XXIV Symposium *Vibrations in Physical Systems,* Poznan – Bedlewo, May 12-15, 2010

Vibrations of a Mechanical System with Friction Clutch

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Abstract

This work presents results of the numerical investigations devoted to clutch system dynamics. The general non-linear wear model has been presented and used during simulations. The influence of the considered wear model on the contact pressure distribution of the clutch discs has been investigated. Dynamics of the system is monitored via standard trajectories of motion in the system's phase space and behaviour of the system around resonance angular velocity is studied. Presented results show interesting phenomena of the investigated system and a key role of the influence of the wear process on its dynamics.

Keywords: Clutch, friction, wear

1. Introduction

Dynamic phenomena in the neighbourhood of the resonance angular velocity have the significant influence on the endurance of elements of the system and its dynamics. The mentioned phenomena can be caused by various factors like friction, wear, heat generation or/and impacts. In this work chosen issues of vibrations of a mechanical system with friction clutch are discussed and investigated. The attention is focussed on the investigation of influence of wear of clutch shields on its dynamics in the neighbourhood and far from the resonance regions.

In many monographs [1], [2], [3], [4], [5] friction and wear essential testing methods and problems of the theory of wear in such systems are described. Empirical models, which let for better understanding occurring processes are studied. However, a general relation between friction and wear has not been formulated so far.

In this work we consider general non-linear differential model of wear w in the form

$$\dot{w} = K^{(w)} P^{\alpha} \left| V_r \right|^{\beta}, \tag{1}$$

where $K^{(w)}$ is a coefficient of material wear, V_r is relative sliding velocity of surfaces touching each other, P is a contact pressure, and α, β are rates dependent on the model of wear, the step of lubricating and spreading on the contacting surfaces. For $\alpha = \beta = 1$ we obtain a particular linear Archard's wear model (see [1]).

2. Model of a System with Friction Clutch

Our investigations are concern of mechanical system with flexible-friction clutch, shown in Figure 1.

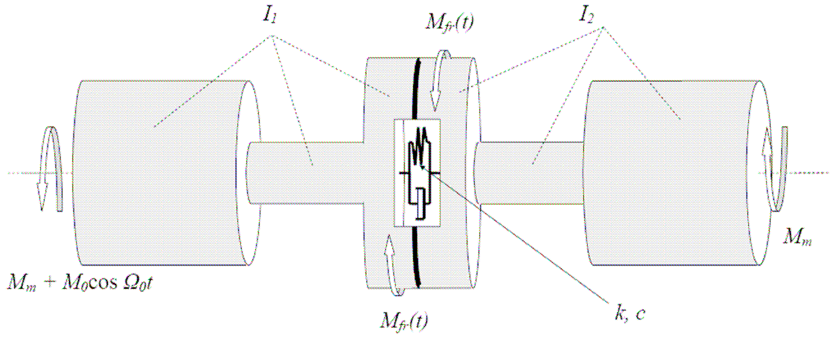

Figure 1. The model of the considered mechanical system

For a study we consider two-masses model of the system. The body 1 have the reduced moment of interia I_1, whereas the body 2 have the reduced moment of interia I_2. Vibrations in the system are caused by harmonic excitation from the side of the motor. The moment of motor is characterized by an average constant value M_m and a harmonic excitation part $M(t) = M_0 \cos\Omega_0 t$. The clutch is characterized by springiness (susceptibility) k and damping c in angular direction, and friction torque $M_{fr}(t)$ moved by the clutch. The system governing equations have the form

$$\begin{aligned} I_1\ddot{\psi}_1 + c(\dot{\psi}_1 - \dot{\psi}_2) + k(\psi_1 - \psi_2) &= M_m + M_0 \cos\Omega_0 t - M_{fr}(t) \\ I_2\ddot{\psi}_2 + c(\dot{\psi}_2 - \dot{\psi}_1) + k(\psi_2 - \psi_1) &= -M_m + M_{fr}(t) \end{aligned}, \tag{2}$$

where ψ_1 and ψ_2 are angles of driving shaft and driven shaft of the clutch, respectively. Taking $\psi = \psi_1 - \psi_2$ as the relative angular displacement of clutch shields and the reduced moment of interia $I_r = I_1 I_2/(I_1 + I_2)$, we obtain

$$\ddot{\psi} + \frac{c}{I_r}\dot{\psi} + \frac{k}{I_r}\psi = \frac{M_m}{I_r} + \frac{M_0}{I_1}\cos\Omega_0 t - \frac{M_{fr}(t)}{I_r}. \tag{3}$$

Friction torque moved by the clutch $M_{fr}(t)$ is

$$M_{fr}(t) = 2\pi \int_{R_1}^{R_2} \mu R^2 P(R,t)\, dR, \tag{4}$$

where μ is a coefficient of friction, R_1 and R_2 are internal and outside radii of contact surfaces, respectively, and $P(R,t)$ is contact pressure between shields pressed by force $Q(t)$.

