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THE METHOD OF VIBRATION CONTROL IN THE POINTS OF CONTINUOUS FLEXIBLE SYSTEMS

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A continuous flexible system harmonically excited is considered. Minimization of vibration in chosen points of the flexible system is realized by means of joining an additional system characterized by suitable receptances, which have to be calculated. The paper presents the method of determining the appropriate receptances of additional system in order to cancel vibrations in the mentioned points of the main system.

Keywords: Vibration control; Continuous flexible systems; Computational algorithm

1. INTRODUCTION

In many technical problems it is necessary to achieve a considerable reduction of vibration in chosen points of flexible continuous system. Places of fixation of sensitive measurement equipments, and a work of machine operators, may be treated as chosen points of a flexible vibrating construction. For solution of these problems the receptances method was used, presented in the Refs. [1–4]. The possibility and advisability of applying this method results from the fact that most of vibrating structures are continuous linear media of complex shapes, which causes that their precise mathematical description is impossible. The receptance is determined as the ratio of complex amplitude of the displacement of any point B in v direction to the amplitude of the force in any point A in u direction.

$$\text{If } F_A^u(t) = F_{A_0}^u \exp(i\omega t), \quad \bar{s}_B^v(t) = \bar{s}_{B_0}^v \exp(i\omega t), \tag{1}$$

where

$$\bar{s}_{B_0}^v = s_{B_0}^v \exp(i\varphi_{BA}),$$

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and a receptance is defined as follows:

$$\alpha_{BA}^{vu}(i\omega) = \frac{\bar{s}_{B_0}^{v}}{F_{A_0}^{u}}. \tag{2}$$

2. THEORETICAL ANALYSIS

The linear continuous flexible system under consideration is presented in Fig. 1. This system is harmonically excited in any point A in u direction with a frequency ω. In the points $D_i (i = 1 \cdots p$ or $\infty)$ the system 1 is connected with the motionless base. This connection can be continuous. A purpose of the consideration is minimization of vibration in any chosen point B of the flexible continuous system 1 is realized by means of joining an additional system 2 shown in Fig. 1. The system 2 is characterized by suitable dynamic charaeteristics (receptance) in a point C, being the connection of the system 1 and 2. Dynamic characteristics of additional system 2 have to be calculated. The connected systems 1 and 2 in Fig. 1 can be divided in point C into two subsystems 1 and 2 (see Fig. 2). Dynamic interaction $R_c^w(t)$ between the subsystems 1 and 2 must be considered. The dynamic interaction of the subsystems is described by the continuity and equilibrium conditions which define the displacements and forces at the division point C. Based on receptance shown earlier, complex amplitudes of vibration in the points B and C of subsystem 1 (Fig. 2) read

$$\bar{s}_{B_0}^{v} = \alpha_{BA}^{vu}(i\omega) F_{A_0}^{u} + \alpha_{BC}^{vw}(i\omega) R_{C_0}^{w}, \tag{3}$$

$$\bar{s}_{C_0}^{w} = \alpha_{CA}^{wu}(i\omega) F_{A_0}^{u} + \alpha_{CC}^{ww}(i\omega) R_{C_0}^{w}. \tag{4}$$

Similarly for the subsystem 2 one gets

$$\bar{s}_{C_0}^{w} = \beta_{CC}^{ww}(i\omega)[-R_{C_0}^{w}]. \tag{5}$$

Suitable receptances in the (3), (4), (5) for simple continuous systems (e.g. beams, plates) can be calculated. The receptances of complex flexible systems cannot be

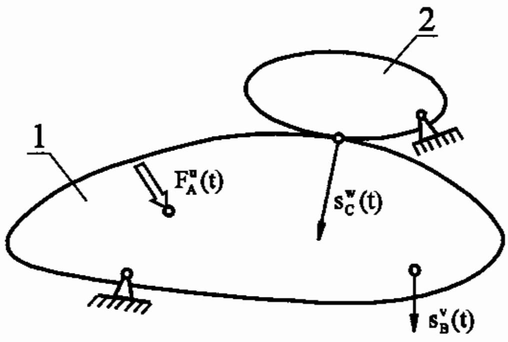


FIGURE 1 The analysed system.

