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Observation of Chaos in the Nonautonomous System with Two Degrees of Freedom

Introduction

The chaotic behaviour of coupled harmonic oscillators has not received so much attention when compared with simple nonlinear sinusoidally driven oscillators. The last are up to now widely referenced in the literature (see for example [1–3]). We will report the influence of control parameters to the chaotic behaviour of each of the two coupled oscillators, when one of them is sinusoidally excited.

Analysis

We consider the mechanical system with two degrees of freedom governed by the equations:

$$\left.\begin{aligned} &m_1\ddot{x}_1 + (c_3 - c_1)\,\dot{x}_1 - c_3\dot{x}_2 + c_2x_1^2\dot{x}_1 \\ &\quad + (k_1 + k_3)\,x_1 - k_3x_2 + k_2x_1^3 = q_1\cos\omega_1 t\,, \\ &m_2\ddot{x}_2 + (c_3 - c_4)\,\dot{x}_2 - c_3\dot{x}_1 + c_5x_2^2\dot{x}_2 \\ &\quad + (k_3 + k_4)\,x_2 - k_3x_1 + k_5x_2^3 = 0\,, \end{aligned}\right\} \qquad (1)$$

where m_1, m_2 are the masses of the vibrating oscillators, $c_1, \ldots, c_5$ and $k_1, \ldots, k_5$ are adequate damping and stiffness coefficients and q_1 is the amplitude of the exciting force with frequency ω_1. By introducting the connections

$$\left.\begin{aligned} &\tau = \omega_1 t\,, \qquad \xi_1 = (\omega_1 m_1^{-1})\,k_2^{1/2}x_1\,, \\ &\xi_2 = (\omega_1 m_2)^{-1}k_5^{1/2}x_2\,, \qquad M = m_1m_2^{-1}\,, \qquad k = k_2k_5^{-1}\,, \\ &B_1 = q_1\omega_1^{-3}m_1^{-3}k_2^{1/2}\,, \qquad \alpha_1 = c_1(m_1\omega_1)^{-1}\,, \\ &\alpha_3 = c_3(m_1\omega_1)^{-1}\,, \qquad \alpha_4 = c_4(m_1\omega_1)^{-1}\,, \\ &\gamma_1 = \omega_1c_2k_2^{-1}\,, \qquad \gamma_2 = \omega_1c_4k_2^{-1}\,, \\ &\varkappa_1 = k_1m_1^{-1}\omega_1^{-2}\,, \qquad \varkappa_3 = k_3m_1^{-1}\omega_1^{-2}\,, \\ &\varkappa_4 = k_4m_1^{-1}\omega_1^{-2}\,, \end{aligned}\right\} \qquad (2)$$

the equations (1) receive the nondimensional form

$$\left.\begin{aligned} &\xi_1'' + (\alpha_3 - \alpha_1)\,\xi_1' - \alpha_3(K/M)^{1/2}\xi_2' + \gamma_1\xi_1^2\xi_1' \\ &\quad + (\kappa_1 + \kappa_3)\,\xi_1 - \kappa_3(K/M)^{1/2}\xi_2 + \xi_1^3 = B_1\cos\tau\,, \\ &\xi_2'' + M(\alpha_3 - \alpha_4)\,\xi_2' - M_{\alpha 3}(M/K)^{1/2}\xi_1' + \gamma_2K\xi_2^2\xi_2' \\ &\quad + M(\kappa_3 + \kappa_4)\,\xi_2 - M\kappa_3(M/K)^{1/2}\xi_1 + \xi_2^3 = 0\,. \end{aligned}\right\} \qquad (3)$$

As the equations (3) have the following symmetry under the transformations

$$T\colon (\xi_1, \xi_1', \xi_2, \xi_2', \tau) \to (\xi_1, \xi_1', \xi_2, \xi_2', \tau + 2\pi)\,,$$

