R-functions method in parametric vibrations analysis of orthotropic plates with complex shape

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Summary. The effective numerically analytical method of parametric vibrations research for orthotropic plate with complex shape is proposed. The novel approach is based on hybrid applications of variation-type methods with R-functions theory. Using proposed method and developed software the regular and chaotic regimes of an orthotropic plates with an arbitrary shape can be analysed.

Introduction

Due to wide application of the elements which are modulated by plate and shell structures in civil, aerospace, ship and transport engineering, etc., the investigation of various problems of plate and shell theory has received a particular interest among scientists. One of the important problems which is studied in modern literature is bifurcation and chaotic dynamics analysis. Such tasks occur when plate or shell is subjected to parametrical load. In recent papers, for example in [1], problems of parametric vibrations of plates and shells including investigation of chaotic regimes are studied. But in these works objects with rectangular plane form are considered. The main reason of limitation in geometry is mathematical difficulties of getting solutions in case of arbitrary form of plate.

Generally in modern literature for plate with non- canonical geometry the universal approaches via FEM (Finite Element Method) or FDM (Finite Difference Method) are used. In the given work we propose the approach based on application of RFM (R-functions Method) proposed by V.L. Rvachev. It is powerful method for investigation of objects with arbitrary form as well as with different boundary conditions. The basic functions constructed by RFM are presented in analytical form and exactly satisfied the boundary conditions for plates with an arbitrary shape.

The main idea of the proposed method is reduction of nonlinear system of the differential equations with partial derivatives (PDEs) to system of ordinary differential equations (ODEs) using Bubnov-Galerkin method and original presentation for displacements in the midsurface. The implementation of the offered algorithm of the discretization requires the solving series linear boundary value problems, which are solved by Ritz approach combined with RFM.

Formulation

We consider an orthotropic thin plate with thickness h. The plate is subjected to the in-plane periodic excitation $p(t)$. The governing equation based on Kirchhoff's hypotheses are taken in the nondimensional operator form [3]

$$L\overline{U} = \overline{Nl} - \varepsilon\frac{\partial \overline{F}}{\partial t} - \frac{\partial^2 \overline{F}}{\partial t^2}, \qquad (1)$$

Here the following notation are introduced

$$\overline{U} = \{u, v, w\}^T, \quad L = \begin{pmatrix} L_{11} & L_{12} & 0 \\ L_{21} & L_{22} & 0 \\ 0 & 0 & L_{13} \end{pmatrix}, \quad \overline{Nl} = \{Nl_1, Nl_2, Nl_3\}^T, \quad \overline{F} = \{0, 0, w\}^T$$

Where elements $L_{ij}, i, j = \overline{1,3}$ of the matrix L are linear operators:

$$L_{11} = C_1\frac{\partial^2}{\partial x^2} + C_2\frac{\partial^2}{\partial y^2}\cdot, \quad L_{12} = L_{21} = C_3\frac{\partial^2}{\partial x \partial y}, \quad L_{22} = C_2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad L_{13} = \frac{1}{12(1-\mu_1\mu_2)}\left(C_1\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + 2(C_2 + C_3)\frac{\partial^4}{\partial x^2 \partial y^2}\right),$$

where $C_1 = \frac{E_1}{E_2}$, $C_2 = \frac{G(1-\mu_1\mu_2)}{E_2}$, $C_3 = \frac{G(1-\mu_1\mu_2)}{E_2} + \mu_1$.

Components of the vector $\overline{Nl}$ are defined as:

$$Nl_1 = \frac{\partial w}{\partial x}L_{11}w + \frac{\partial w}{\partial y}L_{12}w, \quad Nl_2 = \frac{\partial w}{\partial x}L_{21}w + \frac{\partial w}{\partial y}L_{22}w, \quad Nl_3(u,v,w) = \frac{\partial^2 w}{\partial x^2}N_x(u,v,w) + 2\frac{\partial^2 w}{\partial x \partial y}T(u,v,w) + \frac{\partial^2 w}{\partial y^2}N_y(u,v,w),$$

In these formulas and below u, v and w represent the displacements of the point in the middle plane of plate in x, y and z directions, respectively, E_1, E_2 are elasticity (Young) modules, μ_1, μ_2 are Poisson's ratios, G is shear modulus, ρ is the plate density, and ε is a damping coefficient, N_x, N_y, T are membrane stress resultants.

The following relations between nondimensional and dimensional parameters hold

$$\overline{x} = \frac{x}{a}, \ \overline{y} = \frac{y}{a}, \ \overline{w} = \frac{w}{h}, \ \overline{u} = \frac{ua}{h^2}, \ \overline{v} = \frac{va}{h^2}, \ \overline{p} = \frac{a^2 p}{h^3 E_2}, \ \overline{t} = \frac{h}{a^2}\sqrt{\frac{E_2}{\rho}}t, \ \overline{\varepsilon} = \frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}\varepsilon.$$

Boundary and initial conditions are added to the movement system (1). We denote that on loaded part of counter the boundary conditions have the following form

$$N_n = -p(t), \ T_n = 0. \qquad (2)$$

In (2) N_n and T_n normal and tangent forces, n is normal to domain boundary $\partial\Omega$.

