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ANALYTICAL AND NUMERICAL STUDY OF NONLINEAR BEHAVIOUR OF THE ELECTROMECHANICAL SYSTEM

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The nonlinear behaviour of a generator with an amplifier (modelled by a simple oscillator) supplying a current to the string embedded in the electromagnetic field is analyzed. The dynamics of this system is governed by a set of ordinary and partial differential equations and integral equations with a time delay. The method of averaging proposed and the symbolic computations applied lead to the set of four averaged amplitude ordinary differential equations; The numerical analysis reveals interesting nonlinear phenomena.

1. Introduction

In this paper attention is focussed on the derivation of a set of differential nonlinear averaged equations and their numerical analysis. An electromechanical system serves as an example of application of the numerical method proposed. A key part of this model (details are given in the next section) consists of a string, at the ends of which stresses are generated due to the electromagnetic induction. Oscillations of the string are governed by the nonlinear partial differential equation, and the excitation is generated by an amplifier. The theoretical approach is based on the reduction of the original set of equations to a system of ordinary averaged equations (called sometimes envelope or amplitude equations), which is simpler and more suitable for the numerical analysis. Furthermore, this new system of equations allows for discovering the nonlinear phenomena which are very difficult to observe in the analysis of the original set of equations.

A motivation of the presented approach lies in the recently increasing attention paid to the analysis of the averaged equations. Observations of the structures formed in a layer of conducting fluid in the presence of a horizontal magnetic field during the Rayleigh–Bernard convection prove the existence of periodic, quasi-periodic and even chaotic orbits [1–7]. These results have been found as a result of a computer study of the „truncated" system using the appropriate scaling and averaging technique. It is expected that in the present system of averaged equations dynamical behaviour can be also observed.

At the beginning of this paper the averaging method is outlined and the averaged equations are derived. They consist of two first order amplitude equations and two phase equations of the first order. This set is then transformed to the set of four first order amplitude differential equations, which yields (contrary to the first one) the information on the stability of the fixed points determined. The averaging procedure is supported by the symbolic computation (here the„Mathematica" package has been used). It indicates the importance of application of the symbolic computation to the averaging techniques

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in general and, in particular, to the system governed by ordinary and partial nonlinear differential equations. In the latter case, direct derivation of the averaged equations leads to arduous calculations.

In the numerical analysis of the set of equations obtained attention is paid to the computer study of the transient and steady-state oscillations. In the case of a conservative system the analysis is based on the solution of the initial value problem. In the case of a dissipative system, the boundary value problem has been solved.

2. Governing Equations

The detailed discussion of the model and the process of deriving the governing equations is given in [8]. This model consists of a distributed mass system (a stretched string), its transversal oscillations being governed by partial differential equations. The amplifier supplies the current to the string, which is embedded in a magnetic field. The amplifier is modelled by a simple linear oscillator with damping c and frequency $\sqrt{k}$. The amplitude of the current undergoes controlled changes due to the amplifier with a time delay. The electromagnetic induction $B(x)$ acting along the string generates stresses at the ends of the string according to the equation

$$E(t) = \int_0^l B(x) \frac{\partial u(t,x)}{\partial t}\, dx\,, \tag{2.1}$$

where x is the spatial coordinate, t denotes time, $u(t,x)$ is the amplitude of oscillations of the string at point (t,x) and l is the length of the string. The generated stresses are responsible for the force excitation

$$Y(t) = h_1 E(t) - h_2 E^3(t)\,, \tag{2.2}$$

where h_1, h_2 are constant coefficients. Dynamics of the amplifier is governed by the equation

$$\ddot{I}(t) + 2\lambda \dot{I}(t) + kI(t) = \dot{Y}(t-\mu)\,, \tag{2.3}$$

where a dot denotes differentiation with respect to t, λ is the damping coefficient, and μ is the time delay. The changes of time in $I(t)$ and the changes of x in $B(x)$ produce oscillations of the string according to the equation

$$\frac{\partial^2 u(t,x)}{\partial t^2} - c^2 \frac{\partial^2 u(t,x)}{\partial x^2} = -\frac{\varepsilon}{\varrho}\left(2h_0 \frac{\partial u(t,x)}{\partial t} - B(x)I(t)\right), \tag{2.4}$$

where h_0, ϱ are constants, and ε is a small positive parameter. The frequencies of free oscillations of the string are $\omega_s = \Pi cs/l$ and the homogeneous boundary conditions are

$$u(t,0) = u(t,l) = 0\,. \tag{2.5}$$

3. Averaging Equations

For $\varepsilon = 0$ the solution to Eq. (2.4) is given by

$$u_0 = a_1 \cos(\omega_1 t + \theta_1) \sin\left(\pi \frac{x}{l}\right) + a_3 \cos(3\omega_1 t + \theta_3) \sin\left(3\pi \frac{x}{l}\right), \tag{3.1}$$

where a_1, a_3 are the amplitudes, and θ_1, θ_3 — the phases.

For sufficiently small $\varepsilon \neq 0$ the solution to Eq. (2.4) is expected to be of the form

$$u = u_0 + \varepsilon u_1(x, a_1, a_3, \theta_1, \theta_3) + \text{h.o.t.} \tag{3.2}$$

h.o.t. denoting higher order terms.

