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LETTERS TO THE EDITOR

CHAOTIC MOTION OF A CYLINDRICAL CONTAINER ON A NON-LINEAR SUSPENSION: EXPERIMENTAL RESULTS

1. INTRODUCTION

Recently, in the literature, many examples have appeared of irregular motion (chaos) in both discrete and continuous physical systems. Numerical methods are available to allow distinction to be made between irregular and chaotic motion, especially chaotic and quasi-periodic motion. These methods are applied to time histories, phase diagrams, Liapunov exponents and spectrum or auto-correlation functions of systems [1]. Examples from a simple mechanical system with chaotic motion have been given by Moon and Dowell, and their colleagues [2–5]. The investigations of both Moon and Dowell are exceptional in that they have verified their work by comparing the results from experimental and theoretical work in mechanical systems. However, in real mechanical systems, due to their complexity, this approach usually cannot be applied directly. This contribution gives an example of chaotic motion which has been detected in results from an experimental rig modelling the motion of a cylindrical container, partially filled with fluid. The problem of fluid sloshing effects is not new. A comprehensive literature review of the behaviour of fluid in moving containers on aircraft and space vehicles was published in 1966 [6]. An important study of autoparametric coupling of a structure and response at the principal internal resonance, due to the interaction of a limited number of modes of vibration of a structure and the first antisymmetric fluid sloshing mode was conducted by Ibrahim and Barr [7]. In addition to this study the sloshing of fluid in a container has been studied by experiments and digital simulation [8]. Reports on the effects of sloshing in offshore structures, as studied by employing lumped parameter models [9] and the destabilizing effects, due to the sloshing of fluids in various vehicles have been published [10]. In a recent study [11] on small flexible polyester cylindrical containers with fluid it has been shown from experimental work that under horizontal excitation combination unstable resonances of the sum type may occur.

2. EXPERIMENTAL RIG

Figure 1(a) shows a cylindrical container placed on a cast iron table, mass 1300 kg, via four machinery vibration isolators and wooden spacing blocks. The table is suspended on four pneumatic springs, each inflated to an air pressure of 3·4 bar. As shown on Figure 1(b), the motion of the table is sufficient to deflect the pneumatic springs into their non-linear range.

Experiments were carried out with the following conditions: full; $\frac{3}{4}$ full; $\frac{1}{2}$ full of fluid; with the container empty. The experiments were in the form of resonance tests for small amplitudes of vibration and tests at large amplitudes of vibration with harmonic excitation. The direction of excitation is indicated on Figure 1(a). As a means of assessing the response of the system a low frequency displacement transducer was attached to the table, along the axis of excitation and the frequency spectrum of the response from this transducer was recorded on a spectrum analyzer.

3. RESULTS AND DISCUSSION

If only time histories of a system are available it is difficult to distinguish between regular motion with many frequencies and chaotic motion. However, spectrum analysis

