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CHAOS IN SIMPLE MECHANICAL SYSTEMS WITH FRICTION

1. INTRODUCTION

Some simple deterministic non-linear systems with one degree of freedom act "strangely" at some values of the system parameters: i.e., their response to harmonic exciting force is chaotic. To this kind of system belongs, for example, those described by the Duffing equation, or the Duffing–Van der Pol equation [1–3]. The chaotic motion in such systems is connected with the occurrence of a strange attractor in phase space. The strange attractor is a particular region from which no phase trajectories leave; they are unstable in the Lyapunov sense. The criterion for determining the chaos is connected with the main characteristic exponent of Lyapunov [4], or with the occurrence of a continuous frequency spectrum [1].

2. EXAMPLES OF CHAOS IN SYSTEMS WITH FRICTION

Two examples of systems with friction, of one and two degrees of freedom, respectively, which can be in chaotic motion for some parameter patterns are shown in Figures 1(a)

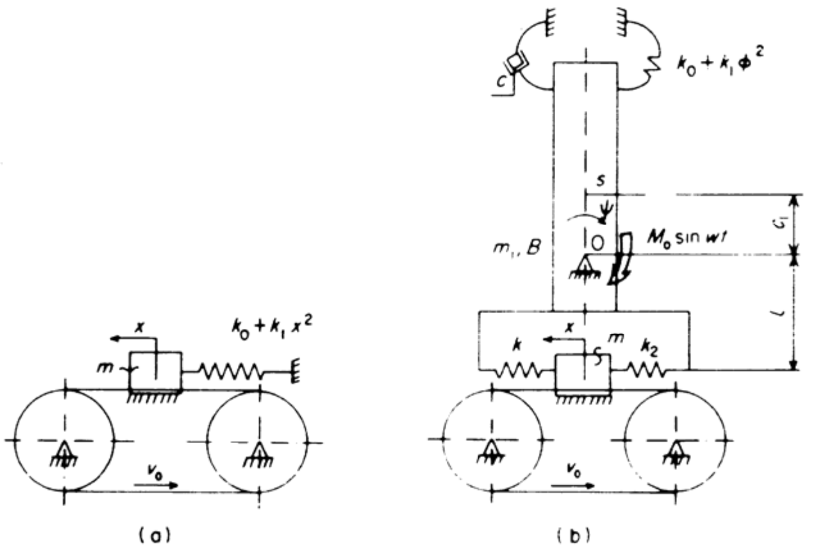

Figure 1. Non-linear frictional systems with (a) one and (b) two degrees of freedom.

and (b). The equation of motion of the system shown in Figure 1(a) is

$$m\ddot{x}+k_0x+k_1x^3=mg[\mu_0 \operatorname{sgn}(v_0-\dot{x})-\alpha(v_0-\dot{x})+\beta(v_0-\dot{x})^3]+P_0\cos \omega t. \tag{1}$$

If $|v_0-\dot{x}|\leq \varepsilon$ (ε being a very small positive number) and $|k_0x(t)+k_2x^3(t)-P_0\cos \omega t|< mg\,\mu_0$, then $x(t)=v_0t$. Otherwise, more complicated solutions of equation (1) have to be found. In the work reported here, numerical solutions have been obtained by means of digital simulation. In this work the parameters in the equation were systematically varied, to provide a variety of solutions. Among these were a periodic and an almost periodic solution, as shown by their respective Poincaré maps; i.e., the point sets on the phase plane $(x, \dot{x})$ with co-ordinates x and $\dot{x}$ such that the time interval between two consecutive

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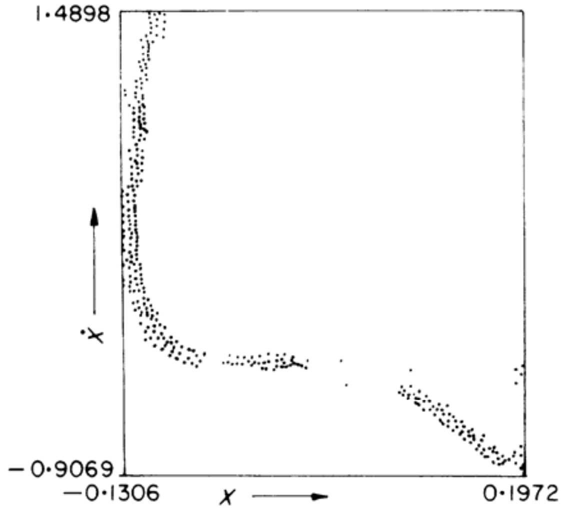

Figure 2. The strange attractor for the system of Figure 1(a).

ones is $T=2\pi/\omega$. The strange attractor shown in Figure 2 was obtained for the following parameters: $m=1$ kg; $k_0=0$; $k_1=1\times10^4$ Nm^{-3}; $\alpha=0{\cdot}05$; $\beta=0{\cdot}02$; $\mu_0=0{\cdot}6$; $v_0=1$ ms^{-1}; $P_0=10{\cdot}5$ N; $\omega=23$ s^{-1}. The initial conditions were $x(0)=0{\cdot}001$ m are $\dot{x}(0)=0$. Figure 3 shows the amplitudes of the Fourier components versus frequency for $x(t)$ (FFT of chaotic motion). The frequency spectrum for the chaotic motion is continuous, whereas for the almost periodic vibrations it is discrete. By means of analysis of Poincaré maps an assessment of the influences of the amplitude, the excitation frequency, the coefficients of the friction characteristic, and the non-linear rigidity, k_1, on the size and position of the strange attractor, has been performed. The strange attractor was found to be most sensitive to the excitation frequency, among these parameters.

