Chaotic parametric vibrations of flexible shells

J. Awrejcewicz, A.V. Krysko, E.Yu. Krylova, I.V. Papkova, V.A. Krysko

Abstract: Chaotic vibrations of rectangular spherical shells subjected to the action of periodic load have been rarely analyzed. This work extends investigations initiated in the works by Awrejcewicz et al. [1-3].

1. Mathematical model

A mathematical model of the flexible rectangular plate with constant stiffness and density under the action of periodic load (Fig. 1) is constructed using the Kirchhoff-Love hypotheses and taking into account the non-linear relations between deformations and displacements in the von Karman form. In the rectangular co-ordinates the 3D plate space is defined as: $\Omega = \{x_1, x_2, x_3 \mid (x_1, x_2) \in [0;a] \times [0;b],$ $x_3 \in [-h;h]\}$, $0 \le t < \infty$. In initial time interval $t \in [0;1]$ we introduce small static load, i.e. its lack defines the governing differential equations as homogenous ones.

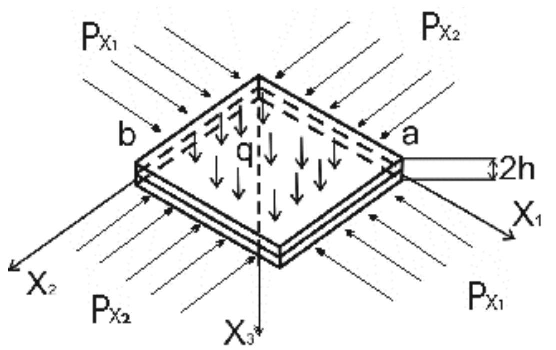


Figure 1. Plate computational scheme

We study the following non-dimensional PDEs governing dynamics of the shallow shells [4]:

$$\begin{aligned}&\frac{1}{12(1-\mu^2)}\left(\nabla_\lambda{}^4 w\right)-\nabla_k^2 F-L(w,F)-\frac{\partial^2 w}{\partial t^2}-\varepsilon\frac{\partial w}{\partial t}-q(x_1,x_2,t)=0,\\&\nabla_\lambda{}^4 F+\nabla_k{}^2 w+\frac{1}{2}L(w,w)=0\end{aligned}\tag{1}$$

where: $\nabla_\lambda{}^4$, $L(w,F)$ and $\nabla_k{}^2$ – known non-linear operators, w and F – functions of deflection and stress, respectively.

The following non-dimensional parameters are introduced: $\lambda = a/b$; $x_1 = a\overline{x}_1$, $x_2 = b\overline{x}_2$; $k_{x_1} = a^2/R_{x_1}(2h)$, $k_{x_2} = b^2/R_{x_2}(2h)$ – non-dimensional shell parameters regarding x_1 and x_2, respectively; $w = 2h\overline{w}$ – deflection; $F = E(2h)^3\overline{F}$ – stress function; $t = t_0\overline{t}$ – time; $q = \frac{E(2h)^4}{a^2b^2}\overline{q}$ – external load; $\varepsilon = (2h)\overline{\varepsilon}$ – damping coefficient, $P = E(2h)^3\overline{P}$ – external longitudinal load. Bars over non-dimensional quantities in the governing equations are already omitted. The following notation is introduced: a,b – plane dimensions in x_1 and x_2 directions, respectively; μ – Poisson's coefficient.

Equations (1) are supplemented by the following boundary conditions [5]:

$$\begin{aligned}&w=0;\frac{\partial^2 w}{\partial x_1{}^2}=0; F=0;\frac{\partial^2 F}{\partial x_1{}^2}=p_{x_2}\quad \text{for}\quad x_1=0;1,\\&w=0;\frac{\partial^2 w}{\partial x_2{}^2}=0; F=0;\frac{\partial^2 F}{\partial x_2{}^2}=p_{x_1}\quad \text{for}\quad x_2=0;1,\end{aligned}\tag{2}$$