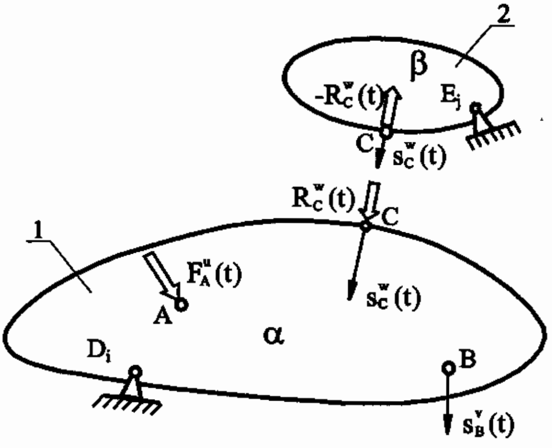


FIGURE 2 Two separated subsystems.

analysed with any sufficient accuracy. Hence, the need to determine the receptances experimentally on real system and under actual condition of operation occurs [5, 6]. Taking into account (3)–(5) one obtains:

$$\bar{s}^{v}_{B_0} = \gamma^{vu}_{BA}(i\omega)F^{u}_{A_0}, \tag{6}$$

where

$$\gamma^{vu}_{BA}(i\omega) = \frac{\alpha^{vu}_{BA}[\alpha^{ww}_{CC} + \beta^{ww}_{CC}] - \alpha^{vw}_{BC}\alpha^{wu}_{CA}}{\alpha^{ww}_{CC} + \beta^{ww}_{CC}}. \tag{7}$$

Equations (6) and (7) yield a vibration at the considered point B, which is caused by nulling of numerator of (7). It leads to condition

$$\beta^{ww}_{CC}(i\omega) = \frac{\alpha^{vw}_{BC}(i\omega)\alpha^{wu}_{CA}(i\omega) - \alpha^{vu}_{BA}(i\omega)\alpha^{ww}_{CC}(i\omega)}{\alpha^{vu}_{BA}(i\omega)}. \tag{8}$$

The value of receptance obtained from (8) of additional subsystem 2 may be realized by both discrete or continuous models. For instance, a combination of mass, stiffness and damping can be used for continuous systems such as beams or plates.

3. EXAMPLE

The presented method is illustrated by the use of constant cross-sectional beams on two supports (pinned-pinned) without damping (Fig. 3). The harmonic force in point A in y direction excites vibration of the beam shown in Fig. 3. To minimize the vibration in any point B two cases of this method are considered:

1. In any point C of the beam additional mass m is attached;
2. In any point C of the beam additional massless spring with stiffness k in y direction is attached.

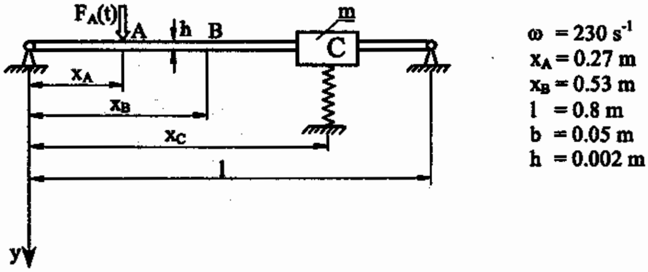


FIGURE 3 Beam on two supports with additional subsystem.

The suitable receptances of considered beam in the A, B, C points can be calculated using expressions:

$$\alpha_{BA}(\omega) = \frac{2}{\rho A l}\sum_{n=1}^{\infty}\frac{\sin(n\pi(x_B/l))\sin(n\pi(x_A/l))}{\alpha_n^2-\omega^2},$$
$$\alpha_{CA}(\omega) = \frac{2}{\rho A l}\sum_{n=1}^{\infty}\frac{\sin(n\pi(x_C/l))\sin(n\pi(x_A/l))}{\alpha_n^2-\omega^2},$$
$$\alpha_{BC}(\omega) = \frac{2}{\rho A l}\sum_{n=1}^{\infty}\frac{\sin(n\pi(x_B/l))\sin(n\pi(x_C/l))}{\alpha_n^2-\omega^2}, \tag{9}$$
$$\alpha_{CC}(\omega) = \frac{2}{\rho A l}\sum_{n=1}^{\infty}\frac{[\sin(n\pi(x_C/l))]^2}{\alpha_n^2-\omega^2}$$

