Chaos caused by hysteresis in 2-dof vibrations of the rotor suspended in a magneto-hydrodynamic field

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Summary. 2-dof nonlinear dynamics of the rotor supported by the magneto-hydrodynamic bearing is studied. In the case of soft magnetic materials the analytical solutions are obtained by means of the method of multiple scales [1]. It was shown that hysteresis generates chaotic vibrations of the rotor under certain conditions. The chaotic behavior regions of the rotor are obtained in various control parameter planes using an approach based on the analysis of wandering trajectories.

Mathematical model

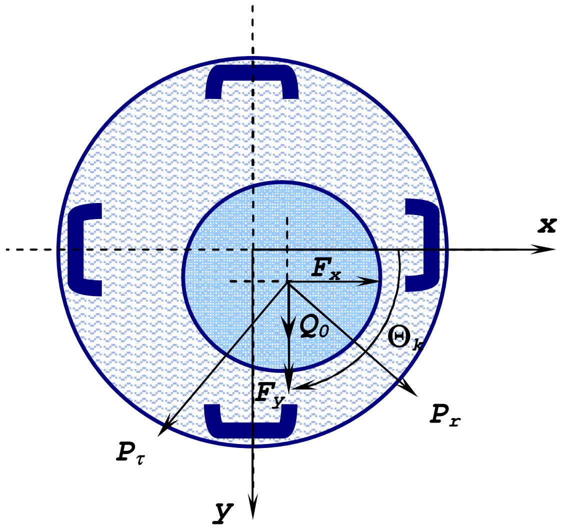

Fig. 1 The cross-section diagram of a rotor symmetrically supported on the magneto-hydrodynamic bearing

A uniform symmetric rigid rotor (Fig. 1) supported by a magneto-hydrodynamic bearing (MHDB) system is considered. The four-pole legs are symmetrically placed in the stator. F_k is the electromagnetic force produced by the kth opposed pair of electromagnet coils. θ_k is the angle between axis x and the kth magnetic actuator. Q_0 is the vertical rotor load identified with its weight, (P_r , P_τ) are the radial and tangential components of the dynamic oil-film action. Equations of motion of the rotor are represented in the dimensionless form [2]

$$\ddot{x} = P_r(\rho,\dot{\rho},\dot{\phi})\cos\phi - P_\tau(\rho,\dot{\phi})\sin\phi + F_x,$$
$$\ddot{y} = P_r(\rho,\dot{\rho},\dot{\phi})\sin\phi + P_\tau(\rho,\dot{\phi})\cos\phi + F_y + Q_0 + Q\sin\Omega t,$$
$$P_r(\rho,\dot{\rho},\dot{\phi}) = -2C\left\{\frac{\rho^2(1-2\dot{\phi})}{p(\rho)q(\rho)} + \frac{\rho\dot{\rho}}{p(\rho)} + \frac{2\dot{\rho}}{\sqrt{p(\rho)}}arctg\sqrt{\frac{1+\rho}{1-\rho}}\right\},$$
$$P_\tau(\rho,\dot{\phi}) = \pi C\frac{\rho(1-2\dot{\phi})}{q(\rho)\sqrt{p(\rho)}}.$$

Here $x = \rho\cos\phi$, $y = \rho\sin\phi$, $\dot{\phi} = \dfrac{\dot{y}x - \dot{x}y}{\rho^2}$, $\dot{\rho} = \dfrac{x\dot{x} + y\dot{y}}{\rho}$,

$$\rho = \sqrt{x^2+y^2}, \quad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}, \quad p(\rho) = 1-\rho^2, \quad q(\rho) = 2+\rho^2 .$$

The magnetic control forces are expressed as follows

$$F_x = -\gamma\dot{x} - \lambda(x - x_0), \quad F_y = -\gamma\dot{y} - \lambda(y - y_0),$$

where (x_0, y_0) are coordinates of the rotor static equilibrium, γ and λ are control parameters.
In the case of rigid magnetic materials the hysteretic properties of the system described can be considered using the Bouc-Wen hysteretic model. It was shown [3] that this modeling mechanism for energy dissipation was sufficiently accurate to model loops of various shapes in accordance with a real experiment, reflecting the behavior of hysteretic systems from very different fields. The hysteretic model of the rotor–MHDB system is as follows

$$\begin{aligned}
\ddot{x} &= P_r(\rho,\dot{\rho},\dot{\phi})\cos\phi - P_\tau(\rho,\dot{\phi})\sin\phi - \gamma_m\dot{x} - \lambda_m[\delta(x-x_0) + (1-\delta)z_1],\\
\ddot{y} &= P_r(\rho,\dot{\rho},\dot{\phi})\sin\phi + P_\tau(\rho,\dot{\phi})\cos\phi - \gamma_m\dot{y} - \lambda_m[\delta(y-y_0) + (1-\delta)z_2] + Q_0 + Q\sin\Omega t,\\
\dot{z}_1 &= \left[k_z - (\gamma + \beta\, sgn(\dot{x})\, sgn(z_1))|z_1|^n\right]\dot{x},\\
\dot{z}_2 &= \left[k_z - (\gamma + \beta\, sgn(\dot{y})\, sgn(z_2))|z_2|^n\right]\dot{y}.
\end{aligned} \tag{1}$$

Here z_1 and z_2 are the hysteretic forces. The case δ=0 corresponds to the maximal hysteretic dissipation and δ=1 corresponds to the absence of hysteretic forces in the system, parameters k_z, β, n and γ govern the shape of the hysteresis loops.