Supposing that $B(x)$ is a symmetric function, i.e. $B(x) = B(1 - x)$, we assume

$$B = B_1 \sin(\Pi x/l) + B_3 \sin(3\Pi x/l)\,. \tag{3.3}$$

For small μ the right-hand side of Eq. (2.3) can be approximated by $(dY/dt)(1 - \mu)$. A solution to the linear equation (2.3) has the form

$$I_0(t) = M_1 \cos\omega_1 t + N_1 \sin\omega_1 t + M_3 \cos 3\omega_1 t + N_3 \sin 3\omega_1 t + \text{h.h.}\,, \tag{3.4}$$

where M_1, N_1, M_3, N_3 are given below, and h.h. denotes higher harmonics, which are not taken into account. The amplitudes are

$$M_i = (m_i b_i + n_i c_i)/d_1\,, \qquad N_i = (-n_i b_i + m_i c_i)/d_3\,, \qquad i = 1, 3\,, \tag{3.5}$$

where

$$\begin{aligned}
m_1 &= \omega_1^2 - k\,,\\
n_1 &= 2\lambda\omega_1\,,\\
m_3 &= 9\omega_1^2 - k\,,\\
n_3 &= 6\lambda\omega_1\,,\\
d_1 &= -(k - \omega_1^2)^2 - 4\lambda^2\omega_1^2\,,\\
d_3 &= -(k - 9\omega_1^2)^2 - 36\lambda^2\omega_1^2\,,
\end{aligned}$$

(3.6)

$$\begin{aligned}
b_1 = (1 - \mu\omega_1)\Big\{\Big(&-\frac{B_1 a_1 h_1 l\omega_1^2}{2} + \frac{27 B_1 B_3^2 a_1 a_3^2 h_2 l^3 \omega_1^4}{16}\Big)\cos\theta_1 +\\
&+\frac{3B_1^3 a_1^3 h_2 l^3 \omega_1^4}{32}\cos^3\theta_3\Big\} + \frac{B_1 a_1 l\mu\omega_1^3}{2}\Big(-h_1 + \frac{27 B_3^2 a_3^2 h_2 l^2 \omega_1^2}{8}\Big)\sin\theta_1 +\\
&+\frac{3B_1^3 a_1^3 h_2 l^3 \mu\omega_1^5}{32}\cos^2\theta_1 \sin\theta_1 + \frac{9B_1^2 B_3 a_1^2 a_3 h_2 l^3 \mu\omega_1^5}{32}\sin 2\theta_1 \cos\theta_3 +\\
&+\frac{3B_1^3 a_1^3 h_2 l^3 \omega_1^4}{32}(1 - \mu\omega_1)\sin^2\theta_1 \cos\theta_1 + \frac{B_1^2 B_3 a_1^2 a_3 h_2 l^3 \omega_1^4}{32}(9 - 63\mu\omega_1)\sin^2\theta_1 \cos\theta_3 +\\
&+\frac{3B_1^3 a_1^3 h_2 l^3 \mu\omega_1^5}{32}\sin^3\theta_1 - \frac{9B_1^2 B_3 a_1^2 a_3 h_2 l^3 \mu\omega_1^5}{32}\cos^2\theta_1 \sin\theta_3 -\\
&-9\frac{B_1^2 B_3 a_1^2 a_3 h_2 l^3 \omega_1^4}{32}(1 - 7\mu\omega_1)\sin 2\theta_1 \sin\theta_3 + \frac{9B_1^2 B_3 a_1^2 a_3 h_2 l^3 \mu\omega_1^5}{32}\sin^2\theta_1 \sin\theta_3\,,\\
c_1 = -\frac{1}{2}B_1 a_1 l\mu\omega_1^3\Big(&h_1 - \frac{27}{8}B_3^2 a_3^2 h_2 l^2 \omega_1^2\Big)\cos\theta_1 + \frac{3}{32}B_1^3 a_1^3 h_2 l^3 \mu\omega_1^5 \cos^3\theta_1 -\\
&-\frac{9}{32}B_1^2 B_3 a_1^2 a_3 h_2 l^3 \mu\omega_1^5 \cos^2\theta_1 \cos\theta_3 +\\
&+\frac{1}{2}(1 - \mu\omega_1)\Big(B_1 a_1 h_1 l\omega_1^2 - \frac{27}{8}B_1 B_3^2 a_1 a_3^2 h_2 l^3 \omega_1^4\Big)\sin\theta_1 -
\end{aligned}$$

(3.6) [cont.]

$$-\frac{3}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\mu\omega_1)\sin\theta_1\cos^2\theta_1 - 9B_1^2B_3a_1^2a_3h_2l^3\omega_1^4(1-7\mu\omega_1)+$$
$$+\frac{3}{32}B_1^3a_1^3h_2l^3\mu\omega_1^5\cos\theta_1 + 3B_1^3B_3a_1^2a_3h_2l^3\mu\omega_1^5\cos\theta_3)\sin^2\theta_1-$$
$$-\frac{3}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\omega_1) + \frac{9}{32}B_1^2B_3a_1^2a_3h_2l^3\omega_1^4(1-7\mu\omega_1)\cos^2\theta_1\sin\theta_3-$$
$$-\frac{9}{32}B_1^2B_3a_1^2a_3h_2l^3\omega_1^5\mu\sin 2\theta_1\sin\theta_3 - \frac{9}{32}B_1^2B_3a_1^2a_3h_2l^3\omega_1^4(1-7\mu\omega_1)\sin^2\theta_1\sin\theta_3,$$

$$b_3 = -\frac{3}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\mu\omega_1)\cos^3\theta_1+$$
$$+(1-3\mu\omega_1)\left(-\frac{9}{2}B_3a_3h_1l\omega_1^2 + \frac{27}{16}B_1^2B_3a_1^2a_3h_2l^3\right)\cos\theta_3+$$
$$+\frac{243}{32}B_3^3a_3^3h_2l^3\omega_1^4(1-3\mu\omega_1)\cos^3\theta_3 + \frac{27}{32}B_1^3a_1^3h_2l^3\omega_1^5\cos^2\theta_1\sin\theta_1+$$
$$+\frac{9}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\mu\omega_1)\cos\theta_1\sin^2\theta_1 + \frac{9}{32}B_1^3a_1^3h_2l^3\mu\omega_1^5\sin^3\theta_1-$$
$$-\frac{27}{2}B_3a_3h_1l\mu\omega_1^3\left(1-\frac{3}{8}\mu\omega_1\right)\sin\theta_3 + \frac{729}{32}B_3^3a_3^3h_2l^3\mu\omega_1^5\cos^2\theta_3\sin\theta_3-$$
$$-\frac{243}{32}B_3^3a_3^3h_2l^3\omega_1^4(1-3\mu\omega_1)\cos\theta_3\sin^2\theta_3 + \frac{729}{32}B_3^3a_3^3h_2l^3\mu\omega_1^5\sin^3\theta_3,$$

