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On the solution of a coupled thermo-mechanical problem for non-homogeneous Timoshenko-type shells

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Abstract

In this work a coupled thermo-mechanical problem of non-homogeneous shells with variable thickness and variable Young modulus (a so-called Timoshenko type model) is considered. The problem is reduced to uniformly correct problem in the form of a first order difference operator equation. In addition, a similar approach can easily be applied to the Kirchhoff–Love model.

Keywords: Thermo-mechanics; Shells; Coupled problem; Self-adjoint operators

1. Introduction and statement of the problem

As it is well known (see, for example [1]), the thermo-mechanical equations governing the dynamics of the Timoshenko shell model have the form

$$\rho h \frac{\partial^2 u}{\partial t^2} - \frac{1}{1-\mu^2}\frac{\partial}{\partial x}\left\{Eh\left[\frac{\partial u}{\partial x} - k_x w + \mu\left(\frac{\partial v}{\partial y} - k_y w\right)\right]\right\}$$

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$$
-\frac{1}{2(1+\mu)}\frac{\partial}{\partial y}\left[Eh\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\alpha_T}{1-\mu}\frac{\partial}{\partial x}E\int_{-\frac{h}{2}}^{\frac{h}{2}}\theta\,dz=p_1,
$$

$$
\rho h\frac{\partial^2 v}{\partial t^2}-\frac{1}{1-\mu^2}\frac{\partial}{\partial y}\left\{Eh\left[\frac{\partial v}{\partial y}-k_y w+\mu\left(\frac{\partial u}{\partial x}-k_x w\right)\right]\right\}
$$

$$
-\frac{1}{2(1+\mu)}\frac{\partial}{\partial x}\left[Eh\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\alpha_T}{1-\mu}\frac{\partial}{\partial y}E\int_{-\frac{h}{2}}^{\frac{h}{2}}\theta\,dz=p_2,
$$

$$
\rho h\frac{\partial^2 w}{\partial t^2}+\frac{1}{2(1+\mu)}\left\{\frac{\partial}{\partial x}\left[k^2Eh\left(\psi_x+\frac{\partial w}{\partial x}\right)\right]\right.
$$

$$
\left.+\frac{\partial}{\partial y}\left[k^2Eh\left(\psi_y+\frac{\partial w}{\partial y}\right)\right]\right\}
$$

$$
-\frac{Eh}{1-\mu^2}\left[(k_x+\mu k_y)\left(\frac{\partial u}{\partial x}-k_x w\right)+(k_y+\mu k_x)\left(\frac{\partial v}{\partial y}-k_y w\right)\right]
$$

$$
+\frac{E\alpha_T}{1-\mu}(k_x+k_y)\int_{-\frac{h}{2}}^{\frac{h}{2}}\theta\,dz=q_1,
$$

$$
\frac{\rho h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}-\frac{\partial}{\partial x}\left[D\left(\frac{\partial\psi_x}{\partial x}+\mu\frac{\partial\psi_y}{\partial y}\right)\right]-\frac{1-\mu}{2}\frac{\partial}{\partial y}\left[D\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right)\right]
$$

$$
+\frac{k^2Eh}{2(1+\mu)}\left(\psi_x+\frac{\partial w}{\partial x}\right)+\frac{\alpha_T}{1-\mu}\frac{\partial}{\partial x}E\int_{-\frac{h}{2}}^{\frac{h}{2}}z\theta\,dz=0,
$$

$$
\frac{\rho h^3}{12}\frac{\partial^2\psi_y}{\partial t^2}-\frac{\partial}{\partial y}\left[D\left(\frac{\partial\psi_y}{\partial y}+\mu\frac{\partial\psi_x}{\partial x}\right)\right]-\frac{1-\mu}{2}\frac{\partial}{\partial x}\left[D\left(\frac{\partial\psi_y}{\partial x}+\frac{\partial\psi_x}{\partial y}\right)\right]
$$

$$
+\frac{k^2Eh}{2(1+\mu)}\left(\psi_y+\frac{\partial w}{\partial y}\right)+\frac{\alpha_T}{1-\mu}\frac{\partial}{\partial y}E\int_{-\frac{h}{2}}^{\frac{h}{2}}z\theta\,dz=0,
$$

$$
\tilde{n}_\varepsilon(1+\varepsilon)\frac{\partial\theta}{\partial t}-\lambda_q\Delta\theta+\frac{E\alpha_T T_0}{1-\mu}\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-(k_x+k_y)w\right.
$$

$$
\left.+z\left(\frac{\partial\psi_x}{\partial x}+\frac{\partial\psi_y}{\partial y}\right)\right]=q_2, \tag{1}
$$

