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Nonlinearity of muscle characteristics

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Abstract: In this paper a comparison between two types of linear and one nonlinear models of skeletal muscle stiffness is shown. Results are compared with experimental data obtained by Soderberg for biceps brachii in the case of muscle stretching. It is shown that results for nonlinear stiffness model in case of length - force relationship fits to the experimental data.

1. Introduction

1.1. Biological muscles properties

Testing and muscle modelling are very important aspects of biomechanics. The first mathematical model of muscle was due to Hill, who created it in the twenties of the twentieth century. Since that time a huge progress in understanding of muscle behaviour has been observed (see for example [1] - [5]). Most experiments show that biological muscles have nonlinear behaviour (see [1] - [3]). It has been observed in experiments that stretching force depends nonlinearly on elongation, muscle internal force depends nonlinearly on velocity of shortening, and velocity of muscle movement as a function of load (called Hill curve) is also nonlinear (see Figures 1 and 2). These graphs have been confirmed repeatedly in experimental studies by many authors for many types of muscles of many species (for example: frog, cat, human), see for example [1], [3] and [5]. In the paper muscle models of stiffness are presented. They are investigated and numerical simulations are caried out. It turns out that nonlinear one fits to the experimental data from the literature in contrary to linear models.

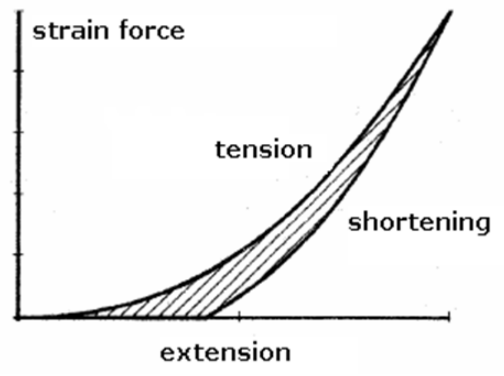

Figure 1. Value of external stretching force as a function of elongation (adopted from [2]), where the values are normalised

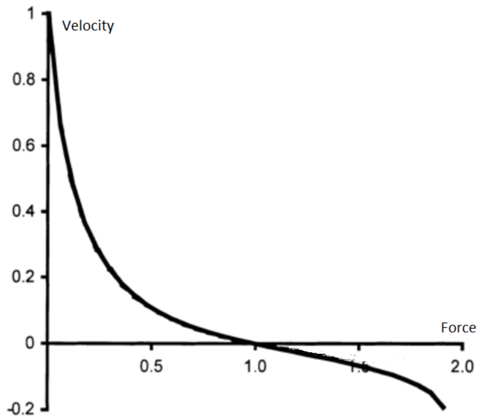

Figure 2. Hill curve exhibiting the value of muscle shortening or elongation velocity as a function of load (from [3])

1.2. Considered muscle model

Mathematical description of the model presented in Fig. 3 is as follows:

$$\begin{cases} m_1\ddot{x}_1 + K_1x_1 + C_1\dot{x}_1 + K_2(x_1 - x_2) + C_2(\dot{x}_1 - \dot{x}_2) = 0 \\ m_2\ddot{x}_2 + K_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) + K_3(x_2 - x_3) + C_3(\dot{x}_2 - \dot{x}_3) = F_{2,3} \\ m_3\ddot{x}_3 + K_3(x_3 - x_2) + C_3(\dot{x}_3 - \dot{x}_2) + K_4(x_3 - x_4) + C_4(\dot{x}_3 - \dot{x}_4) = -F_{2,3} \\ m_4\ddot{x}_4 + K_4(x_4 - x_3) + C_4(\dot{x}_4 - \dot{x}_3) + K_5(x_4 - x_5) + C_5(\dot{x}_4 - \dot{x}_5) = F_{4,5} \\ m_5\ddot{x}_5 + K_5(x_5 - x_4) + C_5(\dot{x}_5 - \dot{x}_4) + K_6(x_5 - x_6) + C_6(\dot{x}_5 - \dot{x}_6) = -F_{4,5} \\ m_6\ddot{x}_6 + K_6(x_6 - x_5) + C_6(\dot{x}_6 - \dot{x}_5) + K_7(x_6 - x_7) + C_7(\dot{x}_6 - \dot{x}_7) = F_{7,8} \\ m_7\ddot{x}_7 + K_7(x_7 - x_6) + C_7(\dot{x}_7 - \dot{x}_6) + K_8(x_7 - x_8) + C_8(\dot{x}_7 - \dot{x}_8) = -F_{7,8} \\ m_8\ddot{x}_8 + K_8(x_8 - x_7) + C_8(\dot{x}_8 - \dot{x}_7) + K_{ext}x_8 + C_{ext}\dot{x}_8 = F_{ext} \end{cases} \tag{1}$$