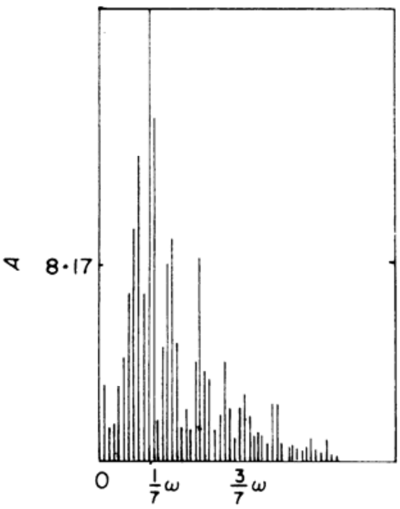

Figure 3. Fast Fourier transform of the chaotic motion for the system with one degree of freedom.

The equations of motion of the system with two degrees of freedom shown in Figure 1(b) are

$$B\ddot{\varphi}+(k+k_2)(l\varphi-x)l+c\dot{\varphi}+k_0\varphi-c_1m_1g\sin\varphi+k_1\varphi^3=M_0\sin\omega t$$

$$m\ddot{x}+(k+k_2)(x-l\varphi)=mg\,\mathrm{sgn}\,(v_0-\dot{x})[a+b\exp(-d(v_0-\dot{x}))]. \tag{2}$$

In this case, particular attention has been paid to the influence of the viscous damping coefficient c on the behaviour of the strange attractor. Figure 4 shows an example of a strange attractor for the following data: $k_0 = 0$; $k_1 = 10\,000$ Nm; $c_1 = 0{\cdot}12$ m; $B = 0{\cdot}0073$ kgm^2; $l = 0{\cdot}1$ m; $m = 10$ kg; $m_1 = 0{\cdot}5$ kg; $k + k_2 = 3000$ Nm^{-1}; $M_0 = 300$ Nm; $v_0 = 0{\cdot}2$ ms^{-1}; $\omega = 10$ s^{-1}; $d = 10$ sm^{-1}; $a = 0{\cdot}36$; $b = 0{\cdot}32$ and $c = 0{\cdot}001$ Nms. For other values as well (c changing from 0·001 to 5·0).

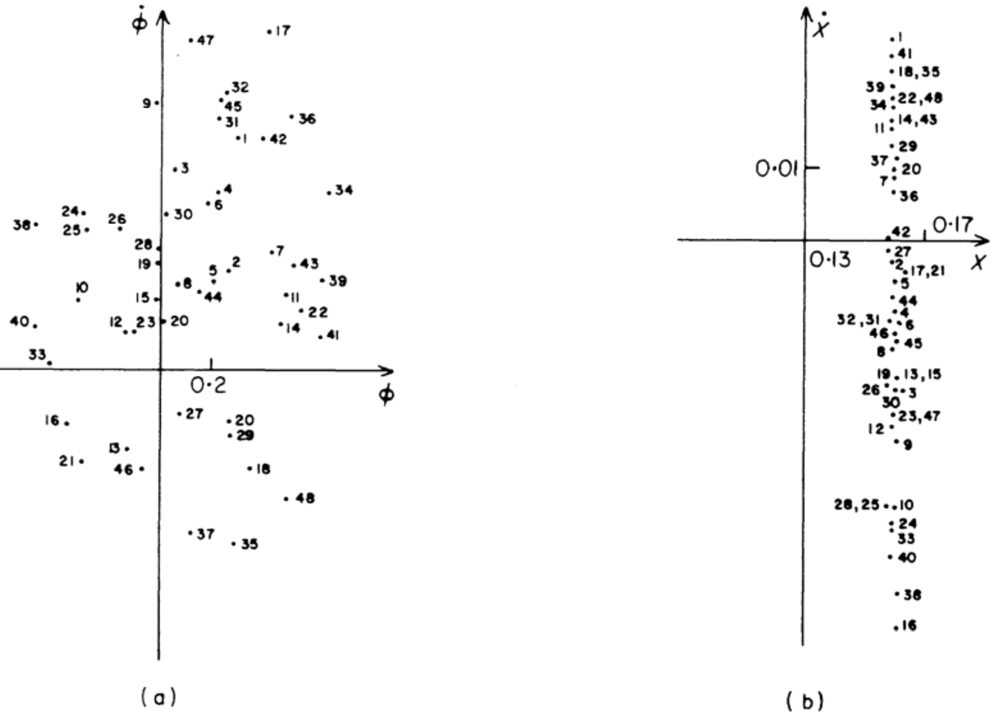

Figure 4. Poincaré maps on the planes (a) $(\varphi, \dot{\varphi})$ and (b) $(x, \dot{x})$ for the system with two degrees of freedom shown in Figure 1(b).

The Poincaré maps on the plane $(\varphi, \dot{\varphi})$ are sets of points in a particular two-dimensional region, while on the plane $(x, \dot{x})$ they are sets of points placed on a straight line. The concentration of the points on the Poincaré maps becomes greater as the damping value c increases.

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