where $u, v, w, \psi_x, \psi_y, \theta$ are known functions. The attached boundary conditions will be defined later. The following standard notation is used: T_0—initial temperature; $E = E(x, y) > 0$—Young modulus; μ—Poisson's coefficient ($0 \leqslant \mu \leqslant 0.5$); $\rho = \rho(x, y) > 0$—material density; λ_q—heat transfer coefficient; $\tilde{n}_\varepsilon$—specific heat capacity corresponding to a constant deformation tensor; α_T—linear thermal expansion coefficient;

$$\frac{1}{k^2} = \frac{1}{h} \int_{-\frac{h}{2}}^{\frac{h}{2}} f^2\left(\frac{z}{h}\right) dz;$$

$f(z/h)$—function describing a distribution of tangent stresses along the thickness of a shell; $h = h(x, y)$—variable shell thickness; $\theta = \theta(x, y, z, t)$—shell temperature increase; $u = u(x, y, t)$, $v = v(x, y, t)$, $w = w(x, y, t)$—components of the displacement vector of the point (x, y) in the mean surface and a deflection at the time instant t; $\psi_x(x, y, t)$, $\psi_y(x, y, t)$—rotation angles of the normal to the mean surface in the planes xz and yz, respectively; k_x, k_y—initial curvatures corresponding to the coordinates x, y; p_1, p_2, q_1—external load intensities along the axes x, y, z; q_2—specific heat capacity power of the sources situated within the shell.

2. The method

The system (1) is considered for $t \geqslant 0$ in a three-dimensional space $\Omega = \Omega_1 \times (-h/2, h/2)$, where $\Omega_1 \subset \mathcal{R}^2$ is the bounded space with a piece-wise boundary (all of the functions appearing in (1) are also assumed to be sufficiently differentiable).

In order to reduce (1) to a difference-operator equation we introduce the following Hilbert spaces $H_{xy} = L_2(\Omega_1)$, $H_{xyz} = L_2(\Omega)$, spanned by measurable functions having integral square norm and defined in the spaces Ω_1 and Ω, respectively. We also use the notation $H_1 = H_{xy} \oplus H_{xy}$, $H_2 = H_{xy}$, $H_3 = H_{xy} \oplus H_{xy}$, and $H_4 = H_{xyz}$.

In addition, we introduce the following differential and matrix-differential expressions (T denotes transposition):

$$\widetilde{K}_1 = -\frac{1}{1+\mu} \begin{pmatrix} \frac{1}{1-\mu}\frac{\partial}{\partial x}Eh\frac{\partial}{\partial x} + \frac{1}{2}\frac{\partial}{\partial y}Eh\frac{\partial}{\partial y} & \frac{\mu}{1-\mu}\frac{\partial}{\partial x}Eh\frac{\partial}{\partial y} + \frac{1}{2}\frac{\partial}{\partial y}Eh\frac{\partial}{\partial x} \\ \frac{\mu}{1-\mu}\frac{\partial}{\partial y}Eh\frac{\partial}{\partial x} + \frac{1}{2}\frac{\partial}{\partial x}Eh\frac{\partial}{\partial y} & \frac{1}{1-\mu}\frac{\partial}{\partial y}Eh\frac{\partial}{\partial y} + \frac{1}{2}\frac{\partial}{\partial x}Eh\frac{\partial}{\partial x} \end{pmatrix}$$
$$= -\frac{1}{2(1-\mu)} \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & 0 \end{pmatrix} Eh \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 0 & 0 \end{pmatrix}$$
$$-\frac{1}{2(1+\mu)} \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} Eh \begin{pmatrix} \frac{\partial}{\partial x} & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix},$$

$$\widetilde{K}_2 = -\frac{k^2}{2(1+\mu)}\left(\frac{\partial}{\partial x}Eh\frac{\partial}{\partial x} + \frac{\partial}{\partial y}Eh\frac{\partial}{\partial y}\right) + \frac{Eh}{1-\mu^2}\left(k_x^2 + k_y^2 + 2\mu k_x k_y\right),$$

$$\widetilde{K}_3 = -\begin{pmatrix} \frac{\partial}{\partial x}D\frac{\partial}{\partial x} + \frac{1-\mu}{2}\frac{\partial}{\partial y}D\frac{\partial}{\partial y} & \mu\frac{\partial}{\partial x}D\frac{\partial}{\partial y} + \frac{1-\mu}{2}\frac{\partial}{\partial y}D\frac{\partial}{\partial x} \\ \mu\frac{\partial}{\partial y}D\frac{\partial}{\partial x} + \frac{1-\mu}{2}\frac{\partial}{\partial x}D\frac{\partial}{\partial y} & \frac{\partial}{\partial y}D\frac{\partial}{\partial y} + \frac{1-\mu}{2}\frac{\partial}{\partial x}D\frac{\partial}{\partial x} \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{k^2Eh}{2(1+\mu)} & 0 \\ 0 & \frac{k^2Eh}{2(1+\mu)} \end{pmatrix}$$

