, we create de facto standards that make it easy to ignore numerical analysis. We regard the existing base as static and invest in the development of problem-solving environments and high-level languages. This is needed, but we also need to maintain our investment in continuing research on algorithms themselves" [30]. An increase of the number of subroutines and program packages developed on the basis of different principles results in an increase of probability of omitting the existence of their limitations and principal systematic errors.

One of the psychological problems of human being is a tendency to avoid responsibility, which fits well with the virtual world and computers. "Computer programs have made to many people believes that all technical problems are readily solvable" [33]. P. R. Halmos [36] reminded us about "permanent human opposite to a creative work", and computer users have a "lucky" possibility to "replace ideas by computations" (instead "replace computations by ideas" [37]).

Finally, the following psychological problem appears. Large computer projects (demographic, genetic, climate modelling, etc.) need extreme efforts due to identification, unification, verification of the number of parameters, and so on. It means that a share of uncreative activities is also extremely high. An individual influence of a researcher becomes washed out, but prise of his mistake may be very high.

## Honesty is the best policy

It is clear that for computer science oriented researches it is rather difficult to be totally honest: grants and funds as well as the advertisements are needed. But compromise between honesty of a researcher and the battle for money should be achieved. It seems to us, position of famous mathematicians concerning using of mathematics must be cited as an example.

This matter has been discussed by N. Wiener and J. Schwarz. In particular, Wiener wrote [38]: "One of the fundamental responsibilities of a mathematician is to persuade to other scientists not to many expecting from mathematics".

In his turn, Schwartz notes [39, p.358]: "Related to deficiency of mathematics, and perhaps more productive of rueful consequence, is the simple-mindedness of mathematics-its willingness, like that of a computing machine, to elaborate upon any idea, however absurd; to dress scientific brilliancies and scientific absurdities alike in the impressive uniform of formulae and theorems. Unfortunately however, an absurdity unclad. The very fact that a theory appears in mathematical form somehow makes us ready to take it seriously".

I. Gel'fand et al. wrote even sharper [40]: "An experience of collaboration with physicians and biologists shows that a new mathematics is needed which does not exist up till now".

It seems that in published papers with computer simulation results stronger emphasis should be put on a choice of parameter sets and the rules for negligibility of other parameters should be exhibited (if there is a lack of such rules, it should be clearly stated) [41]. A honesty of the computer programs creators is required also because: "The engineers who can 'stand up to' a computer will be those who understand that software incorporates many assumptions that cannot be easily detected by its users but that affect the validity of the results. There are thousand points of doubt in every complex computer program" [33]. A user must know from the programmer about all 'dangerous' places, limitations and weak points of a program. On the other hand a program possesses its own life, and even not everything, in some sense, is clear to its creators. The experienced programmers believe that each 'complex programs has errors', and a testing and improvement of a program belong to most complex and expensive parts of this task.

"The metaphysic of *Ada* may thus be defined as follows: what can be controlled is never completely real; what is real can never be completely controlled. The metaphysics is strikingly similar to Einstein's famous aphorism, "As far as the laws of mathematics refer to reality, they are not certain; and as far they are certain, they do not refer to reality" [42]. *Ada* is thus Nabokov's tribute to an idea intrinsic to the field concept, that reality can never be entirely captured in the abstractions or either art or science [43, p.136].

And therefore a "successful computer - aided design requires vigilance and the same visual knowledge and intuitive sense of fitness that successive designers have always depended on when making critical design decisions" [33].

## Computational Mechanics, Computational Fluid Dynamics, etc.

One can see a lot of "new sciences": Computational Physics, Computational Biology, etc. (reader can himself to add word "Computational" before any science title). Are there really "new sciences"? Why one can not see "Algebraic Biology" or "Geometrical Geography"? We refer some opinions of world - known experts.

Batchelor et al. wrote [44]: "This debate was motivated by the increasing importance of CFD (Computational Fluid Dynamics) in research and applications, which make some people believe that in the foreseeable future CFD will dominate fluid mechanics, relegating analysis and experiments to a peripheral role. Has the idea of a "numerical wind-tunnel", replacing real ones, any validity? Does fluid dynamics differ from other physical science centred on experiments and observations?

The conclusion of the debate was that experiment, analysis and computation are of comparable importance and, ideally, one would like to see them interact in the same laboratory. None of these can flourish without the other two".

Novoshilov et al. wrote [45]: "The authors are firm opponents of exchanging the fundamental science - the theory of shells - 6or one of the chapters of applied sciences. This regrettable tendency is a side effect of implementation of numerical methods. On journals (and monographs) pages there were flowing avalanches of works with the opinions of numerical experts, realised sometimes with the use of standard packets of applied programmes. Unfortunately (or perhaps fortunately) there cannot be a pattern set for all life cases. At the same time the most important thing is the understanding of the penetrated problem, and not the number. As for the numerical methods, then when the complicated problems are set, the introductory analytical solutions may turn out to be very useful, and sometimes they are even necessary to quicken the realisation of numerical algorithm. In the domain of mechanics of deformed solid body it is primary to accept initial hypothesis and assumptions based on deep understanding of work of the material in construction.

The estimation of errors in the accepted hypothesis and assumptions - the formation of the set of equations adequately describing the construction working.

The position of the authors: sensible combination of analytical and numerical methods with the understanding of the mechanical side of the considered problem".

By the way, "Accurate numerical computation of shells with arbitrarily small thickness is impossible in practice. Standard finite element codes usually fail to give accurate results for h/R less then 0.01 or 0.001" [46].

Is that so that there is an insurmountable precipice between the "analyst" and the "computer expert"? We are sure that this is not the case. We know from our own experiences that it is much easier to reach an agreement with a clever and extensively educated "computer expert" than with a limited "analyst". Moreover, it can be assumed that the boundary between the analysts and the "followers of the number theory" will violently obliterate together with the implementation of the programme packets of the "Mathematica" type. The latter allows forming symbolic explanation, and also the packets giving the possibility of analytical presentation of the bulk of numerical data.

## Brave virtual world*

*The speed of computation as a limiting factor in simulating physical systems has largely been replaced by the difficulty of extracting useful information from large data sets.*
*J. Guckenheimer [29]*

Wide using of computer models creates the world of new reality, and we need experts for analysis this world. The situation is similar to that of the status of experimental and theoretical physics: an expert of computer information analysis (further referred to as an analyst) should translate the simulation results into a language of human oriented science.

The fundamental goal of an analyst working in a virtual world is the same as in the real one. First of all we must remember, that "The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms" [42, p.256]. That's why "the search for symmetries and regularities" [31, p.846] (as well as evident asymmetry and irregularity!) plays the key role for any researcher. Here is an important place for asymptotical methods, based on decreasing dimensions and decompositions, multiple scale analysis, and homogenization. In our opinion, in analysis of virtual world a key in place should play an asymptotic analysis.

Naturally, the asymptotic analyst and the programmer users should work in close collaboration. An advantage of computer models lies in a possibility of quick verification of hypotheses, detaching small (large) parameters, estimation of validity of different characteristic, etc. A role of an analyst is to formulate important and properly stated questions as well adequate conclusions on the basis of the obtained results.

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* Compare [47].