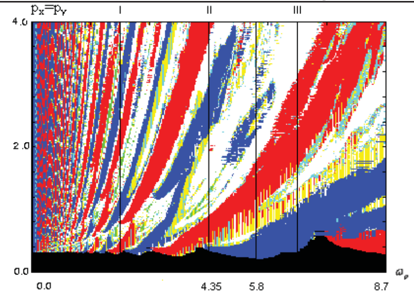
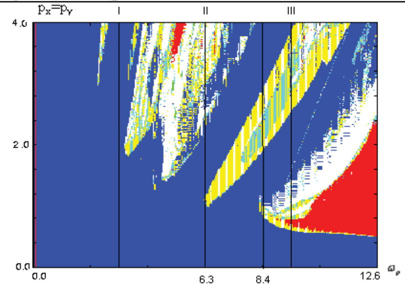
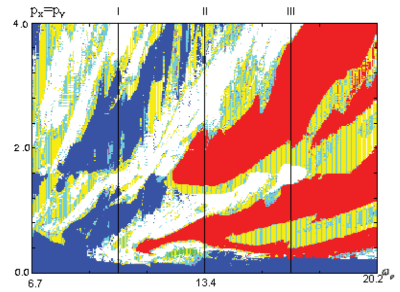
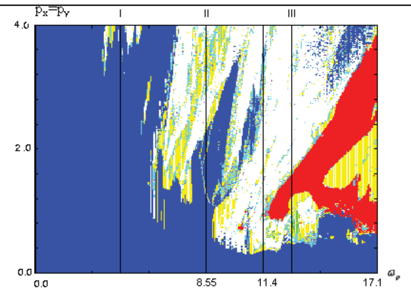
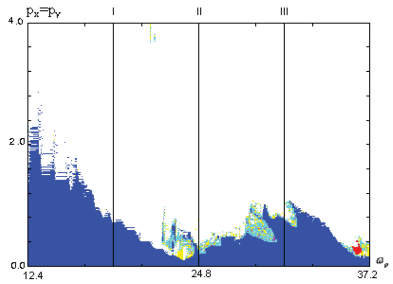
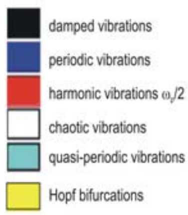
and the following initial conditions:

$$w(x_1,x_2)\,|_{t=0}=0,\ \frac{\partial w}{\partial t}=0.\tag{3}$$

2. Method of solutions and results

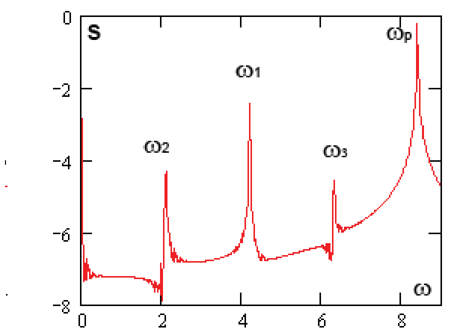
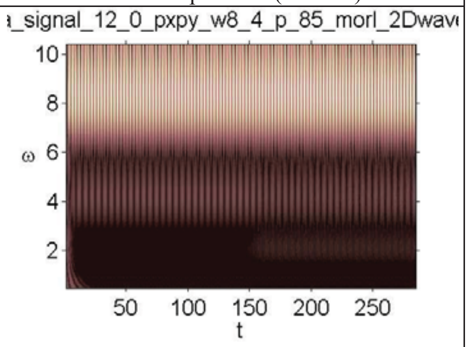
Our mechanical object is studied keeping fixed the following parameters: $\lambda = 1$, Poisson's coefficient $\mu = 0.3$. We apply the following longitudinal load $p_{x_1} = p_{x_2} = p_0\sin(\omega_p t)$. After the application of FDM with approximation $O(h^2)$ regarding spatial co-ordinates the differential problem (1-3) is solved by the Runge-Kutta method of the fourth order. In addition, on each time step we need to solve a large system of linear algebraic equations regarding the stress function. Time step is yielded through the Runge principle. The number of partitions of spatial co-ordinates is n=14. The number of partitions choice and convergence of the obtained numerical results is discussed by Awrejcewicz et al. [6].

Table 1. Charts of the shell vibration regimes versus geometric parameters.