$$= -\frac{1+\mu}{2}\begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & 0 \end{pmatrix} D \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 0 & 0 \end{pmatrix}$$

$$-\frac{1-\mu}{2}\begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} D \begin{pmatrix} \frac{\partial}{\partial x} & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} + \begin{pmatrix} \frac{k^2Eh}{2(1+\mu)} & 0 \\ 0 & \frac{k^2Eh}{2(1+\mu)} \end{pmatrix},$$

$$\widetilde{K}_4 = -\frac{\lambda_q}{T_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right),$$

$$\widetilde{P}_{12} = \frac{1}{1-\mu^2}\left(\frac{\partial}{\partial x}Eh(k_x + \mu k_y), \frac{\partial}{\partial y}Eh(k_y + \mu k_x)\right)^{\mathrm{T}},$$

$$\widetilde{P}_{21} = -\frac{1}{1-\mu^2}\left(Eh(k_x + \mu k_y)\frac{\partial}{\partial x}, Eh(k_y + \mu k_x)\frac{\partial}{\partial y}\right),$$

$$\widetilde{P}_{23} = -\frac{1}{2(1+\mu)}\left(\frac{\partial}{\partial x}k^2Eh, \frac{\partial}{\partial y}k^2Eh\right),$$

$$\widetilde{P}_{32} = \frac{1}{2(1+\mu)}\left(k^2Eh\frac{\partial}{\partial x}, k^2Eh\frac{\partial}{\partial y}\right)^{\mathrm{T}},$$

$$\widetilde{Q} = -\frac{E\alpha_T}{1-\mu}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right).$$

Analysis of the expressions $\widetilde{K}_1$, $\widetilde{K}_2$, $\widetilde{K}_3$, $\widetilde{K}_4$ leads to the conclusion that they are formally self-adjoint. The minimal operators generated by those operators in the spaces H_1, H_2, H_3, H_4 are positive well defined. We are going to attach to the system (1) the boundary conditions in such a way that the operators K_1, K_2, K_3, K_4 generated by the expressions $\widetilde{K}_1$, $\widetilde{K}_2$, $\widetilde{K}_3$, $\widetilde{K}_4$ and the mentioned boundary conditions in the spaces H_1, H_2, H_3, H_4 will be self-adjointed and positive defined (observe that, for instance, Dirichlet type conditions satisfy our requirement).

The maximal operators generated by the expressions $\widetilde{P}_{21}$, $\widetilde{P}_{12}$, $\widetilde{P}_{23}$, $\widetilde{P}_{32}$, $\widetilde{Q}$ serve as the operators P_{21}, P_{12}, P_{23}, P_{32}, Q. The operators P_{21}, P_{23}, Q map from $H_{xy} \oplus H_{xy}$ into H_{xy}, whereas the operators P_{12}, P_{32} map from H_{xy} into $H_{xy} \oplus H_{xy}$. Note, that expressions $\widetilde{P}_{21}$ and $\widetilde{P}_{12}$, as well as $\widetilde{P}_{23}$ and $\widetilde{P}_{32}$ are formally self-adjoint to each other. The expression formally self-adjoint to Q has the form

$$\frac{\alpha_T}{1-\mu}\left(\frac{\partial}{\partial x}E, \frac{\partial}{\partial y}E\right)^{\mathrm{T}}.$$

By G_i we denote the product operators on the spaces H_i $(i = 1,2,3,4)$ with the functions ρh, ρh, $\rho h^3/12$, $\tilde{n}_\varepsilon(1+\varepsilon)/T_0$, respectively. It is obvious, that G_i are bounded and possess everywhere bounded inverse operators. In addition, we introduce the following bounded operators from H_{xy} into H_{xyz}: the operator B_1 assigns to each function $f \in H_{xy}$ the same function f, but considered as an element from H_{xyz}, B_2—the product operator of the functions from H_{xy} with z. It is not difficult to check that self-adjoint operators B_1^*, B_2^* to the operators B_1 and $B_2 : H_{xyz} \to H_{xy}$ map according to the formulas:

$$B_1^* g(x,y,z) = \int_{-\frac{h}{2}}^{\frac{h}{2}} g(x,y,z)\,dz,$$

$$B_2^* g(x,y,z) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z g(x,y,z)\,dz, \qquad g \in H_{xyz}.$$