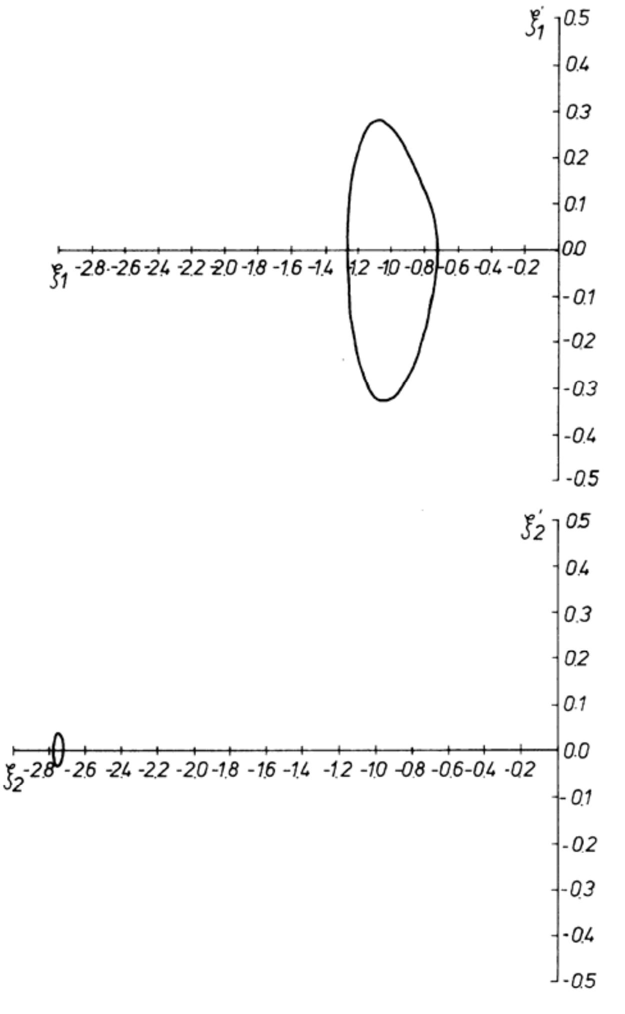

Fig. 1. Phase portraits of the periodic orbit.

we have computed the following Poincaré map $M \subset R^4$ defined as the following set:

$$P = \{\xi_1(\ ), \xi_1'(\ ), \xi_2(\ ), \xi_2'(\ ) \mid \tau = 2\pi k, k = 1, 2, 3, \dots\},$$

where $\xi_{1,2}(\tau)$ is a solution of equations (3). Finite approximations of P have been calculated numerically by the use of Gear's method.

We begin with the periodic orbits obtained for the following fixed parameters: $\alpha_1 = \alpha_4 M = -0.2857$, $\alpha_3 = \alpha_3 M = 0.1$, $\gamma_1 = \gamma_2 K = 0$, $\varkappa_1 = \varkappa_4 M = -0.8163$, $\varkappa_3 = \varkappa_3 M = 0.1$, $B_1 = 0.3219$ and $M = K = 10$ (Fig. 1). This periodic attractor persists and the situation has not qualitatively changed with the decrease of $M = K$. For $M = K = 2$ we have found a strange chaotic attractor which is presented in Fig. 2. The broad band power spectra and the points of the Poincaré map testify that the solution of (3) is chaotic. The broad band power spectra lie to the left of the pointed value $\omega = 1$ – in the subharmonic region. With the decrease of $M = K$ only slight changes are found. For example for $M = K = 1$ the Poincaré map and power spectra are shown in Fig. 3. The share of the second oscillator (ξ_2) of the component $\omega = 1$ is more evident than in the previously illustrated case. Further decrease of the control parameter causes the evolution of the investigated attractor which, however, remains chaotic (Fig. 4). There are four groups of spectra – near the values of 0.25, 0.5, 0.75 and 1.0. For $M = K = 0.2$ we have discovered an interesting fact presented in Fig. 5. The first oscillator has broad-band spectra in the interval from 0.0 to 1.0 with a large amplitude for $\omega = 1$ (where $\omega = 1$ is the frequency of the exciting force). The second also has clearly visible broad-band spectra which are shifted into the region of the low frequencies and without the amplitude corresponded to $\omega = 1$.

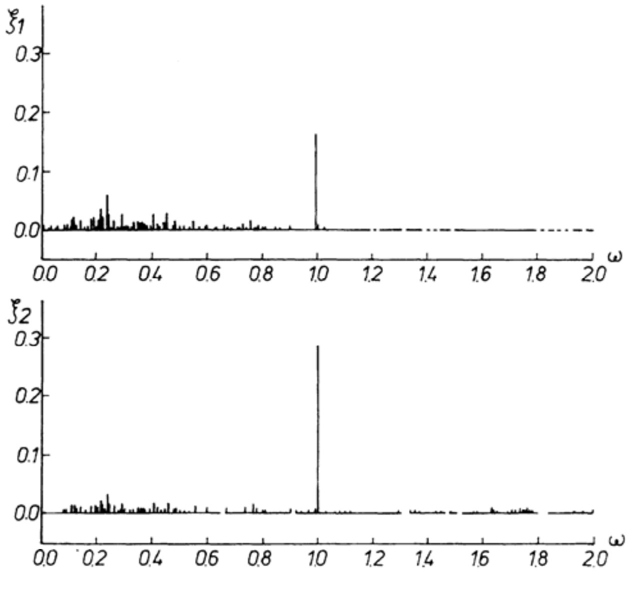

Fig. 2b.