	
$k_{x_1} = k_{x_2} = 0$	$k_{x_1} = 12$, $k_{x_2} = 0$
	
$k_{x_1} = 24$, $k_{x_2} = 0$	$k_{x_1} = k_{x_2} = 12$
 $k_{x_1} = k_{x_2} = 24$	Notation 

In this work we take amplitude and frequency of the exciting longitudinal load acting on the shell perimeter. Our aim was to construct charts displaying the system vibration regimes with parameters 300×300. In order to construct each of the charts 90 000 differential problems have been solved. Each of the mentioned problem required analysis of signals (time histories), phase and modal portraits, Poincaré cross-sections and maps, Fourier and wavelets frequency power spectra, autocorrelation functions, and signs of the Lyapunv exponents. Table 1 gives charts of the shell vibration regimes depending on the geometric shell parameters. In the first sub-harmonic regime the zones of Hopf bifurcations are wide. It is seen that an increase of the geometric parameters implies the increase of bifurcation zones and chaotic dynamics zones, and a decrease of the zones of periodic vibrations. Besides, one may observe that in the case of shell curvatures $k_{x_1}=k_{x_2}=24$ a transition from periodic into chaotic vibrations appears suddenly without any other transitional zones. The following general remarks are based on the computational results. Small exciting load amplitudes generate damped vibrations. Low values of the applied frequencies $\omega \le 2$ of sub-harmonic zones of vibrations (excitation frequency is doubled in comparison to the shell vibration frequency) are mixed with the zones of periodic vibrations. An increase of the excitation frequency implies extension of these zones into higher frequencies, and they are interlaced with rather large chaotic zones.

Table 2. Fourier and wavelet spectra ($k_{x_1}=12$, $k_{x_2}=0$, $\omega_p=8.4$, $p_0=8.5$).

<table>
<tr><td>Fourier spectrum (all time)</td><td>Wavelet spectrum (all time)</td></tr>
<tr><td>
</td><td>
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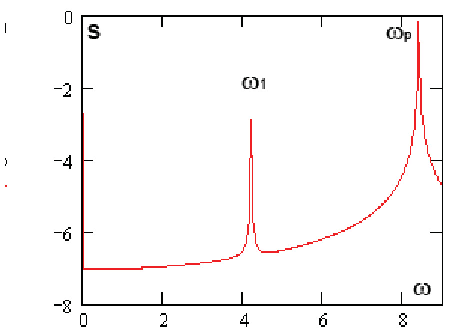
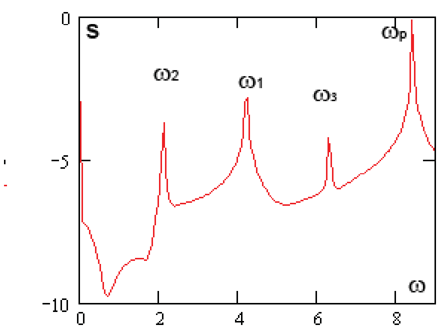
<tr><td>Fourier spectrum $t<150$</td><td>Fourier spectrum $t>150$</td></tr>
<tr><td>
</td><td>
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While constructing the shell vibration charts chaotic dynamics occurred already after the second or even first Hopf bifurcation. Additionally, the wavelet spectra imply that these bifurcations appear even at fixed amplitude and frequency of the excitation for $t \ge t_0$. In the numerical example with parameters $k_{x_1}=12$, $k_{x_2}=0$ (cylindrical panel), $\omega_p=8.4$, $p_0=8.5$, $t\in[0;300]$, using both Fourier and wavelets spectra it is evidently demonstrated how power spectrum essentially changes in time.

For instance, at $t < 150$ the first Hopf bifurcation takes place ($\omega_1 = \omega_p / 2$), at $t > 150$ the second Hopf bifurcation appears ($\omega_2 = \omega_p / 4$) (Table 2). A more detailed analysis is provided by the wavelet analysis which allows us to monitor local particularities of the studied signal.

3. Conclusions

The charts reported in this work allow us to control the investigated continuous mechanical systems. The choice of the control parameters should be made in a way to keep the system within a safe (periodic) zone. Otherwise, transition into the chaotic zone implies the system stability loss and its catastrophe.

Acknowledgements

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