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FRICTIONAL AUTO-VIBRATIONS IN A CONTACT THERMOELASTIC PROBLEM

The solution of a contact thermoelastic problem of a solid in the form of an elastic layer moving between two rigid walls subject to friction and heating is presented. It is assumed that the friction coefficient depends on the relative velocity between the contacting bodies. A stability of the stationary solution is studied. A computation of contact parameters during heating of the bodies is performed. A possibility of existence of frictional auto-vibrations is shown.

Frictional auto-vibration is well-known phenomenon of intermittent motion caused by a velocity dependent friction force combined with elasticity of a mechanical system [4]. There are many examples in a literature focused on analysis of autonomous systems exhibiting regular non-linear self-excited vibrations [1–4]. Our new proposed model does not have any elastic part, but it can exhibit self-excited stick-slip vibrations (Fig. 1).

1. Statement of the problem. Let us consider one-dimensional model of the thermo-elastic contact of a body with a surrounding medium. Assume, that this body is represented by a rectangular plate $b_1 \times b_2 \times 2L$ (Fig. 1). The plate has the mass M_1 subject to the force $F = F_* h_F(t)$ and moves vertically along walls in direction z_1 of the rectangular co-ordinates $0x_1y_1z_1$. In the initial instant the body is situated in the distance Z_0 and possesses the velocity $\dot{Z}_0$. The distance between walls is always equal to initial plate thickness $2L$.

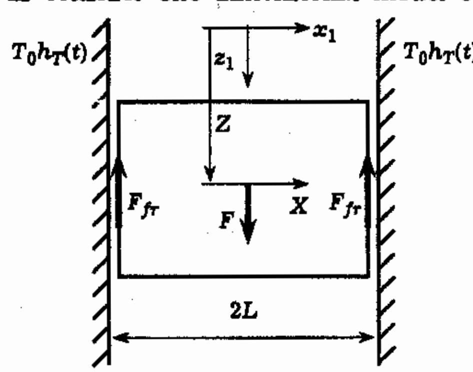

Fig. 1. The anelyzed system.

It is assumed that the heat conduction between the layer and the walls obeys Newton's law. In the initial instant the temperature is governed by the formula $T_0h_T(t)$ $(h_T(t) \to 1,\ t \to \infty)$. It causes heat extension in direction of $0x_1$, and the body starts to contact with walls. In the result of this process, a frictional contact on the parallelepiped sides $X = \pm L$ occurs (see Fig. 1). The simple frictional model is further applied, i.e. friction force F_{fr} is approximated a product of a normal reaction force $N(t)$ and a friction coefficient. It means, that $F_{fr} = f(\dot{Z})N(t)$ is the friction force defining a resistance of two sliding bodies. Here, contrary to the assumption made in reference [6], the kinematic friction coefficient $f(\dot{Z})$ depends on the relative velocity of the sliding bodies [2].

The friction force $\sigma_{XZ}(X,t)$ per unit contact surface $X = -L$, $X = L$ generates a heat. According to the Ling [5] assumptions, the friction forces work is transmitted into a heat energy. Note that the non-contacting plate surfaces are heating isolated and have the dimensions of $L/b_1 \ll 1$, $L/b_2 \ll 1$, which stands in agreement with the assumption of our one-dimensional modelling.

In what follows the problem is reduced to determination of the mass plate centre displacement $Z(t)$, plate velocity $\dot{Z}(t)$, contact pressure $P(t) = N(t)/b_1b_2 = -\sigma_{XX}(-L,t) = -\sigma_{XX}(L,t)$; plate temperature $T_1(X,t)$, and displacement $U(X,t)$ in the X axis direction.

