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Letter to the Editor

Dynamics of folded shells

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1. Introduction

It is well known that the computation of periodic structures is carried out using a homogenization approach [1–4]. In this work the Fourier homogenization method is applied [5,6] to analyze the dynamics of folded shells.

The dynamics of a regular closed folded shell composed of the same isotropic cylindrical panels is governed by the following equations [7–8]:

$$\frac{1}{Eh}\nabla^4\varphi - \left[k + \gamma\sum_{i=0}^{n-1}\delta(y-bi)\right]\frac{\partial^2 w}{\partial x^2} = 0,$$

$$D\nabla^4 w + \left[k + \gamma\sum_{i=0}^{n-1}\delta(y-bi)\right]\frac{\partial^2 w}{\partial x^2} + \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

and the attached boundary conditions.

In the above w denotes the normal deflection; φ is the stress function; k is the panel curvature ($k \equiv const \geqslant 0$, but a generalization into the case $k \equiv k(y) \geqslant 0$ can be easily realized); γ is the rotation angle of a normal to the surface during transition through $y = bi$ ($i = 0, 1, \ldots, n-1$), and b is the panel width; D is the cylindrical stiffness; E is the Young's modulus of the panel material; h denotes thickness; ρ is material density and t denotes time.

The occurrence in Eq. (1) of the generalized function $\sum_{i=0}^{n-1}\delta(y-bi)$ is substituted by its Fourier approximation

$$\sum_{i=0}^{n-1}\delta(y-bi) = \frac{1}{b}\left(1 + 2\sum_{j=1}^{\infty}\cos\left(\frac{2\pi j}{b}y\right)\right). \tag{2}$$

Assume that the period of vibrations is essentially larger than b. Taking into account the introduced assumptions, in the first approximation the coefficients appearing in Eq. (1) are

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averaged, which corresponds to omitting the variable terms in series (2). This results in the following equations:

$$\frac{1}{Eh}\nabla^4\varphi_0 - (k+\gamma/b)\frac{\partial^2 w_0}{\partial x^2} = 0,$$
$$D\nabla^4 w_0 + (k+\gamma/b)\frac{\partial^2\varphi_0}{\partial x^2} + \rho h\frac{\partial^2 w_0}{\partial t^2} = 0. \quad (3)$$

Note that the boundary conditions attached to Eq. (3) are found via averaging the given original boundary conditions. A transition from Eqs. (1)–(3) possesses simple physical interpretation. It corresponds to the transition from the initial system with discrete stringers to an equivalent orthotropic system. The initial folded shell is substituted by a smooth shell with the reduced curvature $\tilde{k} = k + \gamma/b$.

A solution to system (3) is sought in the following form:

$$w_0 = F_1(x,t)\cos\left(\frac{2\pi m}{nb}y\right),$$
$$\varphi_0 = F_2(x,t)\cos\left(\frac{2\pi m}{nb}y\right), \quad (4)$$

where $F_1(x,t)$ and $F_2(x,t)$ are the functions satisfying the given boundary conditions. In order to improve the obtained approximate solution, the following solution of Eq. (1) is assumed:

$$w = w_0 + w_1, \qquad \varphi = \varphi_0 + \varphi_1. \quad (5)$$

Substituting Eq. (5) to Eq. (1) and taking into account Eq. (3), one obtains

$$\frac{1}{Eh}\nabla_1^4\varphi_1 - \varepsilon^{-2}\left(\frac{b}{2\pi}\right)^2\left[k + \frac{2\pi\gamma\varepsilon}{b}\sum_{i=0}^{n-1}\delta(\eta - 2\pi i\varepsilon)\right]\frac{\partial^2 w_1}{\partial\xi^2}$$
$$= \frac{\gamma b}{2\pi^2\varepsilon^2}\frac{\partial^2 F_1(\xi)}{\partial\xi^2}\cos\eta\sum_{j=1}^{\infty}\cos\varepsilon^{-1}j\eta, \quad (6)$$

$$D\nabla_1^4 w_1 + \varepsilon^{-2}\left(\frac{b}{2\pi}\right)^2\left[k + \frac{2\pi\gamma\varepsilon}{b}\sum_{i=0}^{n-1}\delta(\eta - 2\pi i\varepsilon)\right]\frac{\partial^2\varphi_1}{\partial\xi^2} + \rho h\frac{\partial^2 w_1}{\partial t^2}$$
$$= -\frac{\gamma b}{2\pi^2\varepsilon^2}\frac{\partial^2 F_2(\xi)}{\partial\xi^2}\cos\eta\sum_{j=1}^{\infty}\cos\varepsilon^{-1}j\eta, \quad (7)$$

where

$$\nabla_1^4 = \frac{\partial^4}{\partial\xi^4} + 2\frac{\partial^4}{\partial\xi^2\partial\eta^2} + \frac{\partial}{\partial\eta^4}\{\xi;\eta\} = \frac{2\pi}{b}\varepsilon\{x;y\}.$$

The right-hand sides of Eqs. (6) and (7) change quickly with respect to η, since the assumption $\varepsilon \ll 1$ has been introduced. This observation means that one can apply various asymptotical approaches to solve system (6) and (7) (see Ref. [9]).