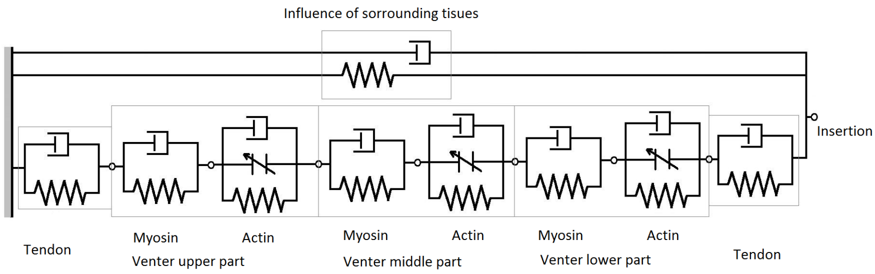

Figure 3. Muscle model scheme

where $x_i,\ i = 1, ..., 8$ is location of centre of gravity of the masses of the different muscle parts; K_i and C_i denote stiffness and dumping parameter of i-th element, respectively the last equation describes influence of surrounding tissues on muscle behaviour.

2. Simulation

2.1. Simulation conditions

Presented model was simulated with three different types of stiffness functions. The simulation time was 50 seconds and the tensile force was applied according to the formula: $F(t) = 5 \cdot t$, where t is time of simulation in seconds.
First simulation was done with following stiffness parameters:

$$K_i := k_i \cdot (k \cdot (x_i - x_{i-1}))^2,\ i = 1, ..., 8, \tag{2}$$

where k_i - beginning stiffness of i-th element (for $i = 1,\ x_{i-1} = x_0 = 0$, k - correction factor). This type of stiffness coefficient was adjusted by the authors of this paper to the presented model. Second simulation was done with constant stiffness parameters

$$K_i := k_i := const,\ i = 1, ..., 8. \tag{3}$$

The last simulation was done with stiffness given by

$$K_i := k_i \cdot (k \cdot (x_i - x_{i-1})),\ i = 1, ..., 8. \tag{4}$$

2.2. Results

Following results were obtained for previously defined stiffnesses.

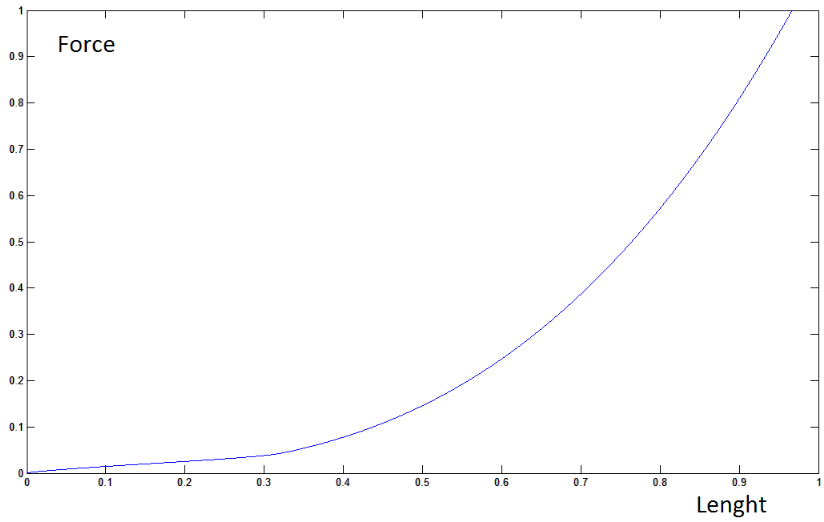

Figure 4. Value of external stretching force as a function of elongation for nonlinear stiffness parameter (see Eq. 2)

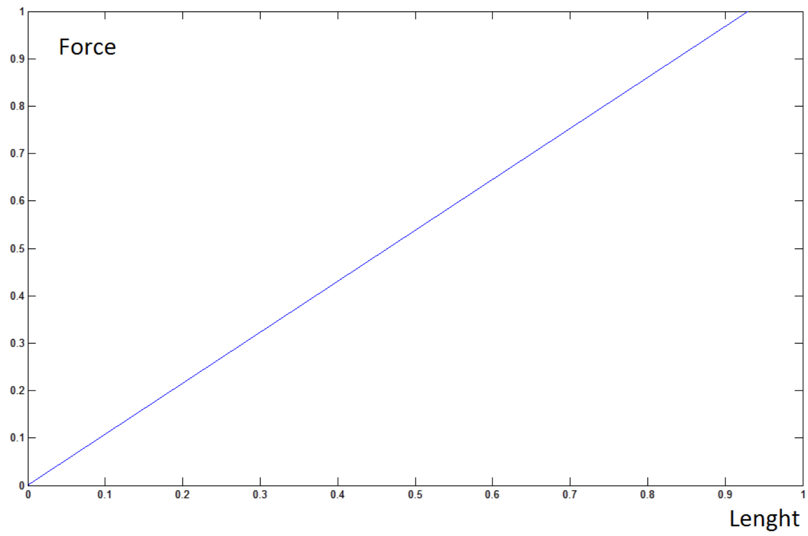

Figure 5. Value of external stretching force as a function of elongation for linear - constant stiffness parameter (see Eq. 3)