In the considered case, the studied problem is governed by dynamics of the plate mass centre

$$m\ddot{Z}(t) = F_* h_F(t) - 2f(\dot{Z})P(t), \tag{1}$$

and equations of the heat stress theory for an isotropic body

$$\frac{\partial}{\partial X}\left[\frac{\partial}{\partial X}U(X,t) - \alpha_1\frac{1+\nu_1}{1-\nu_1}T(X,t)\right] = 0,$$

$$\frac{\partial^2}{\partial X^2}T(X,t) = \frac{1}{a_1}\frac{\partial}{\partial t}T(X,t), \qquad X \in (-L,L), \tag{2}$$

with the attached mechanical

$$U(-L,t) = 0,\ U(L,t) = 0, \tag{3}$$

heat

$$\mp\lambda_1\frac{\partial T_1(\mp L,t)}{\partial X} + \alpha_T\left(T_1(\mp L,t) - T_0h_T(t)\right) = f(\dot{Z})\dot{Z}(t)P(t), \tag{4}$$

and initial

$$T(X,0) = 0,\ X \in (-L,L),\quad Z(0) = Z_0,\quad \dot{Z}(0) = 0 \tag{5}$$

conditions. Normal stresses occurred in plate are defined through

$$\sigma_{XX}(X,t) = \frac{E_1}{1-2\nu_1}\left[\frac{1-\nu_1}{1+\nu_1}\frac{\partial U}{\partial X} + \alpha_1T_1\right]. \tag{6}$$

In the above, the following notation is applied: E_1 – elasticity modulus, ν_1, λ_1, a_1, α_1, α_T are Poisson's ratio, thermal conductivity, thermal diffusivity, thermal expansion an heat transfer coefficients, respectively; $m = M_1/b_1b_2$. Let us introduce the following coefficients

$$t_* = L^2/a_1\ [s],\quad v_* = a_1/L\ [m/s],\quad P_* = T_0E_1\alpha_1/(1-2\nu_1)\ [N/m^2],$$

and the following non-dimensional parameters $x = X/L$, $\tau = t/t_*$, $z = Z/L$,

$$p = \frac{P}{P_*},\ \theta = \frac{T_1}{T_0},\ \varepsilon_1 = \frac{2P_*t_*^2}{mL},\ \gamma = \frac{E_1\alpha_1a_1}{(1-2\nu_1)\lambda_1},\ \mathrm{Bi} = \frac{L\alpha_T}{\lambda_1},\ \ m_0 = \frac{F_*}{2P_*}.$$

2. Solution of the problem. Applying the Laplace transformation, the following system of equations is obtained:

$$p(\tau) = \mathrm{Bi}\int_0^\tau \dot{h}_T(\xi)G_p(\tau-\xi)\,d\xi + \gamma\int_0^\tau F(\dot{z})\dot{z}(\xi)p(\xi)\dot{G}_p(\tau-\xi)\,d\xi, \tag{7}$$

$$\ddot{z}(\tau) = \varepsilon_1\left[m_0\,h_F(\tau) - F(\dot{z})p(\tau)\right], \tag{8}$$

which yields the non-dimensional pressure $p(\tau)$ and velocity $\dot{z}(\tau)$. The temperature is defined through the following formula

$$\theta(x,\tau) = \mathrm{Bi}\int_0^\tau \dot{h}_T(\xi)G_\theta(x,\tau-\xi)\,d\xi + \gamma\int_0^\tau F(\dot{z})\dot{z}(\xi)p(\xi)\dot{G}_\theta(x,\tau-\xi)\,d\xi, \tag{9}$$

where: $F(\dot{z}) = f(v_* \dot{z})$,

$$\{G_p(\tau), G_\theta(1,\tau)\} = \frac{1}{\mathrm{Bi}} - \sum_{m=1}^{\infty} \frac{\{2\mathrm{Bi},\, 2\mu_m^2\}}{\mu_m^2[\mathrm{Bi}(\mathrm{Bi}+1) + \mu_m^2]} e^{-\mu_m^2 \tau}, \tag{10}$$

and μ_m are the roots of the following characteristic equation: $\mathrm{tg}\mu_m = \mathrm{Bi}/\mu_m$, $m = 1, 2, \ldots$.