where

$$\alpha_n = n^2\frac{\pi^2}{l^2}\sqrt{\frac{EI}{\rho A}}.$$

The receptance of mass m is given by

$$\beta_{CC_m}(\omega) = \frac{1}{-m\omega^2}, \tag{10}$$

whereas the receptance of spring has the form

$$\beta_{CC_k}(\omega) = \frac{1}{k}. \tag{11}$$

After putting (10) and (11) into (8) and transformations one obtains:

$$m(\omega) = \frac{\alpha_{BA}(\omega)}{\omega^2[\alpha_{BA}(\omega)\alpha_{CC}(\omega) - \alpha_{BC}(\omega)\alpha_{CA}(\omega)]}, \tag{12}$$

$$k(\omega) = \frac{\alpha_{BA}(\omega)}{\alpha_{BC}(\omega)\alpha_{CA}(\omega) - \alpha_{BA}(\omega)\alpha_{CC}(\omega)}. \tag{13}$$

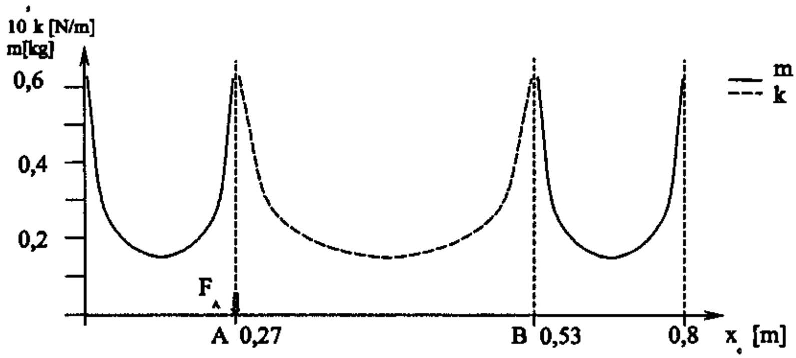


FIGURE 4 Calculation results.

Numerical calculations of expressions (12) and (13) were performed for the considered beam for the dates given in Fig. 3. Obtained results of calculations are presented in Fig. 4. The calculation results of m and k shown in Fig. 4 of vibration cancelling are positive or negative. Negative values are not practically realized. Optimal cases in a chosen point B of the beam for the presented dates are fixed (the mass $m = 0.15\,\text{kg}$ in the half-length between point A and left support or in the half of length between point B and right support). The best choice of a spring stiffness corresponds to the value $k = 12\,\text{kN/m}$.

The presented method may be supplied to minimization of vibration of plates, floors or the flexible three-dimensional constructions.

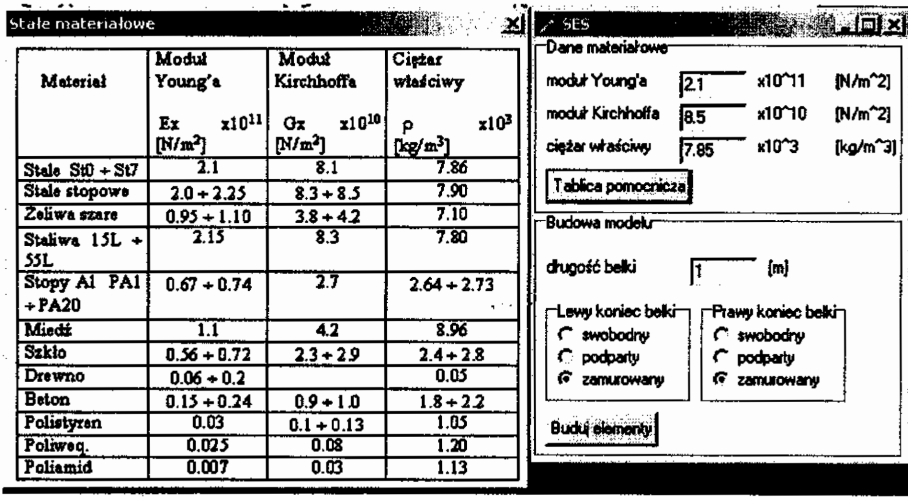
4. COMPUTATIONAL ALGORITHM

In order to carry out a practical implementation of the developed earlier method of beam systems more complex in comparison to that illustrated in Example 3, an algorithm for numerical computations has been prepared using the DELPHI language.