$$c_3 = -\frac{9}{32}B_1^3a_1^3h_2l^3\mu\omega_1^5\cos^3\theta_1 - \frac{27}{2}B_3a_3h_1l\mu\omega_1^3\left(1-\frac{3}{8}\omega_1\right)\cos\theta_3+$$
$$+\frac{729}{32}B_3^3a_3^3h_2l^3\mu\omega_1^5\cos^3\theta_3 + \frac{9}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\mu\omega_1)\sin\theta_1\cos^2\theta_1+$$
$$+\frac{27}{32}B_1^3a_1^3h_2l^2\mu\omega_1^5\cos\theta_1\sin^2\theta_1 - \frac{3}{32}B_1^3a_1^3h_2l^3\omega_1^4(1-\mu\omega_1)\sin^3\theta_1+$$
$$+\frac{1}{2}\left(B_3a_3h_1l\omega_1^2 - \frac{27}{8}B_1^2B_3a_1^2a_3h_2l^3\omega_1^4\right)(1-3\mu\omega_1)\sin\theta_3-$$
$$-\frac{243}{32}B_3^3a_3^3h_2l^3\omega_1^4(1-3\mu\omega_1)\cos^2\theta_3\sin\theta_3 + \frac{729}{32}B_3^3a_3^3h_2l^3\mu\omega_1^5\cos\theta_3\sin^2\theta_3-$$
$$-\frac{243}{32}B_3^3a_3^3h_2l^3\omega_1^4(1-\mu\omega_1)\sin^3\theta_3.$$

Further procedure is typical for the perturbation technique and its details can be found e.g. in [9–16]. Since $B(x)$ and $I(t)$ are defined, Equation (2.4) can be solved using the classical perturbation approach (it is assumed that $u_1(x, a_1, a_3, \theta_1, \theta_3)$ is a bounded and periodic function). Substituting Eq. (3.2) into Eq. (2.4) and taking into account that $a_i = a_i(t)$ and $\theta_i = \theta_i(t)$ $(i = 1, 3)$, the following averaged equations are obtained

$$\frac{da_i}{dt} = \varepsilon P_i(a_1, a_3, \eta),$$
$$\frac{d\theta_i}{dt} = \varepsilon Q_i(a_1, a_3, \eta). \tag{3.7}$$

The right-hand side of Eq. (2.4) (denoted by R) is used to determine the following resonance terms

(3.8)
$$R_{ic} = \frac{2}{\Pi l} \int_0^l \int_0^{2\Pi} R \sin \frac{\Pi i x}{l} \cos \psi_{i0} d\psi_{i0} = \frac{B_i}{\varrho}(M_i \cos\theta_i - N_i \sin\theta_i),$$
$$R_{is} = \frac{2}{\Pi l} \int_0^l \int_0^{2\Pi} R \sin \frac{\Pi i x}{l} \sin \psi_{i0} d\psi_{i0} = \frac{B_i}{\varrho}(N_i \cos\theta_i + M_i \sin\theta_i),$$
$$\psi_{i0} = i\omega_1 + \theta_i, \qquad i = 1, 3,$$

where R_c, R_s correspond to the terms multiplying $\cos\psi_{i0}$ and $\sin\psi_{i0}$, respectively. Comparison of these terms generated by the left-hand side of Eq. (2.4) with those defined by Eq. (3.8) leads to the following formulae

(3.9)
$$P_i = -\frac{1}{2}\frac{B_i(N_i \cos\theta_i + M_i \sin\theta_i}{i\varrho\omega_1},$$
$$Q_i = -\frac{1}{2}\frac{B_i(M_i \cos\theta_i - N_i \sin\theta_i)}{i\varrho a_i \omega_1}.$$

It can be seen from Eq. (3.7) that we are dealing with one variable η instead of two variables θ_1 and θ_3. This is the key point of the averaging procedure presented here. Variable η results from the relation

(3.10)
$$\eta = \theta_3 - 3\theta_1,$$

and the averaging procedure is applied to Eq. (3.7) in a special manner, e.g. for $i = 1$ we take $\theta_3 = \eta + 3\theta_1$, whereas for $i = 3$ we take $\theta_1 = (1/3)(\theta_3 - \eta)$. The final form of the averaged terms appearing in Eq. (3.7) is obtained,

$$Q_1 = \frac{1}{2d_1\varrho}\Bigg\{\Bigg(B_1^2 h_1 l\omega_1 - \frac{3}{16}B_1^4 a_1^2 h_2 l^3\omega_1^3 - \frac{27}{8}B_1^2 B_3^2 a_3^2 h_2 l^3 \omega_1^3\Bigg)(m_1(1-\mu\omega_1) + \mu n_1\omega_1) +$$
$$+ \frac{9}{16}B_1^3 B_3 a_1 a_3 h_2 l^3 \omega_1^3((m_1(1-7\mu\omega_1) + \mu n_1\omega_1)\cos\eta - (n_1(1-7\mu\omega_1) - \mu m_1\omega_1)\sin\eta)\Bigg\},$$
$$Q_3 = \frac{1}{2d_3\varrho}\Bigg\{\Bigg(3B_3^2 h_1 l\omega_1 - \frac{9}{8}B_1^2 B_3^2 a_1^2 h_2 l^3\omega_1^3 - \frac{81}{16}B_3^4 a_3^2 h_2 l^3\omega_1^3\Bigg)(m_3(1-3\mu\omega_1) + 3\mu n_3\omega_1) +$$
$$+ \frac{1}{16a_3}B_1^3 B_3 a_1^3 h_2 l^2 \omega_1^3((m_3(1-\mu\omega_1) + 3\mu n_3\omega_1)\cos\eta + (n_3(1-\mu\omega_1) - 3\mu m_3\omega_1)\sin\eta)\Bigg\},$$