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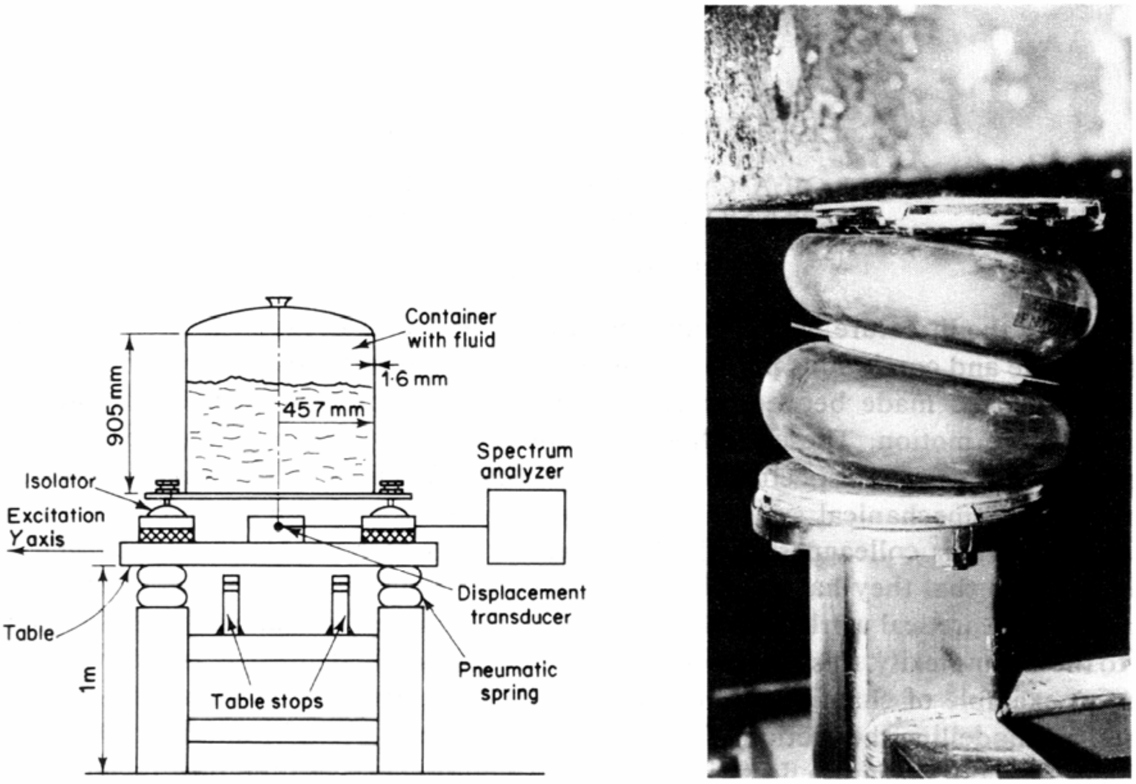

(a) (b)

Figure 1. (a) Test rig; (b) pneumatic spring deflection during table motion.

may be carried out on either simulated or experimental results by using the Fast Fourier Transform method, allowing the two types of motion to be classified. For example, in the case of regular motion with many frequency components the frequency spectrum is discrete, and when chaotic motion is established the frequency spectrum is continuous.

Consider the system of Figure 1, with harmonic excitation along the Y axis, at an amplitude of vibration of approximately 50 mm. The frequency components of the response of the system were recorded by a spectrum analyzer for the following fluid conditions: full; $\frac{3}{4}$ full; $\frac{1}{2}$ full; empty. They are shown on Figures 2(a), 2(b), 2(c) and 2(d) respectively. With reference to Figure 2(b) for the $\frac{3}{4}$ full condition a discrete spectrum is found with six frequency components. One feature of note is the close natural frequencies in the region of ω. Standard resonance tests identified both of these frequencies as resonances of the system, the tests being done for small amplitudes of vibration. For large amplitudes of vibration, in the sufficient non-linear range of the suspension, the discrete frequency components on Figures 2(a), 2(b) and 2(c) were recorded after excitation was well established and after the transient effects had decayed. In general for the container with fluid, each frequency spectrum on these figures is similar to the others, containing frequency components below and above the main exciting frequency ω of the system, due to the non-linear suspension. It is assumed that the behaviour of the fluid in the container is in the linear region because the excitation frequency is low. The results on Figures 2(a), 2(b) and 2(c) give evidence of regular motion. However, from Figure 2(d), this qualitative result for the container without fluid indicates that during vibration the frequency spectrum of the response is continuous, with two distinct frequency components close to the excitation frequency ω. This gives evidence of irregular (chaotic) motion. Therefore this result in relation to the others indicates that the influence of the fluid is sufficient to ensure that chaotic motion of the system does not occur.