The program calculates the receptances of the chosen beam finite elements (it has been discussed earlier during theoretical analysis). Next, using the obtained results the additional parameters are calculated (mass m, stiffness k) of an attached discrete system, which serves as a vibrational eliminator in an arbitrary chosen beam point.

The program computational abilities, a way of introducing input data and some computational examples are given below.

- Using the material table (see Fig. 5) one chooses a beam material and its properties (Young and Kirchoff moduli and material density). Then a beam length in meters and the boundary conditions are chosen. In the latter case the following options are possible: free, freely supported and clamped beam ends.
- Using the next table (see Fig. 6) a shape of a beam cross section is chosen and its diameters are identified.
- In this step a beam model is chosen. A number of finite elements can be defined, with an option of their different shapes and geometrical diameters. In addition, each of the finite elements can be supported (see example given in Fig. 7).
- Now two computational paths can be taken: Either cigenfrequencies of thc taken beam model and the corresponding modes (see Fig. 8) or the path leading to vibration



Stałe materiałowe

Materiał	Moduł Young'a Ex $x10^{11}$ [N/m²]	Moduł Kirchhoffa Gx $x10^{10}$ [N/m²]	Ciężar właściwy ρ $x10^{3}$ [kg/m³]
Stale St0 ÷ St7	2.1	8.1	7.86
Stale stopowe	2.0 ÷ 2.25	8.3 ÷ 8.5	7.90
Żeliwa szare	0.95 ÷ 1.10	3.8 ÷ 4.2	7.10
Staliwa 15L ÷ 55L	2.15	8.3	7.80
Stopy Al PA1 ÷ PA20	0.67 ÷ 0.74	2.7	2.64 ÷ 2.73
Miedź	1.1	4.2	8.96
Szkło	0.56 ÷ 0.72	2.3 ÷ 2.9	2.4 ÷ 2.8
Drewno	0.06 ÷ 0.2		0.05
Beton	0.15 ÷ 0.24	0.9 ÷ 1.0	1.8 ÷ 2.2
Polistyren	0.03	0.1 ÷ 0.13	1.05
Poliweq.	0.025	0.08	1.20
Poliamid	0.007	0.03	1.13

FIGURE 5 Initial material and geometrical properties, and the boundary conditions of a beam.

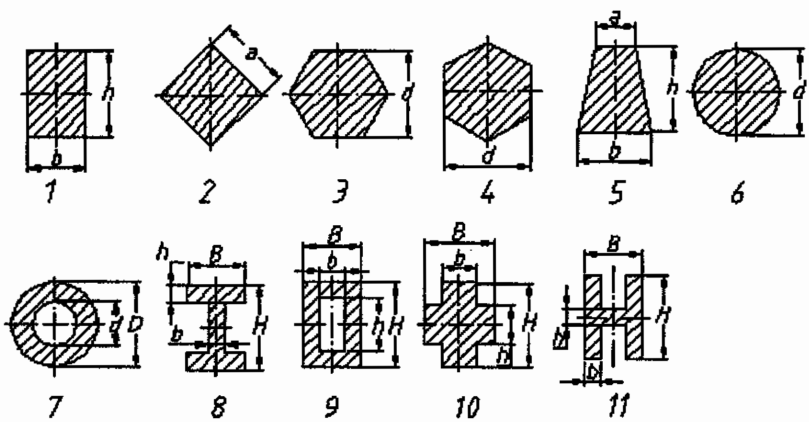


FIGURE 6 Shapes of the cross sections of the beam finite elements.