(3.11)
$$P_1 = -\frac{a_1 h_0}{\varrho} + \frac{1}{2d_1\varrho}\Bigg\{\Bigg(-B_1^2 h_1 a_1 l\omega_1 + \frac{3}{16}B_1^4 a_1^3 h_2 l^3 \omega_1^3 +$$
$$+ \frac{27}{8}B_1^2 B_3^2 a_1 a_3^2 h_2 l^3\omega_1^3\Bigg)(n_1(1-\mu\omega_1) - \mu m_1\omega_1) -$$
$$- \frac{9}{16}B_1^3 B_3 a_1^2 a_3 h_2 l^3\omega_1^3((n_1(1-7\mu\omega_1) - \mu m_1\omega_1)\cos\eta + (m_1(1-7\mu\omega_1) + \mu n_1\omega_1)\sin\eta)\Bigg\},$$
$$P_3 = -\frac{a_3 h_0}{\varrho} + \frac{1}{2d_3\varrho}\Bigg\{\Bigg(-3B_3^2 a_3 h_1 l\omega_1 + \frac{9}{8}B_1^2 B_3^2 a_1^2 a_3 h_2 l^3\omega_1^3 +$$
$$+ \frac{81}{16}B_3^4 a_3^3 h_2 l^3 \omega_1^3\Bigg)(n_3(1-3\mu\omega_1) - 3\mu m_3\omega_1) +$$
$$+ \frac{1}{16}B_1^3 B_3 a_1^3 h_2 l^3\omega_1^3((m_3(1-\mu\omega_1) + 3\mu n_3\omega_1)\sin\eta - (n_3(1-\mu\omega_1) - 3\mu m_3\omega_1)\cos\eta)\Bigg\}.$$

The first attempt to derive the averaged set of equations was made by RUBANIK [17]. However, we have applied here a different method of averaging, and we have found some numerical errors in the set of equations presented in [17].

The analyzed set of equations has some properties which make the numerical analysis very difficult. First of all it is a stiff set of equations (a_3 appears in the denominator of Q_3 in Eq. (3.11)). As it is assumed in the averaging procedure, amplitudes a_i and θ_i change with time very slowly, and a long integration is required to trace the behaviour of the system.

Furthermore, the obtained set of equations does not allow for the determination of stability of the fixed points of Eq. (3.7). In order to emit these difficulties we assume

$$u_0 = (Y_1 \cos\omega_1 t + Y_2 \sin\omega_2 t)\sin\left(\pi\frac{x}{l}\right) + (Y_3 \cos\omega_3 t + Y_4 \sin\omega_3 t)\sin\left(3\pi\frac{x}{l}\right). \tag{3.12}$$

Comparison with Eq. (3.1) yields the following relations:

$$\begin{aligned} Y_1(t) &= a_1(t)\cos\theta_1(t), \\ Y_2(t) &= -a_1(t)\sin\theta_1(t), \\ Y_3(t) &= a_3(t)\cos\theta_3(t), \\ Y_4(t) &= -a_3(t)\sin\theta_3(t). \end{aligned} \tag{3.13}$$

In what follows, the set of the amplitude differential equations has the form

$$\begin{aligned} \dot Y_1 &= \dot a_1(t)\cos\theta_1(t) - a_1(t)\dot\theta_1(t)\sin\theta_1(t), \\ \dot Y_2 &= -\dot a_2(t)\sin\theta_1(t) - a_1(t)\dot\theta_1(t)\cos\theta_1(t), \\ \dot Y_3 &= \dot a_3(t)\cos\theta_3(t) - a_3(t)\dot\theta_3(t)\sin\theta_3(t), \\ \dot Y_4 &= -\dot a_3(t)\sin\theta_3(t) - a_3(t)\dot\theta_3(t)\cos\theta_3(t), \end{aligned} \tag{3.14}$$

where a_i and θ_i are given by Eq. (3.7), and

$$\begin{aligned} \theta_1 &= \operatorname{arctg}(-Y_2/Y_1), \\ \theta_3 &= \operatorname{arctg}(-Y_4/Y_3), \\ a_1 &= \sqrt{Y_1^2 + Y_2^2}, \\ a_3 &= \sqrt{Y_3^2 + Y_4^2}. \end{aligned} \tag{3.15}$$

4. Numerical Calculations and Results

As it has been already mentioned, the averaged equations (3.14) are stiff. For this reason the backward differentiation formula up to the fifth order (called Gear's method) is used to simplify the solution.

Let us begin with the numerical values of the following parameters: $l = 0.1$, $\omega = 1.7$, $\lambda = 0.01$, $k = 25$, $h_1 = 0.01$, $h_2 = 0.6$, $\varrho/\varepsilon = 1$, $B_1 = 4.0$, $B_2 = 0.2$; the damping coefficient $h_0 = 0.0$. The time delay u is used as a control parameter. For $\mu = 0.0$ the solutions Y_i oscillate with very small amplitudes, as shown in Fig. 1. The situation drastically changes with the increasing time delay (see Fig. 2). Not only the magnitude of the oscillations of each solutions increases but, in spite of the evident regularity of $Y_{1,2}(t)$, sudden changes with a very low frequency and large amplitudes of $Y_{3,4}(t)$ are observed.