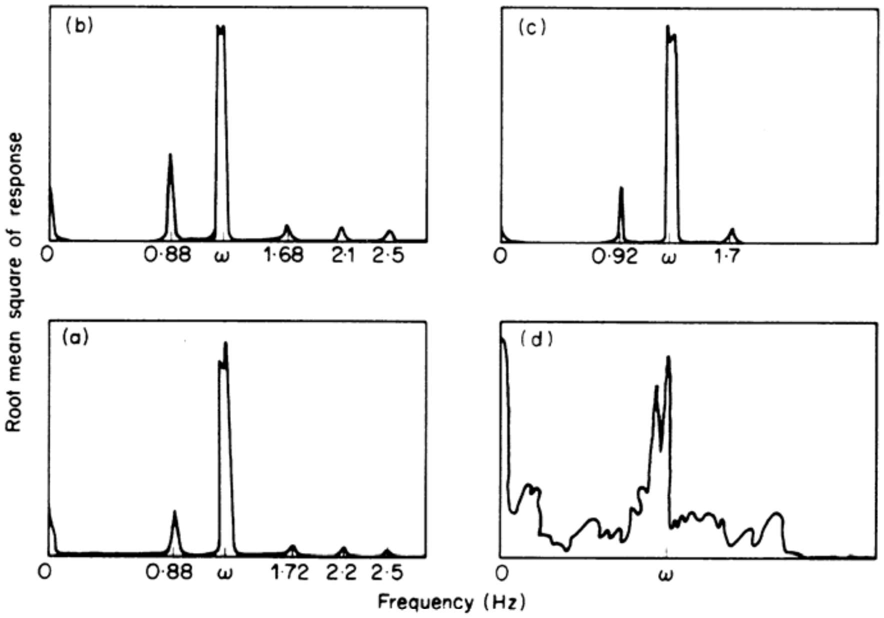

Figure 2. Frequency spectra of system response: (a) full; (b) $\frac{3}{4}$ full; (c) $\frac{1}{2}$ full; (d) empty.

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Institute of Applied Mechanics,
Technical University of Lodz
Lodz 1/15, Poland
J. AWREJCEWICZ

Division of Dynamics and Control,
University of Strathclyde,
Glasgow G1 1XJ, Scotland
R. BARRON

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REFERENCES

1. A. K. LICHTENBERG and M. A. LIEBERMAN 1983 *Regular and stochastic motion.* Berlin: Springer.
2. F. C. MOON 1980 ASME *Journal of Applied Mechanics* **47**, 638-644. Experiments on chaotic motions of a forced nonlinear oscillator: strange attractors.
3. P. J. HOLMES and F. C. MOON 1983 ASME *Journal of Applied Mechanics* **108**, 1021-1037. Strange attractors and chaos in nonlinear mechanics.
4. E. H. DOWELL 1982 *Journal of Sound and Vibration* **85**, 333-344. Flutter of a buckled plate as an example of chaotic motion of a deterministic autonomous system.
5. E. M. DOWELL and C. PEZESHKI 1986 *Transaction of the American Society of Mechanical Engineers* **53**, 5-9. On the understanding of chaos in Duffing's equation including a comparison with experiment.
6. H. N. ABRAMSON 1966 *NASA Report SP-106.* The dynamic behaviour of liquids in moving containers.
7. R. A. IBRAHIM and A. D. S. BARR 1975 *Journal of Sound and Vibration* **42**, 159-179 (Part 1), 181-200 (Part 2). Autoparametric resonances in a structure containing fluid.
8. R. BARRON 1986 *International Conference on System Modelling and Control, Zakopane, Poland.* A study of sloshing of fluids in containers by digital simulation.

9. J. K. VANDIVER and S. MITOME 1978 *Proceedings of the 10th Annual Offshore Technology Conference, Society of Petroleum Engineers, Houston, Texas.* The effect of liquid storage tanks on the dynamic response of offshore platforms.
10. H. F. BAUER 1972/73 *Journal of Vehicle System Dynamics* **1** and **2**, 227–260 (Part 1), 33–48 (Part 2). On the destabilising effects of liquids in various vehicles.
11. M. DHIBA, J. TANI, H. HASHIMOTO and S. SUDO 1986 *Journal of Sound and Vibration* **104**, 301–319. Dynamic stability of liquid-filled cylindrical shells under horizontal excitation, Part 1: experiment.