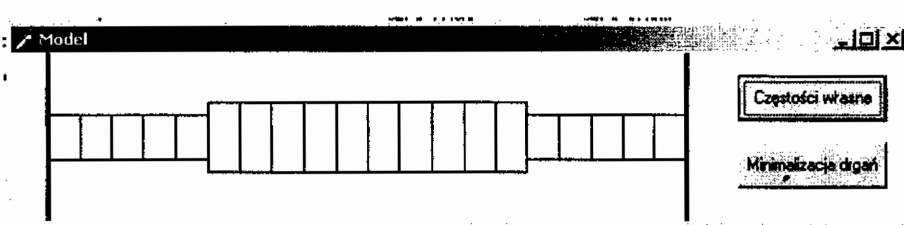


FIGURE 7 Example of the chosen beam model.

minimization procedure of a beam harmonically excited by an appropriate attachment of an additional system Fig. 8. In the last case there is an option of a choice of arbitrary beam point of a harmonic excitation, its amplitude and frequency (Fig. 9). One can also trace a beam mode just before minimization procedure (Fig. 10).

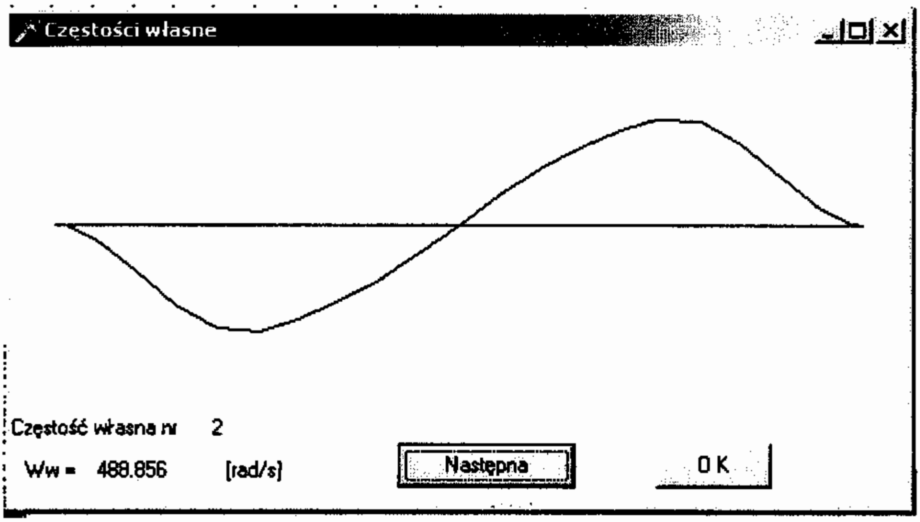


FIGURE 8 Eigenfrequencies and the corresponding beam modes.

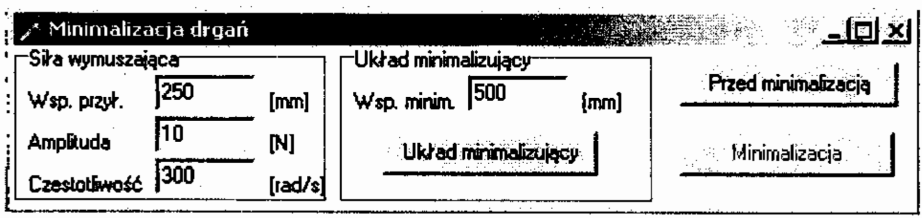


FIGURE 9 Parameters of thc harmonic excitation and a minimization point.

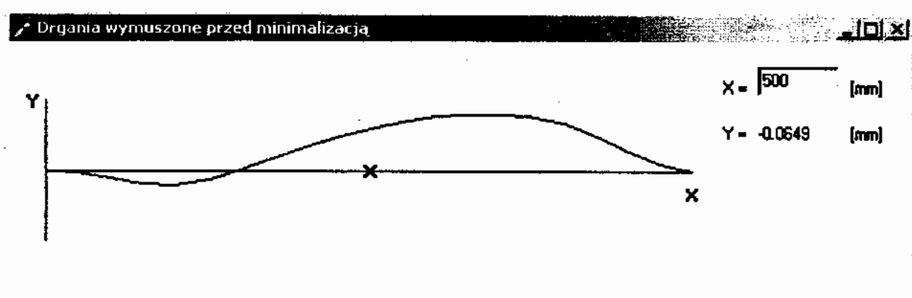


FIGURE 10 Beam mode before minimization procedure.

In the next step a model of an additionally attached system is chosen. Here also there are some options, i.e. different combinations of the systems including mass m and stiffness k (Fig. 11). For example, if one of the control parameters is taken (say, stiffness k) then the second one (mass m) can be calculated and presented together with the corresponding beam mode after the minimization procedure in a chosen beam point (see Fig. 12).