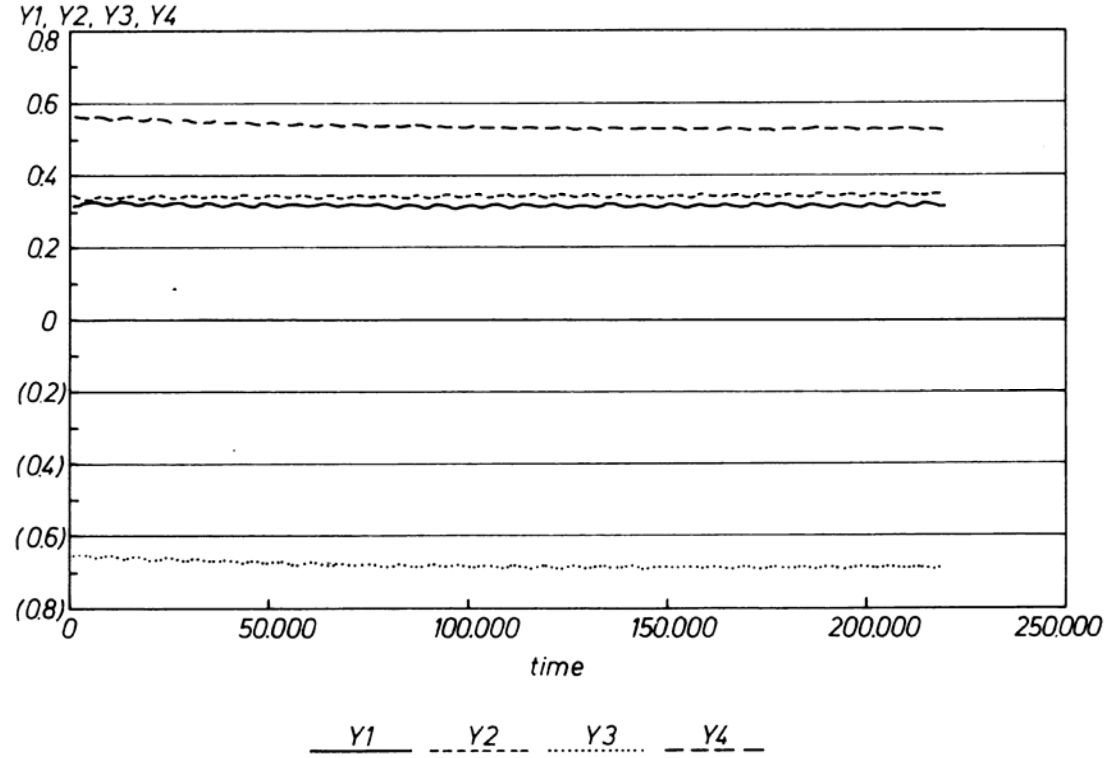


FIG. 1. Solutions to the averaged system of equations for $\mu = 0.0$.

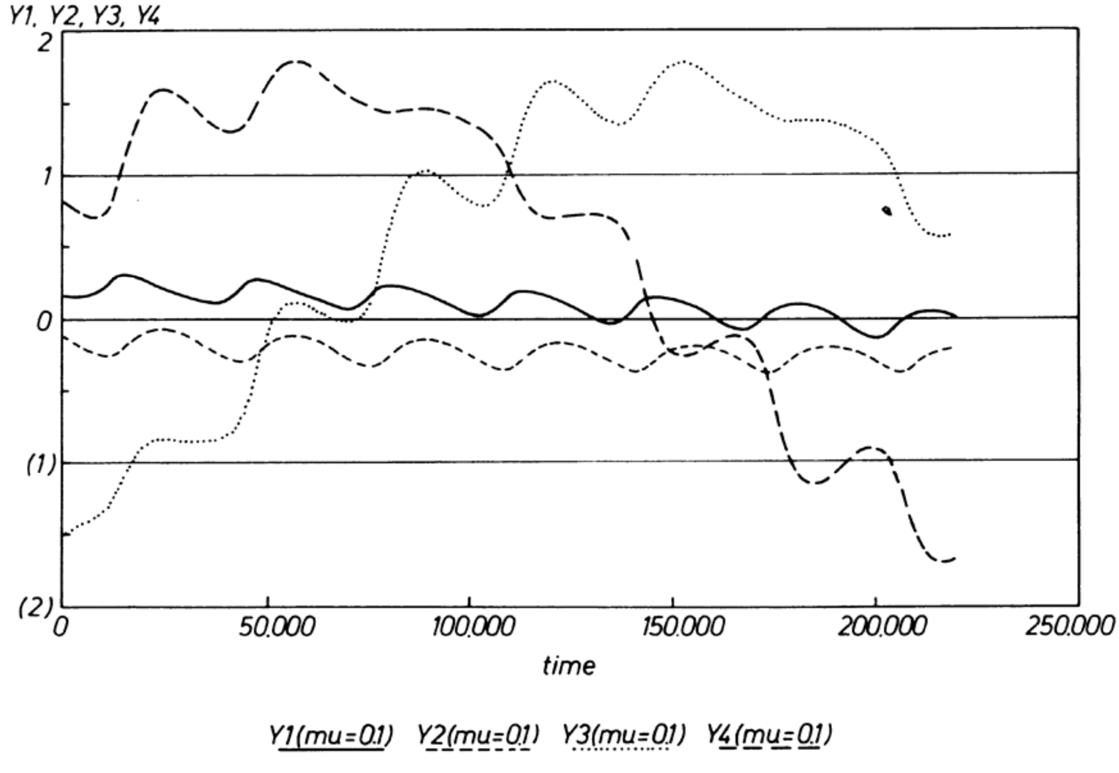


FIG. 2. Solutions to the averaged system of equations for $\mu = 0.1$.