A particular solution of system (6) and (7), which varies with respect to η more significantly than the external load, is found by retaining the first approximation on the left-hand sides the terms having the derivatives of the maximal order with respect to η [9]:

$$\frac{1}{Eh}\frac{\partial^4\varphi_{10}}{\partial\eta^4}=\frac{\gamma b}{2\pi^2\varepsilon^2}\frac{\partial^2F_1(\xi)}{\partial\xi^2}\cos\eta\sum_{j=1}^{\infty}\cos\varepsilon^{-1}j\eta,$$

$$D\frac{\partial^4w_{10}}{\partial\eta^4}=-\frac{\gamma b}{2\pi^2\varepsilon^2}\frac{\partial^2F_2(\xi)}{\partial\xi^2}\cos\eta\sum_{j=1}^{\infty}\cos\varepsilon^{-1}j\eta. \quad (8)$$

The following simple form of solution is obtained (the terms of ε order with respect to Eq. (1) are omitted):

$$\varphi_{10}=Eh\frac{\varepsilon^2\gamma b}{2\pi^2}\frac{\partial^2F_1(\xi)}{\partial\xi^2}\cos\eta\,\Phi(\eta),$$

$$w_{10}=-\frac{1}{D}\frac{\varepsilon^2\gamma b}{2\pi^2}\frac{\partial^2F_2(\xi)}{\partial\xi^2}\cos\eta\,\Phi(\eta), \quad (9)$$

where $\Phi(\eta)$ is the periodic function with period $2\pi\varepsilon$, which in the interval $0\leqslant\eta\leqslant2\pi\varepsilon$ has the form

$$\Phi(\eta)=\frac{\pi^4}{90}-\frac{\pi^2}{12}\varepsilon^{-2}\eta^2+\frac{\pi}{12}\varepsilon^{-3}\eta^3-\tfrac{1}{48}\varepsilon^{-4}\eta^4.$$

Note that the derived solution (9) does not satisfy the boundary condition, and the boundary inaccuracy changes quickly with respect to η. Consequently, this error is compensated by a solution of the homogeneous system (6) and (7), which might be treated as a boundary layer [9]. To construct the boundary layer one may use the approximate system consisting only of maximal order derivatives with respect to ξ and η [3]:

$$\nabla_1^4\varphi_{11}=0, \quad (10)$$

$$\nabla_1^4w_{11}=0. \quad (11)$$

Observe that Eq. (11) governs the plate deflection, whereas Eq. (10) describes the plate deformation on its surface. Both equations may be easily solved using known methods (also a static problem may be solved in a similar way).

2. Example

As an example consider the computation of a prismatic closed folded shell supported by balls on its edges $x=0,L$ where

$$w=\frac{\partial^2w}{\partial x^2}=\varphi=\frac{\partial^2\varphi}{\partial x^2}=0 \quad (12)$$

which is loaded by a non-uniform surface load of the form

$$g=P\sin\frac{\pi x}{L}\cos\frac{2\pi m}{nb}y.$$

For this case, a solution to Eqs. (3) is sought in the form:

$$w_0 = C_1 \sin a\xi \cos \eta,$$
$$\varphi_0 = C_2 \sin a\xi \cos \eta,$$

where

$$C_1 = \frac{\bar{P}d}{\left[Dd^2 + Eh\varepsilon^{-4}\left(\frac{b}{2\pi}\right)^4 (k+\gamma/b)^2 a^4\right]},$$

$$C_2 = -\frac{Eh}{d}\varepsilon^{-2}\left(\frac{b}{2\pi}\right)^2 (k+\gamma/b)a^2 C_1,$$

$$d = (a^2-1)^2, \qquad \bar{P} = \varepsilon^{-4}\left(\frac{b}{2\pi}\right)^4 \qquad a = \frac{b}{2L}\varepsilon^{-1}.$$

One finds from Eq. (9) the particular solution of system (6), which reads

$$\varphi_{10} = -Eh\frac{\varepsilon^2\gamma b a^2}{2\pi^2}C_1 \sin a\xi \cos \eta \Phi(\eta),$$
$$w_{10} = \frac{1}{D}\frac{\varepsilon^2\gamma b a^2}{2\pi^2}C_2 \sin a\xi \cos \eta \Phi(\eta). \qquad (13)$$

In this example solution (13) satisfies exactly the boundary conditions (12), and hence there is no need to construct a boundary layer state governed by Eqs. (10) and (11).

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