Conditions for chaotic vibrations of the rotor

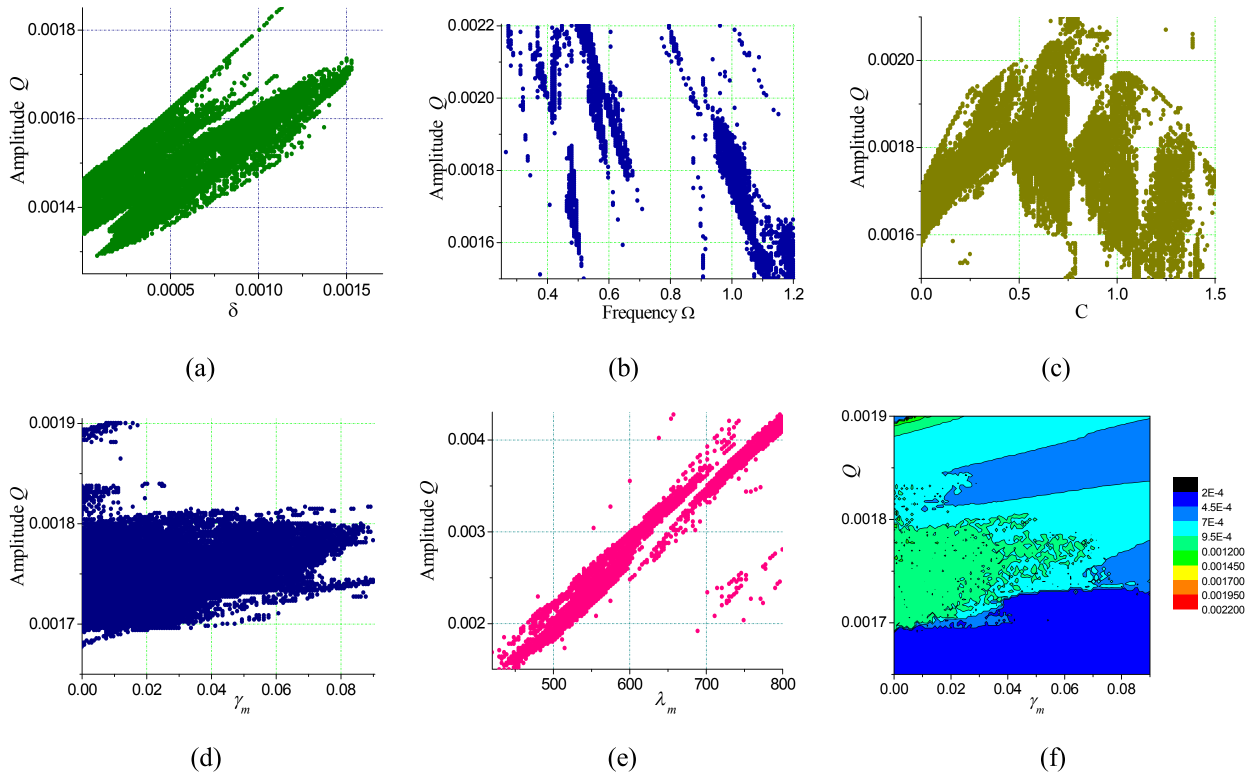

Fig. 1 Chaotic regions (a)-(e) for vertical vibrations of the rotor (1) in the various control parameter planes and amplitude level contours (f) in (γ_m,Q) plane.

Conditions for chaotic vibrations of the rotor have been found using the approach based on the analysis of wandering trajectories. The description of the approach, its advantages over standard procedures and a comparison with other approaches can be found, for example, in [3]. All domains have complex structure which is characteristic of domains where chaotic vibrations are possible. For each aggregate of control parameters there is some critical value of hysteretic dissipation ($1-\delta_{cr}$) that, if $(1-\delta)<(1-\delta_{cr})$, chaos is not observed in the system considered. Phase portraits and hysteretic loops of the rotor motion agree well with the regions obtained. In order to see if the rotor chaotic motion is accompanied by increasing of the amplitude of vibrations, the amplitude level contours of the horizontal and vertical vibrations of the rotor have been obtained. Although some "consonance" between the chaotic vibrations regions and the amplitude level contours is observed, it can not be concluded that chaos leads to essential increasing of the rotor vibrations amplitude.

Conclusions

2-dof non-linear dynamics of the rotor supported by the magneto-hydrodynamic bearing was studied by means of the method of multiple scales. The cases of primary resonances with and without an internal resonance were investigated. The frequency-response curves were obtained. The saturation phenomenon was demonstrated. When the amplitude of the external excitation increases, after some critical value the energy pumping between various submotions of the rotor occurs. Using methodology based on the analysis of wandering trajectories, it was shown, that hysteresis may be a cause of chaotic vibrations of the rotor. The regions of chaotic vibrations of the rotor as well as the amplitude level contours were found in various control parameter planes.

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