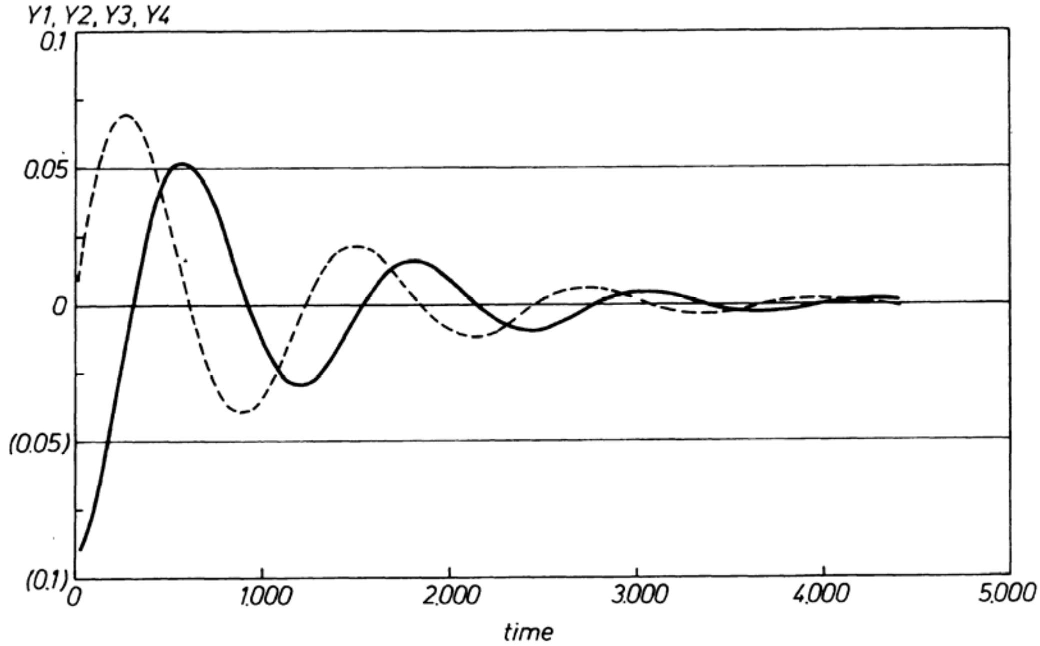


FIG. 3. Transient phenomena of $Y_1(t)$, $i = 1, 4$.

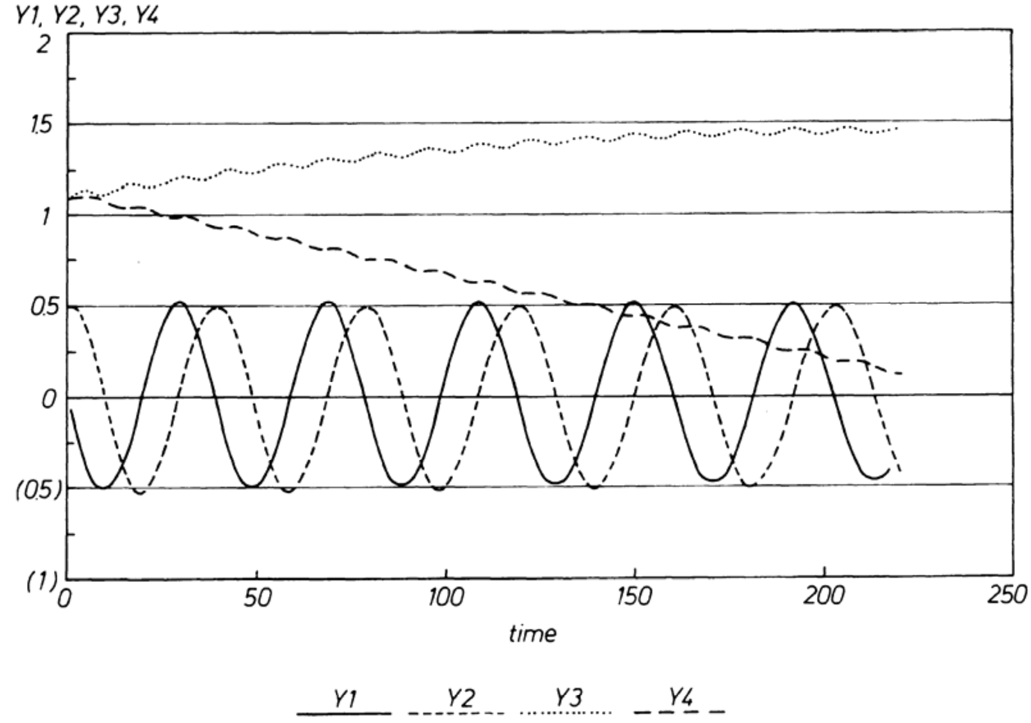


FIG. 4. Time evolution of $Y_i(t)$, $i = 1, 4$ for $B_1 = 15.0$.

Such a behaviour may be termed„unexpected", since the magnitude of the excitation amplitudes B_1, B_2 suggest a different result. Moreover it is found that by confining the solution of the averaged amplitude equations to the first harmonic of oscillations, the results can be made completely incorrect. The observed changes are very slow and