3. Characteristic properties of the solution. A stationary solution to the problem reads:

$$p_{st} = \frac{1}{1-v}, \ \theta_{st} = \frac{1}{1-v}, \ v = F(v_{st})\frac{v_{st}\gamma}{\mathrm{Bi}}, \tag{11}$$

where v_{st} is the solution of the non-linear equation

$$F(v_{st}) = \frac{m_0}{1 + \gamma m_0 v_{st}/\mathrm{Bi}}. \tag{12}$$

For a determination of the solution's behavior the linearization of the problem has been performed in a vicinity of the steady-state point (11). The right-hand sides of Eq. (7), (8), have been linearized. Graphical solution of equation (12) is presented in Fig. 2 for various parameters m_0 and Bi. Recall that for steel $\gamma = 1.87$.

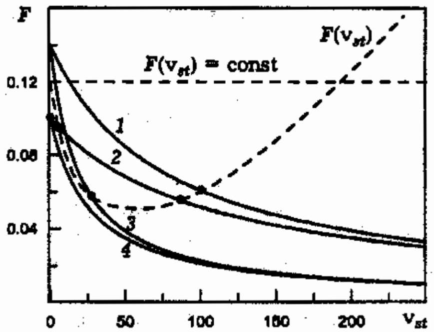

Fig. 2. Graphical solution of equation (12) (solid curves: 1 – $m_0 = 0.14$, $\mathrm{Bi} = 20$; 2 – $m_0 = 0.1$, $\mathrm{Bi} = 20$; 3 – $m_0 = 0.1$, $\mathrm{Bi} = 5$; 4 – $m_0 = 0.14$, $\mathrm{Bi} = 5$; dashed curve corresponds to $F(v_{st})$).

In this case the steel made parallelepiped type plate ($\alpha_2 = 14\cdot10^{-6}\,{}^\circ\mathrm{C}^{-1}$, $\lambda_1 = 21W/(\mathrm{m}\cdot{}^\circ\mathrm{C}^{-1})$, $a_1 = 5.9\cdot10^{-6}\mathrm{m}^2/s$, $\nu_1 = 0.3$, $E_1 = 19\cdot10^{10}\mathrm{Pa}$) with $T_0 = 5^\circ C$, $L = 0.01\,\mathrm{m}$, $z^\circ = \dot{z}^\circ = 0$ and with non-constant friction coefficient is studied. One gets $v_* = 0.59\cdot10^{-3}\,\mathrm{m/s}$, $t_* = 16.95\,s$, $P_* = 3.3\cdot10^7\,\mathrm{Pa}$. The function $F(\dot{z}) = f(v_*\dot{z})$ is defined through the formula taken from the reference [2]. The case of constant friction presented in Fig. 2 by the dashed horizontal line $F(v_{st}) = f = \mathrm{const}$ has been earlier considered in [6], where $v_{st} = \mathrm{Bi}(m_0/f - 1)/(m_0\gamma)$. In the fourth case ($m_0 = 0.14$, $\mathrm{Bi} = 5$) we have one solution of the form: $v_{st} = 27.8$, $p_{st} = \theta_{st} = 2.45$. It is unstable if the parameter ε_1 is larger than its critical value ($\varepsilon_1 \geq \tilde{\varepsilon}$)

$$\tilde{\varepsilon} = (1-v)\left(-B - \sqrt{B^2 - 4AC}\right)/(2A),$$

where: $A = c_1c_2 - c_0c_3$, $B = c_1d_1 + c_2d_0 - c_0d_2$, $C = d_0d_1$, $\beta_1 = F(v_{st})/v_{st}$, $\beta_2 = F'(v_{st})$, $d_m = d_m^{(1)} - \mathrm{Bi}\,v d_m^{(2)}$, , $c_m = \beta_2 d_m^{(1)} + \mathrm{Bi}\,v\beta_1 d_m^{(2)}$, $d_m^{(1)} = (2m + \mathrm{Bi})/(2m)!$, $d_m^{(2)} = 1/(2m+1)!$, $m = 0,\ 1,\ 2,\ 3$