Fig. 2. Two projections of the Poincaré map (a) and power spectra (b) of the strange chaotic attractor ($M = K = 2$).

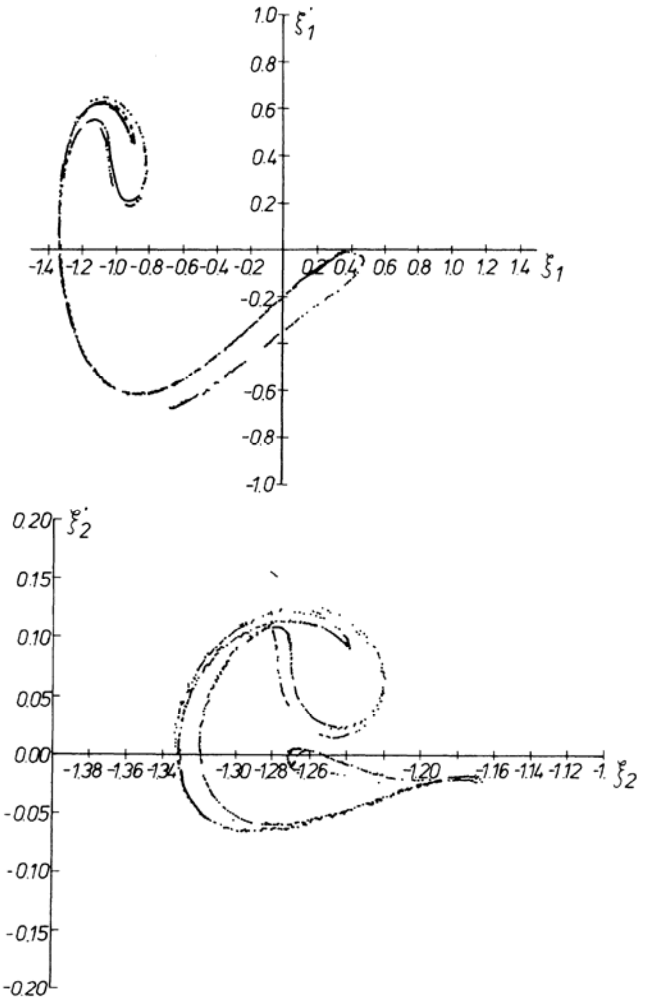

Fig. 2 a.

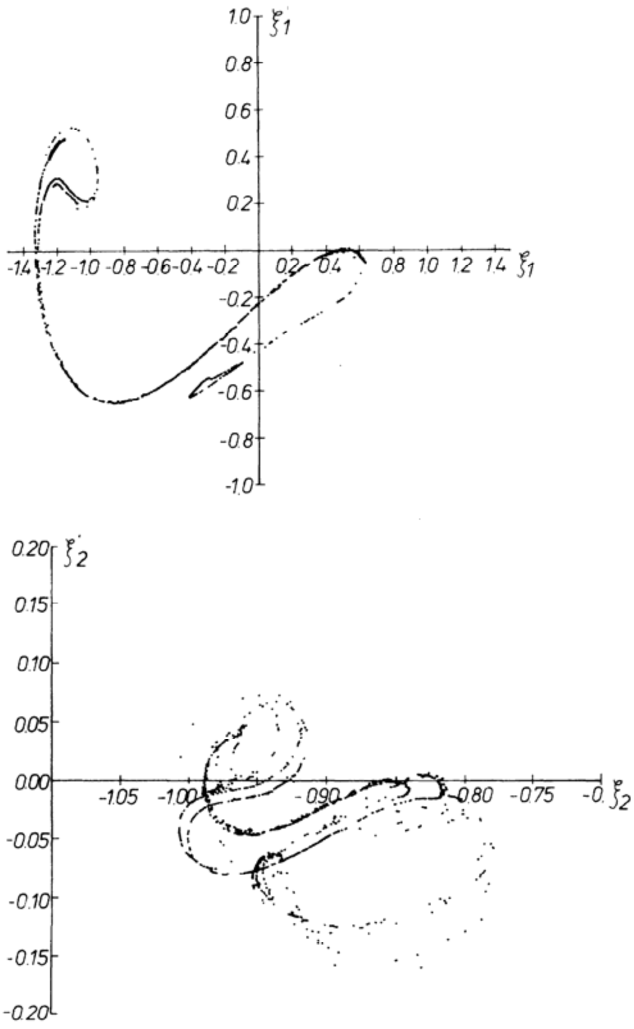

Fig. 3a.