Let us denote

$$S_2 = \frac{E\alpha_T}{1-\mu}(k_x + k_y)B_1, \qquad S_1 = B_1 Q_1, \qquad S_3 = B_2 Q_1.$$

S_1 is regarded as the operator mapping from H_1 into H_4, and S_3 is regarded as the operator mapping from H_3 into H_4. Since B_1, B_2 are bounded operators, then it follows from the results given in the monograph [2, Chapter 8, §1] that $S_1^* = Q_1^* B_1^*$, $S_3^* = Q_1^* B_2^*$.

Let us denote by U, Ψ the column vectors $U = (u, v)^{\mathrm{T}}$, $\Psi = (\psi_x, \psi_y)^{\mathrm{T}}$ and using the operators, the homogeneous system (1) can be presented in the following form introduced earlier:

$$\begin{aligned}
&G_1 \frac{d^2}{dt^2} U(t) + K_1 U(t) + P_{12} W(t) + S_1^* \theta(t) = 0,\\
&G_2 \frac{d^2}{dt^2} w(t) + K_2 w(t) + P_{21} U + P_{23} \Psi + S_2^* \theta(t) = 0,\\
&G_3 \frac{d^2}{dt^2} \Psi(t) + K_3 \Psi(t) + P_{32} w + S_3^* \theta(t) = 0,\\
&G_4 \frac{d}{dt}\theta(t) + K_4 \theta(t) - \frac{d}{dt}\big[S_1 U(t) + S_2 w(t) + S_3 \Psi(t)\big] = 0.
\end{aligned} \tag{2}$$

Observe that for the system (2) the spaces defining the operators are linked by the expressions

$$D(P_{12}) \supset D(K_2^{1/2}), \qquad D(P_{21}) \supset D(K_1^{1/2}), \qquad D(P_{32}) \supset D(K_2^{1/2}),$$
$$D(P_{23}) \supset D(K_3^{1/2}), \qquad D(S_i) \supset D(K_i^{1/2}) \quad (i = 1, 2, 3),$$

and that the system (2) represents a particular case of the more general system

$$F_i \frac{d^2\xi_i}{dt^2} = -N_i\xi_i + \sum_{j=1}^{n} P_{ij}\xi_j - S_i^*\theta_0 \quad (i = 1, 2, \ldots, n),$$
$$F_0 \frac{d\theta_0}{dt} = -N_0\theta_0 + \sum_{j=1}^{n} P_j\xi_j + \frac{d}{dt}\sum_{j=1}^{n} S_j\xi_j \tag{3}$$

with known functions $\xi_i = \xi_i(t)$, $\theta_0 = \theta_0(t)$. Here F_i, N_i are the positive defined self-adjoint operators on the Hilbert spaces H_i $(i = 0, 1, 2, \ldots, n)$, the F_i are bounded, the P_{ij} are linear and closed operators acting from H_j into H_i $(i, j = 1, 2, \ldots, n)$, S_i, P_i are linear and closed operators from H_i into H_0 $(i = 1, 2, \ldots, n)$. The defining spaces of those operators satisfy the conditions

$$D(P_{ij}) \supset D(N_j^{1/2}), \qquad D(S_i) \supset D(N_i^{1/2}),$$
$$D(P_i) \supset D(N_i^{1/2}) \quad (i, j = 1, 2, \ldots, n).$$

Let $H' = H_1 \oplus H_2 \oplus \cdots \oplus H_n$, and by N, F we denote the operators generated in H' by matrices with the diagonals N_i, F_i $(i = 1, 2, \ldots, n)$, respectively (other elements are equal to zero). By $\widetilde{P}$ we denote the operator generated by the matrix $(P_{ij})_{i,j=1}^n$, $\widetilde{S} = (S_1, \ldots, S_n)$, $\widetilde{P}_0 = (P_1, \ldots, P_n)$. Finally, denoting by $\tilde{\xi}$ the column of unknown functions $(\xi_1, \ldots, \xi_n)^{\mathrm{T}}$ we reduce (3) to the system of two equations

$$F \frac{d^2\tilde{\xi}}{dt^2} = -N\tilde{\xi} + \widetilde{P}\tilde{\xi} - \widetilde{S}^*\theta,$$
$$F_0 \frac{d\theta_0}{dt} = -N_0\theta_0 + \widetilde{P}_0\tilde{\xi} + \frac{d}{dt}(\widetilde{S}\xi). \tag{4}$$