4. Numerical solution and analysis. In order to confirm the given conclusions, numerical analysis is carried out for the fourth case for $\mathrm{Bi} = 5$ (now $\varepsilon_1 \approx \tilde{\varepsilon} = 586.5$), and the computational results are shown in Fig. 3 for a few values of the parameter $\varepsilon_1 = 400$; 586.5; 800. Time evolution of both non-dimensional contact pressure $p(\tau)$ and temperature $\theta(-1,\tau) = \theta(1,\tau)$ is reported in Fig. 3.

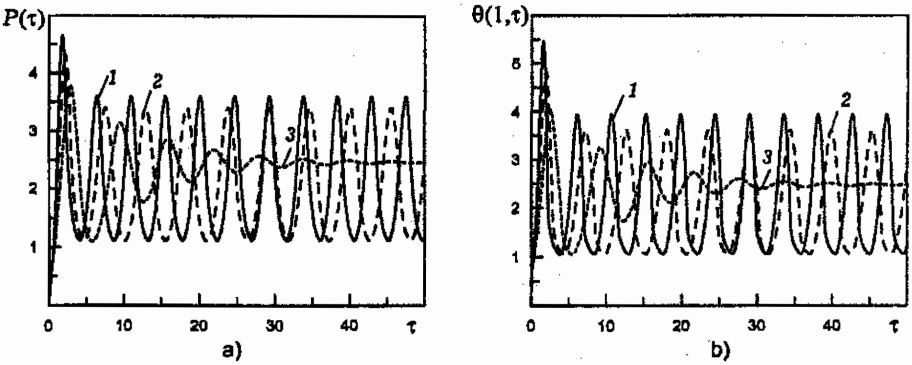

Fig. 3. Time history of non-dimensional contact pressure (a) and temperature (b) for various values of ε_1 (curve 1: $\varepsilon_1 = 800$; curve 2: $\varepsilon_1 = 586.5$; curve 3: $\varepsilon_1 = 400$).

It this work the results devoted to a novel problem of the mechanical system exhibiting frictional thermoelastic contact of a moving body subject to both non-constant friction coefficients are presented and discussed. It is worth noticing that in the case of non-constant friction coefficient and heating, the self-excited vibration can appear in our system without an elastic part (stiffness). The last phenomenon is caused by body heating while accelerating, friction increase, and than braking and cooling of the system.

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ФРИКЦІЙНІ АВТОКОЛИВАННЯ ЗА ТЕРМОПРУЖНОГО КОНТАКТУ ТІЛ

Розглянуто одновимірну модель термопружного контакту інерційного тіла у формі шару з навколишнім середовищем в умовах фрикційного нагріву. Прийнято, що коефіцієнт тертя залежить від відносної швидкості контактуючих тіл. Вивчено стабільність стаціонарних розв'язків. Проаналізовано вплив вхідних параметрів моделі на характеристики контакту (швидкість, контактний тиск, температуру). Показана можливість існування фрикційних автоколивань.

ФРИКЦИОННЫЕ АВТОКОЛЕБАНИЯ ПРИ ТЕРМОУПРУГОМ КОНТАКТЕ ТЕЛ

Рассмотрено одномерную модель термоупругого контакта инерционного тела в форме слоя с окружающей средой в условиях фрикционного нагрева. Принято, что коэффициент трения зависит от относительной скорости контактирующих тел. Изучена стабильность стационарных решений. Проведен анализ влияния входных параметров модели на характеристики контакта (скорость, контактное давление, температуру). Показана возможность существования фрикционных автоколебаний.

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