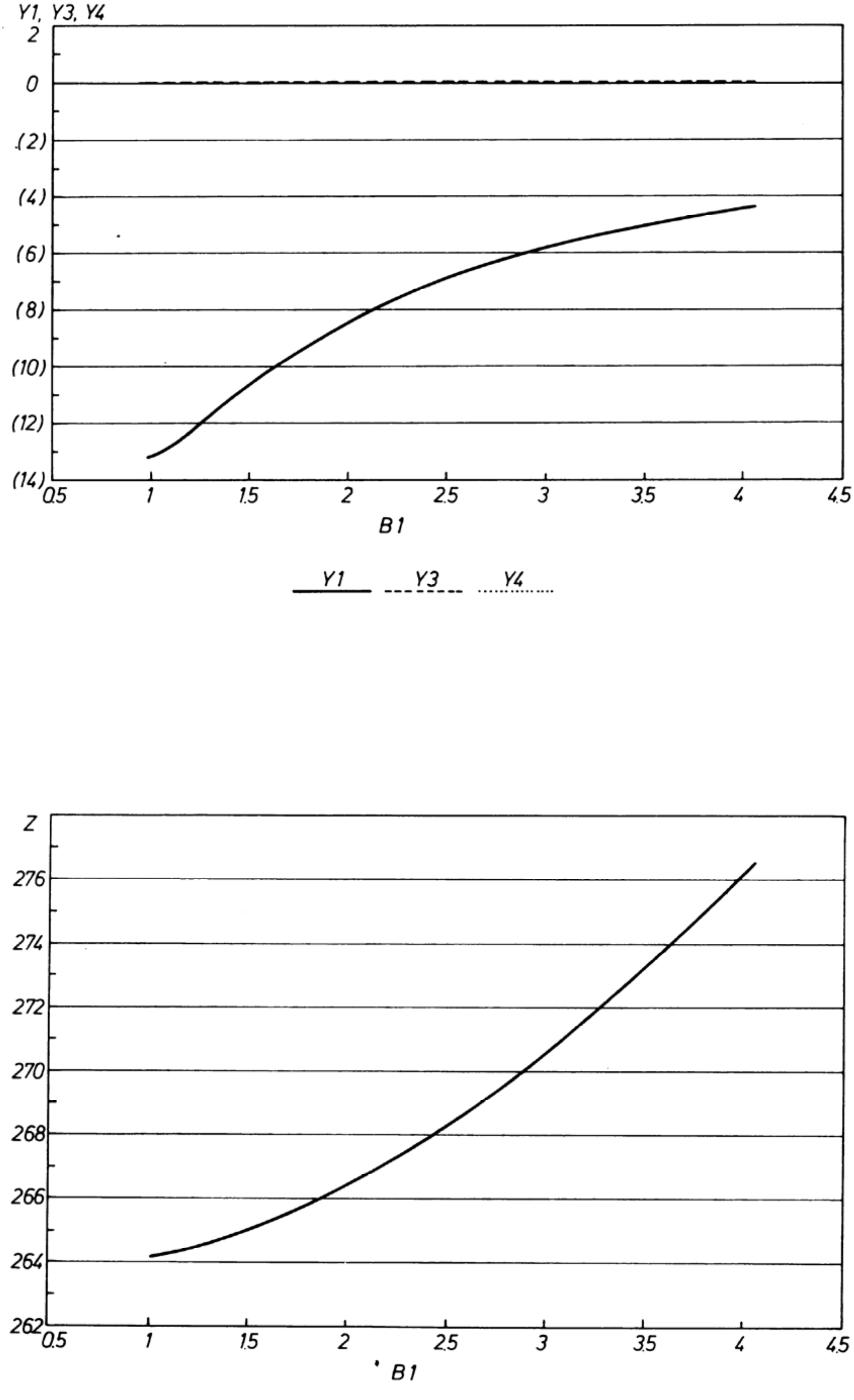
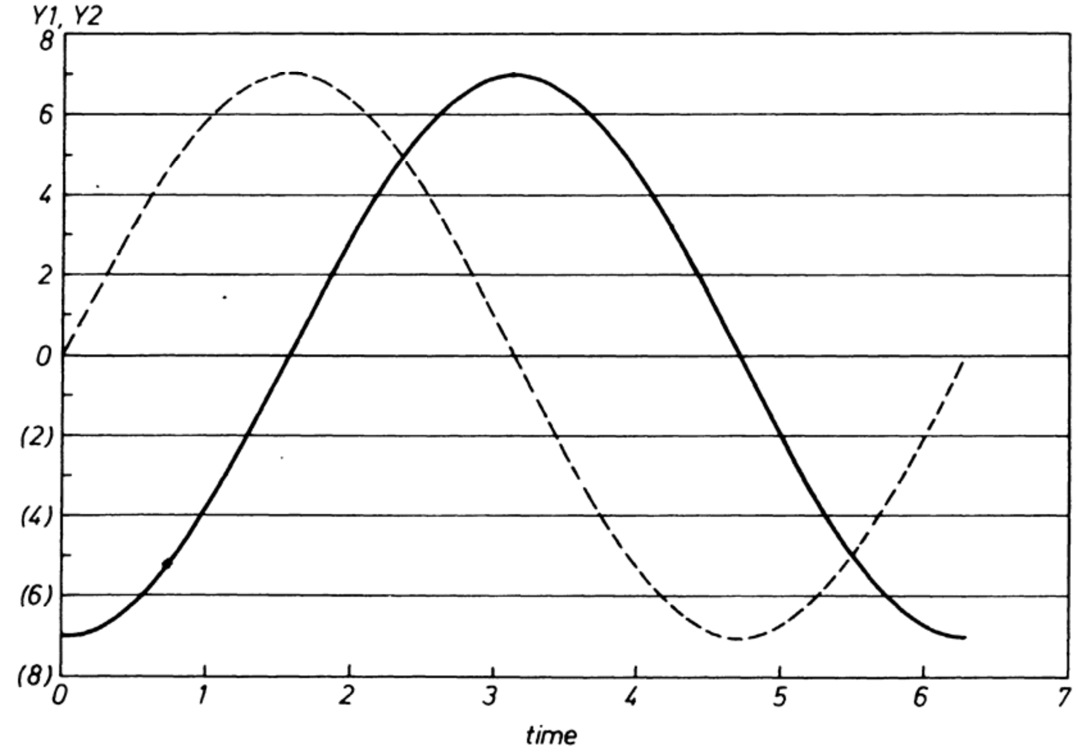


FIG. 5. Fixed points of Y_1, Y_3, Y_4 (a), and variable $Z = 1/\omega$ (b) against the control parameter B_1.