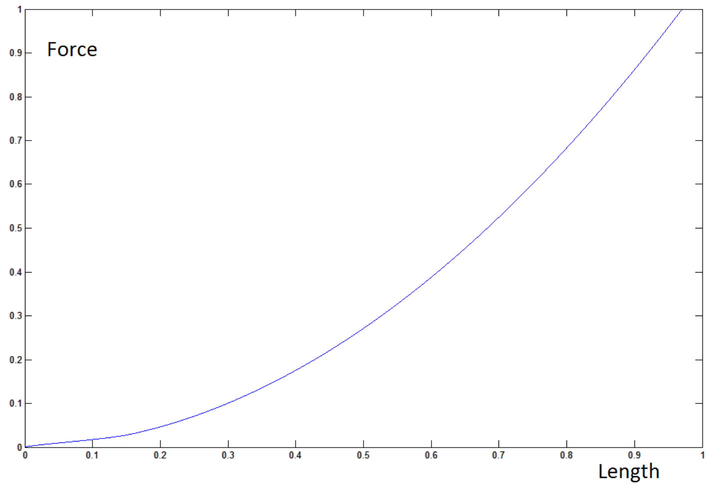

Figure 6. Value of external, stretching force as a function of elongation for linear stiffness parameter (see eq. 4)

2.3. Comparison of results

As it can be seen in Fig. 5 muscle characteristics is linear, whereas the stiffness coefficient is constant. However, experimental suggests (see Fig. 1), that it should be nonlinear. It is easy to observe that graph in Fig. 4 fits to the graph in Fig 1 in the best way. The graph presented in Fig. 6 has similar shape as that in figure Fig. 4. However, comparing normalised results in one figure, see Fig. 7, with experimental data (Soderberg curve for biceps brachii - data taken from [2]) it is clearly seen, that curves from Fig. 5 and Fig.5 do not fit well to the experimental curve than that presented in Fig.4. The standard deviations of the difference between the experimental data and that from the simulations is of the ordert: $0,0045$ in the first model - nonlinear; $0,058$ for the third model - linear; $0,089$ for the second model - linear (constant). It can be expected that our model with nonlinear parameters can be more adequate in other cases.

3. Conclusions

Compution of a model with nonlinear parameters is much more complicated and more time consuming than a simple model with linear parameters. However, nonlinear models are better biocompatibile, because biological tissues (like muscles) are characterized by nonlinear parameters. Another problem is to find adequate function, which will describe correctly the dependance (like in this case length - force dependance). In this case quadratic function form

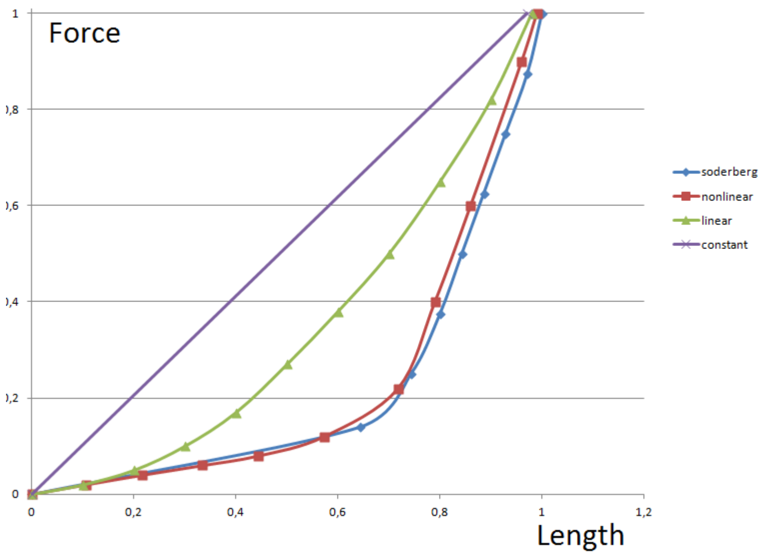

Figure 7. Results comparison with experimental data obtained by Soderberg (triangels line)

of stiffness was taken under consideration, because many authors describe muscle characteristics as an inverse parabolic characteristics (see an example Fig. 2).

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