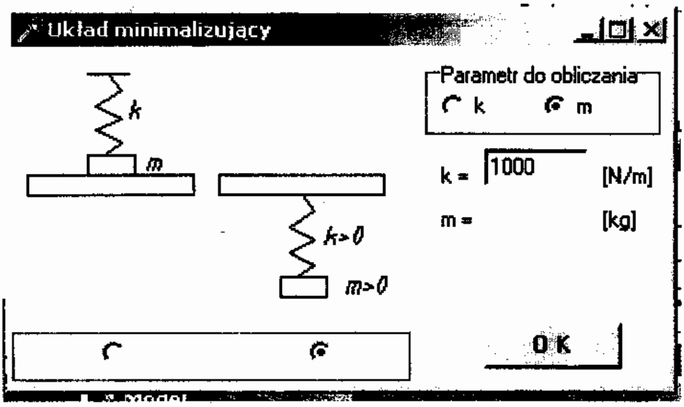


FIGURE 11 Examples of attached minimizing systems.

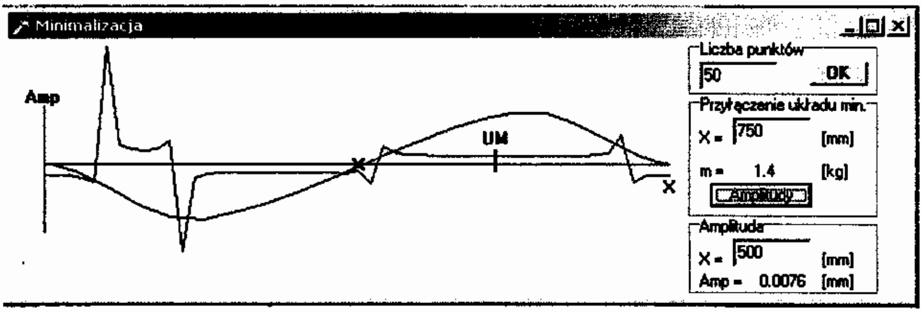


FIGURE 12 Mass distribution of a chosen attached system (k, m) and vibration modes after minimization procedure.

5. CONCLUSIONS

The work is concerned with passive control of vibration level in arbitrary chosen points of continuous flexible systems (for instance beams or plates) using a structural modification method. A target (elimination of vibrations) is achieved by an attachment to a being analysed system an additional system composed of mass, spring or mass and spring.

During dynamical analysis the method of receptances, has been applied.

As an example a simple beam model with constant cross section has been considered.

In order to analyse dynamical behaviour of more complex beam systems (for instance those with changeable and different cross sections) an original program has been developed in the DELPHI language. It opens a very promising research to analyse continuous and non-linear beam systems.

NOMENCLATURE

u, v, w	directions of the displacements (forces),
$s_B^v(t), s_C^w(t)$	displacements of the points B and C,

$\bar{s}^{v}_{B_0}, \bar{s}^{w}_{C_0}, s^{v}_{B_0}, s^{w}_{C_0}$	complex and the real amplitudes of the displacements (vibrations),
$F^{u}_{A}(t), R^{w}_{C}(t)$	forces in the points A and C,
$F^{u}_{A_0}, R^{w}_{C_0}$	amplitudes of the forces,
φ_{BA}	phase (angle) of a displacement with respect to excitation,
$\alpha^{vu}_{BA}(i\omega), \alpha^{uw}_{BC}(i\omega)$,	suitable receptances between the points BA, BC,
$\alpha^{wu}_{CA}(i\omega), \alpha^{ww}_{CC}(i\omega)$	CA, CC of main system
$\beta^{ww}_{CC}(i\omega)$	receptances of the additional system in the point C,
ω	frequency of excitation,
α_n	successive frequency of the free vibration of a beam ($n = 1, 2, 3, \ldots$),
l, b, h, A	length, width, height, cross-sectional area of the beam,
x_A, x_B, x_C	co-ordinates of the points A, B, C of a beam,
E, I	Young's modulus and moment of inertia of a beam,
ρ	mass density of a beam

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