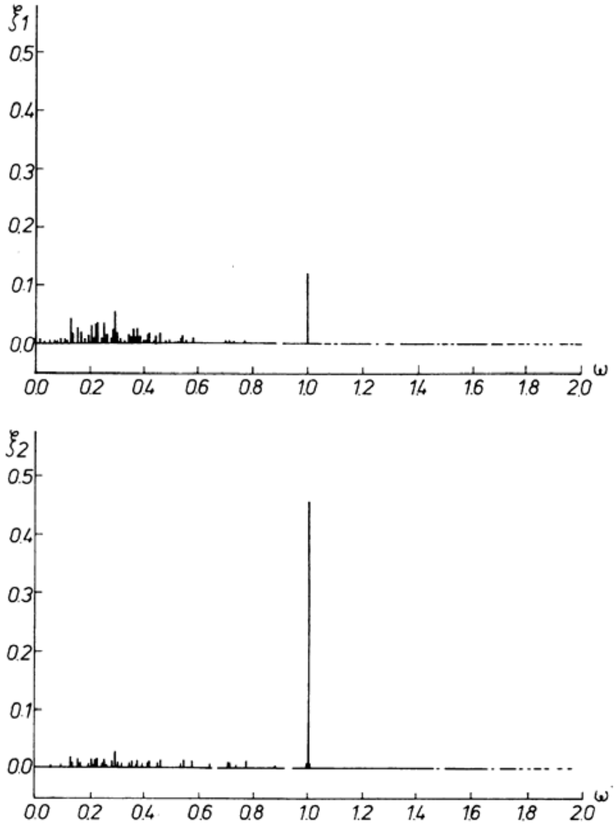

Fig. 3b.

Fig. 3. The same as in Fig. 2 ($M = K = 1$).

We have discussed and illustrated the evolution of chaotic orbits with the change of the control parameters. We have shown that the first oscillator has broad-band spectra with a large amplitude for the frequency of the exciting force, whereas in the spectra of the second oscillator this amplitude is the negligible one.

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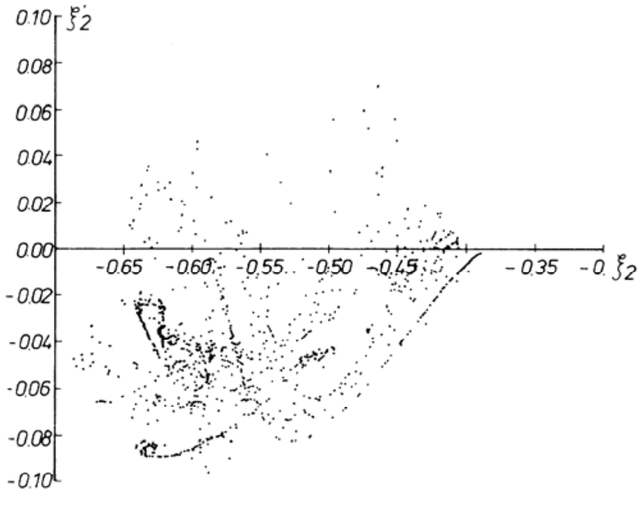

Fig. 4a

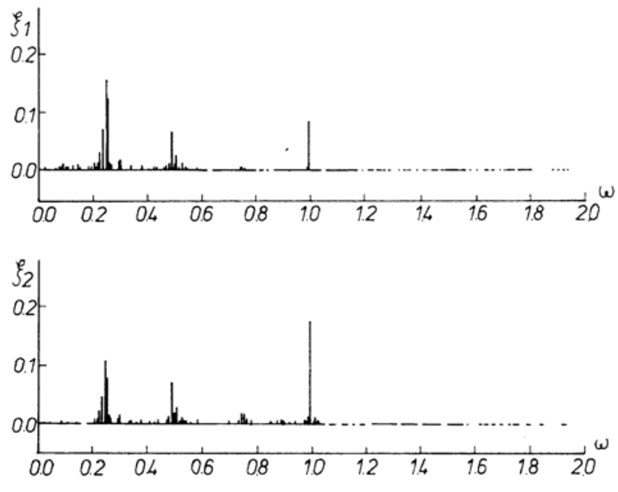

Fig. 4b.