We introduce the following change of variables in (4): $\xi = F^{1/2}\tilde{\xi}$, $\theta = F_0^{1/2}\theta_0$ and we denote $K = F^{-1/2}NF^{-1/2}$, $P = F^{-1/2}\widetilde{P}F^{-1/2}$, $M = F^{-1/2}N_0F^{-1/2}$, $P_0 = F^{-1/2}\widetilde{P}_0F^{-1/2}$, $S = F^{-1/2}\widetilde{S}F^{-1/2}$. The system (4) then takes the form

$$\frac{d^2\xi}{dt^2} = -K\xi + P\xi - S^*\theta,$$
$$\frac{d\theta}{dt} = -M\theta + P_0\xi + \frac{d}{dt}(S\xi), \quad 0 \leqslant t < \infty. \tag{5}$$

The system (5) is analyzed in the space $H' \oplus H_0$. In this system K, M are the positive defined self-adjoint operators in the spaces H', H_0, respectively. P is the linear closed operator on H', and S, P_0 are linear closed operators from H' into H_0. In addition, the following relations are specified: $D(P) \supset D(K^{1/2})$, $D(S) \supset D(K^{1/2})$, $D(P_0) \supset D(K^{1/2})$. Observe, that the system (5) has been considered already in reference [3] but from a different point of view. We propose now a different approach to solve (5) than that given in [3].

In order to reduce (5) to a first order equation let us introduce in the space $H = H' \oplus H' \oplus H_0$ the matrix operator

$$A_0 = \begin{pmatrix} 0 & E & 0 \\ -K+P & 0 & -S^* \\ P_0 & S & -M \end{pmatrix}$$

(here and below E denotes the identity operator from a corresponding space). By η we denote the column vector $\eta = (\xi, \xi', \theta)^{\mathrm{T}}$. Then the system (5) is reduced to the following equation

$$\eta' = A_0 \eta. \tag{6}$$

Let $\widetilde{K}$ be the operator on H, defined by the matrix

$$\widetilde{K} = \begin{pmatrix} K & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}.$$

The change $\varsigma = \widetilde{K}^{-1/2}\eta$ transforms (6) into equation

$$\varsigma' = A_1 \varsigma, \tag{7}$$

where $A_1 = \widetilde{K}^{1/2} A_0^{-1/2}$. Denote by A_2, A_3 the operators, defined by the matrices

$$A_2 = \begin{pmatrix} 0 & K^{1/2} & 0 \\ -K^{1/2} & 0 & -S^* \\ 0 & S & -M \end{pmatrix}, \qquad A_3 = \begin{pmatrix} 0 & 0 & 0 \\ PK^{-1/2} & 0 & 0 \\ P_0K^{-1/2} & 0 & 0 \end{pmatrix}.$$

It is easy to check that $A_1 = A_2 + A_3$. The conditions applied to P, P_0, lead to the conclusion that A_3 is a bounded operator. On the other hand, the properties of the operators S, M, K allow us to show that A_2 is a dissipative operator [4, Chapter 1, §4], (i.e., for all $x \in D(A_2)$ $\operatorname{Re}(A_2 f, f) \leqslant 0$). The following lemma leads to the conclusion, that the closure $\bar{A}_2$ of the operator A_2 is a maximal dissipative operator.

Lemma. *The operator $\bar{A}_2$ possesses everywhere defined and bounded inverse.*

Proof. Observe first that the operator $SK^{-1/2}$ is bounded. The equation $(K^{-1/2} \times S^*)^* = S^{**}K^{-1/2}$ [2, Chapter 8, §4] yields the bound of $(K^{-1/2}S^*)^*$. Therefore, the operator $K^{-1/2}S^*$ allows for a continuous extension into the whole space. In what follows the operator-matrix

$$\begin{pmatrix} -K^{-1/2}S^*M^{-1}SK^{-1/2} & -K^{-1/2} & K^{-1/2}SM^{-1} \\ K^{-1/2} & 0 & 0 \\ M^{-1}SK^{-1/2} & 0 & M^{-1} \end{pmatrix}$$

defines a bounded operator on H, and hence it allows for a continuous extension into the whole space H. It can be checked directly that its product (of an arbitrary order) with the operator-matrix defining the operator A_2, yields the unit operator matrix. Hence the lemma is proved. □

Taking into account the maximal dissipative extension of the operator $\bar{A}_2$, the bound of A_3 and the results given in reference [4, Chapter 1, §4], one obtains the following theorem.

Theorem. *The Cauchy problem for Eq.* (7), *where the operator* A_1 *is replaced by its closure, is uniformly correct.*

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