Method of solution

For discretization of initial PDEs (1) let us present the deflection w as a truncated Fourier series

$$w(x,y,t)=\sum_{i=1}^{n} f_i(t)w_i(x,y). \tag{3}$$

Here w_i are eigenfunctions which correspond to linear vibrations problem. In-plane displacement u,v we propose to present in the following form

$$u(x,y,t)=u_0(x,y)p(t)+\sum_{i,j=1}^{n} f_i(t)f_j(t)u_{ij}(x,y),\ v(x,y,t)=v_0(x,y)p(t)+\sum_{i,j=1}^{n} f_i(t)f_j(t)v_{ij}(x,y). \tag{4}$$

In expressions (4) vector $\overline{U}_0=(u_0,v_0,0)^T$ is the solution of the system

$$L\overline{U}_0=0, \tag{5}$$

supplemented by boundary conditions $N_n^L(u_0,v_0)=-1$, $T_n^L(u_0,v_0)=0$ on loaded part of boundary domain (N_n^L,T_n^L are normal and tangent linear forces). Vector $\overline{U}_{ij}=(u_{ij},v_{ij},0)^T$ is solution of the inhomogeneous system

$$L\overline{U}_{ij}=\overline{Nl}^{(2)}(w_i,w_j), \tag{6}$$

supplemented with boundary conditions $N_n^L(u_{ij},v_{ij})=-F_1$, $T_n^L(u_{ij},v_{ij})=-F_2$ on loaded part of counter. Here vector $\overline{Nl}^{(2)}=(Nl_1,Nl_2,0)^T$ has following components

$$Nl_1=\frac{\partial w_i}{\partial x}L_{11}w_j+\frac{\partial w_i}{\partial y}L_{12}w_j,\quad Nl_2=\frac{\partial w_i}{\partial x}L_{21}w_j+\frac{\partial w_i}{\partial y}L_{22}w_j,$$

F_1,F_2 are expressions depending on eigenfunctions w_i,w_j. Boundary conditions for functions u_0,v_0 and u_{ij},v_{ij} on unloaded part of counter depend on fixed conditions.

Should be noted, that using such presentation for displacements (3), (4) the first two equations of system (1) are satisfied identically. To find the functions w_i in (3), and vectors $\overline{U}_0=(u_0,v_0,0)^T$ and $\overline{U}_{ij}=(u_{ij},v_{ij},0)^T$ in (4) it is necessary to solve sequences of linear problems: linear vibrations problem and plane elasticity problems (5), (6). Listed tasks are solved by variational Ritz's method combined with R-functions theory (R-functions method-RFM). More detail about application of RFM for plates vibrations problems one can find in [2,4].

Substituting (3) and (4) into third equation of system (1) and applying Bubnov-Galerkin method, we obtain ODEs which is presented in matrix form

$$\mathbf{f}''+\varepsilon\mathbf{f}'+\mathbf{C}\mathbf{f}-p(t)\mathbf{A}\mathbf{f}+\mathbf{B}(\mathbf{f})=0, \tag{7}$$

where

$$\mathbf{A}=\begin{pmatrix} a_1^{(1)} & a_2^{(1)} & \dots & a_n^{(1)} \\ a_1^{(2)} & a_2^{(2)} & \dots & a_n^{(2)} \\ \dots & \dots & \dots & \dots \\ a_1^{(n)} & a_2^{(n)} & \dots & a_n^{(n)} \end{pmatrix},\ \mathbf{C}=\begin{pmatrix} \omega_{L,1}^2 & 0 & \dots & 0 \\ 0 & \omega_{L,2}^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \omega_{L,n}^2 \end{pmatrix},\ \mathbf{f}=\begin{pmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{pmatrix},\ \mathbf{f}'=\begin{pmatrix} f_1' \\ f_2' \\ \dots \\ f_n' \end{pmatrix},\ \mathbf{f}''=\begin{pmatrix} f_1'' \\ f_2'' \\ \dots \\ f_n'' \end{pmatrix}, \tag{8}$$

and $\mathbf{B}(\mathbf{f})$ is vector with components $b_m=\sum_{i,j,k=1}^{n} f_i f_j f_k \beta_{ijk}^{(m)}$. In (8) $\omega_{L,i}$ are eigenfrequencis corresponding to w_i mode of plate vibration. Elements of matrix $\mathbf{A}$ and vector $\mathbf{B}(\mathbf{f})$ are defined as double integrals over the investigated domain. Obtained ODEs (7) is solved through Runge-Kutt method.

Presented approach was applied by developed software to study the behavior of H-shaped plate (Figure 1). Effect of various parameters (geometry, boundary conditions, property of materials, parameters of load, dumping) on investigated characteristics (dependencies $w(t)$, $w(\dot{w})$, etc.) is analysed using one and three mode approximation.

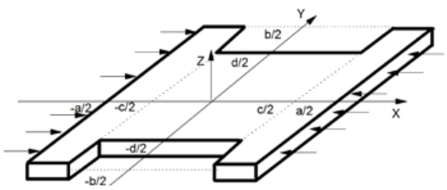

Fig. 1

References

[1] Awrejcewicz J., Krys'ko A.V. *Analysis of complex parametric vibrations of plates and shells using the Bubnov-Galerkin approach.* Applied Mechanics, 73, 2003, p.495- 504.

[2] Awrejcewicz J., Kurpa L., Mazur O. *Research of Stability and Nonlinear vibration of plates by R-Functions Method.* Modeling Simulation and control of Nonlinear Engineering Dynamical Systems. – UK, Springer, 2009, p.179-189.

[3] Ambartsumian S.A., *Theory of Anisotropic Shells*. NASA Report, TTF-118.

[4] Kurpa L., Mazur O. *Parametric vibrations of orthotropic plates with complex shape.* Int. Applied Mechanics, 46, 2010 , p. 438-449(12).