Fig. 4. The same as in Fig. 2 ($M = K = 0.5$).

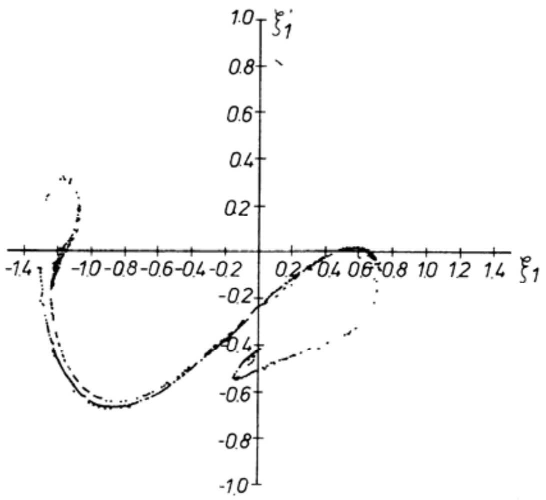

Fig. 4a.

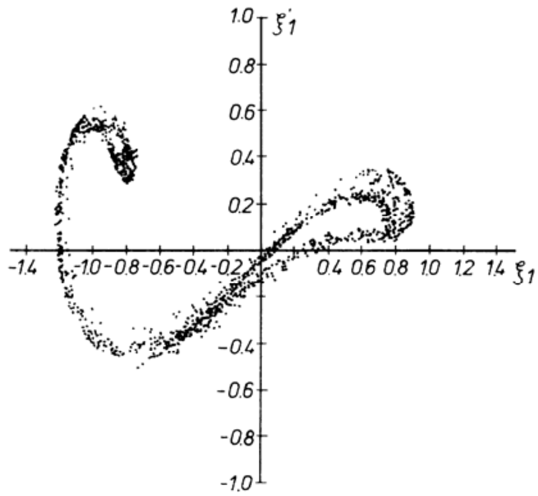

Fig. 5a.

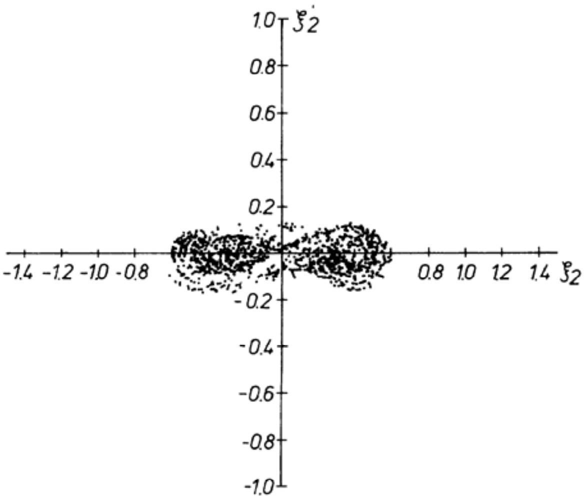

Fig. 5a

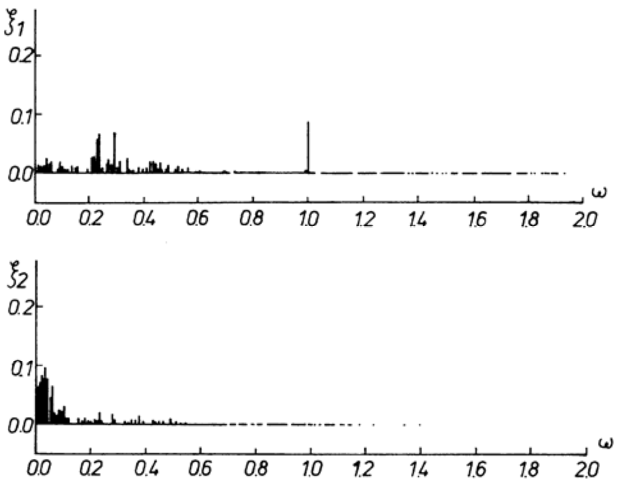

Fig. 5b.

Fig. 5. The same as in Fig. 2 ($M = K = 0.1$).

References

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