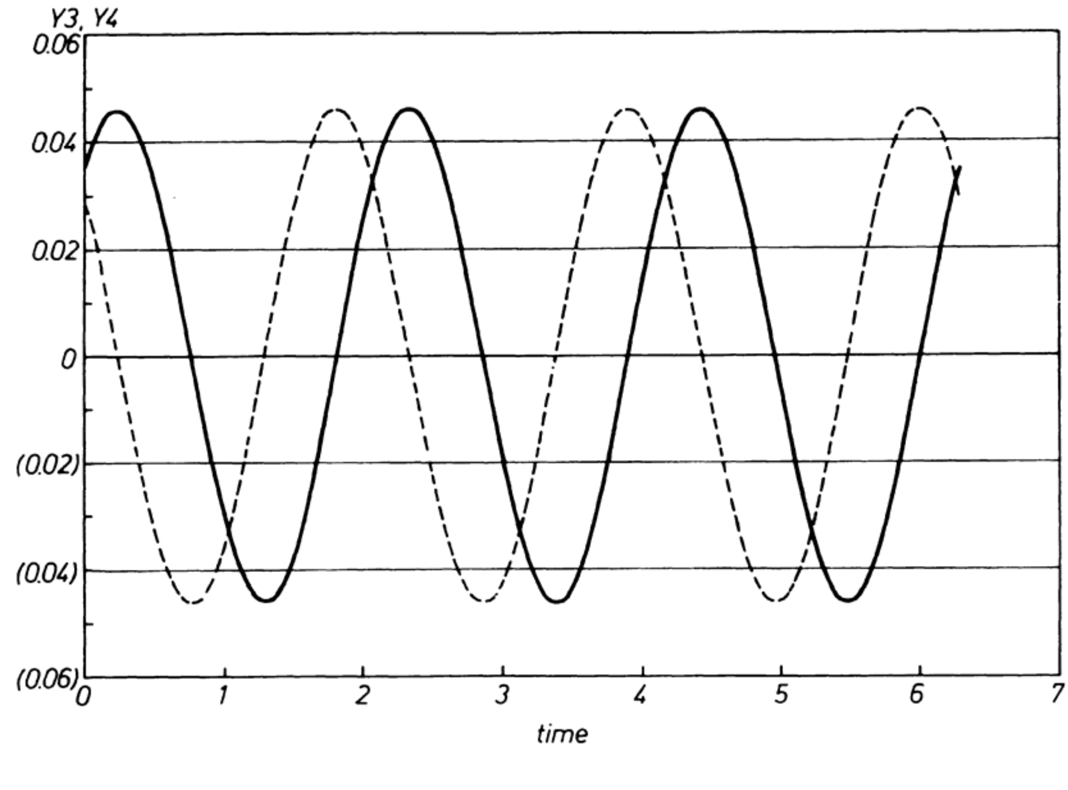


FIG. 6. Time evolution of the periodic orbit $Y_{1,2}(t)$ (a), and $Y_{3,4}(t)$ (b) with the normalized 2π period.

they vanish quickly after introducing of damping h_0. This is presented in Fig. 3 for the following parameters: $l = 0.1$, $\mu = 0.1$, $\omega = 1.7$, $\lambda = 0.01$, $k = 25$, $h_1 = 0.1$, $h_2 = -0.001$, $\varrho/\varepsilon = l/h_0 = 0.002$, $B_1 = 4.0$, $B_3 = 0.2$. In this case the oscillations of $Y_{3,4}(t)$ vanish very quickly, whereas the transient damped oscillations of $Y_{3,4}(t)$ approach zero much more slowly.

With an increase of the amplitude B_1 (the other parameters are the same as in the first example) the oscillations $Y_{1,2}$ also increase, whereas $Y_{3,4}$ exhibit monotonic increase and decrease, respectively, accompanied by small high frequency oscillations. This is illustrated in Fig. 4 for $B_1 = 15.0$.

Let us now proceed with observation of the periodic orbit. To this end consider an approximate position of the fixed point $\mathbf{Y}_0^{(k)}$ (close to the unknown exact one) and perform the numerical integration over the estimated period $T^{(k)}$. Actually we have rescalled the equations according to the rule $\tau = \Omega t$, where Ω serves as an unknown to be determined and the period is equal to 2π. The error $\mathbf{E} = \mathbf{Y}_0^{(k)} - \mathbf{G}_0^{(k)}$ (where $\mathbf{G}_0^{(k)} = \mathbf{G}(\mathbf{Y}_0^k)$) is a point mapping) shows the accuracy of calculations. Then, after a perturbation of the fixed point, its stability is determined. Further details of this approach can be found in [18–20]. The results of calculation of the fixed points as functions of B_1 for $l = 0.1$, $\mu = 0.1$, $\omega = 2.9$, $\lambda = 0.01$, $k = 9.0$, $h_1 = 0.05$, $h_2 = 0.01$, $\varrho/\varepsilon = 1.0$, $h_0 = 0.02$, $B_3 = 0.8$ are presented in Fig. 5. In this Figure Y_3 and Y_4 are much smaller than Y_1 (Y_2 is assumed to be zero). Variation of the period following the change of B_1 is evident (see Fig. 5b, where $Z = 1/\Omega$). The observed periodic orbit is „strongly" stable, i.e. the multipliers are lying close to the origin. As an example, one of the periodic orbits is presented in Fig. 6. It is seen that one revolution of variables $Y_{1,2}$ corresponds to two revolutions of $Y_{3,4}$ during the period 2π.

5. Summary and Conclusions

The results and methods presented in this paper can be reduced to the following three stages.

1. The averaging procedure is proposed to study the original set of partial, integral and ordinary equations governing the dynamics of the electromechanical system.

2. Applications of the symbolic computation to the derivation of the averaged equations is successfully performed indicating the advantage of the application of symbolic computations in the averaging procedures.

3. Numerical study based on both the initial and boundary value problems is carried out showing the interesting nonlinear phenomena. In the case of absence of damping ($h_0 = 0.0$), in spite of the evidens regularity in $Y_{1,2}(t)$, sudden changes of $Y_{3,4}(t)$ with the very low frequency and large amplitudes are found. Growth of the amplitude B_1 causes the increase of the magnitude of oscillations $Y_{1,2}(t)$ and the respective monotonic increase and decrease of $Y_3(t)$ and $Y_4(t)$ with considerably damped oscillations.

A periodic orbit in the averaged system of equations has been also found and analyzed on the basis of the solution of the boundary value problem. An interesting observation is that one revolution of $Y_{1,2}(t)$ corresponds to two revolutions of $Y_{3,4}(t)$. Also the period varies with the change of the control parameter B_1. It means that in the original system periodic and quasi-periodic oscillations of $u(x,t)$ are possible (see Eqs. (3.1) and (3.2)).

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