Let us enrol equations on wear for the left shield and the right shield $\dot{U}_1^{(w)}(R,t) = K_1^{(w)} \left| V_r(R,t) \right|^{\beta} P^{\alpha}(R,t)$, $\dot{U}_2^{(w)}(R,t) = K_2^{(w)} \left| V_r(R,t) \right|^{\beta} P^{\alpha}(R,t)$, and axial displacements $U^{(1)}(R,t) = k_1 P(R,t)$, $U^{(2)}(R,t) = k_2 P(R,t)$ with coefficients of stiffness of shields k_1 and k_2, respectively. In what follows we obtain conditions of the contact of shields of the clutch in the form

$$U^{(1)}(R,t) + U^{(2)}(R,t) + U_1^{(w)}(R,t) + U_2^{(w)}(R,t) = \mathrm{E}(t) , \tag{5}$$

where $\mathrm{E}(t)$ is a function describing distance between shields. After differentiating of equation (5) with respect to the time, taking $k_{12} = k_1 + k_2$, $K^{(w)} = K_1^{(w)} + K_2^{(w)}$, $V_r(R,t) = \Omega_r(t) R$, $\Omega_r = \dot{\psi}$, next multiplying by RdR, integrating over interval $R \in [R_1, R_2]$, and taking into account differentiation regarding time of the equation

$$Q(t) = 2\pi \int_{R_1}^{R_2} RP(R,t) dR , \tag{6}$$

we finally obtain

$$\begin{gathered} k_{12} \frac{\partial P(R,t)}{\partial t} + K^{(w)} R^{\beta} \left| \Omega_r(t) \right|^{\beta} P^{\alpha}(R,t) = \\ = \frac{2K^{(w)}}{R_2^2 - R_1^2} \left| \Omega_r(t) \right|^{\beta} \int_{R_1}^{R_2} R^{1+\beta} P^{\alpha}(R,t) dR + \frac{k_{12}}{\pi (R_2^2 - R_1^2)} \frac{dQ(t)}{dt} . \end{gathered} \tag{7}$$

3. Non-Dimensional Form

Let us introduce the following similarity coefficients: t_*, P_*; non-dimensional time: $\tau = t/t_*$; non-dimensional radius: $r = (R - R_1)/(R_2 - R_1)$; non-dimensional geometrical parameter: $\rho = R_1/(R_2 - R_1)$; other non-dimensional parameters: $\omega_0 = \Omega_0 t_*$, $l_1 = K^{(w)} R_2^{\beta} t_*^{1-\beta} P_*^{\alpha-1} / (k_{12}(1+\rho)^{\beta})$, $d = ct_*/I_r$, $\omega_k^2 = kt_*^2/I_r$, $F_m = M_m t_*^2 / I_r$, $F_0 = M_0 t_*^2 / I_1$, $k_{fr} = 2\pi t_*^2 P_* \mu R_2^3 / (I_r (1+\rho)^3)$, and then following non-dimensional functions: $p(r,\tau) = P((R_2 - R_1)(r+\rho), t_*\tau)/P_*$, $F_{fr}(\tau) = M_{fr}(t_*\tau) t_*^2 / I_r$, $\varphi = \psi$, $\omega = \dot{\psi} t_*$, $\omega_r = \dot{\psi}_r t_*$. Let us take $Q(t) = Q = const$, $t_* = \sqrt{I_r/k}$ and $P_* = (1+\rho)^2 Q / (\pi R_2^2 (1+2\rho))$. Then, replacing integrals appearing in equations (4) and (7), after enrolling them in a non-dimensional form using method of trapezia by dividing the length of non-dimensional radius on m even segments, and taking $\Delta_r = 1/m$, $r_i = \Delta_r i$, $r_j = \Delta_r j$, $p(r_i, \tau) = p_i(\tau)$, $p(r_j, \tau) = p_j(\tau)$ (rates of the method of trapezia are: $a_0 = a_m = 1/2$, $a_j = 1$, $j = 1,2,...m-1$), we obtain the following system of $m+3$ first order ODEs

$$\begin{cases} \dot{\omega} = -\varphi - d\omega + F_m + F_0 \cos\omega_0\tau - k_{fr}\Delta_r \sum_{j=0}^{m} a_j (r_j + \rho)^2 p_j^\alpha(\tau), \\ \dot{\varphi} = \omega, \\ \dot{p}_i(\tau) = -l_1 |\omega_r(\tau)|^\beta (r_i + \rho)^\beta p_i^\alpha(\tau) + \dfrac{2l_1}{(1+2\rho)} |\omega_r(\tau)|^\beta \Delta_r \sum_{j=0}^{m} a_j (r_j + \rho)^{1+\beta} p_j^\alpha(\tau). \end{cases} \tag{8}$$

4. Numerical Computations

Numerical calculations are carried out using the fourth order Runge-Kutta method with constant time step. We assumed the following initial non-dimensional parameters: $\omega_0 = 0.2$, $d = 0.0001$, $F_m = 1$, $F_0 = 0.5$, $\rho = 0.2$, $l_1 = 0.5$, $\alpha = 1$, $\beta = 1$, $m = 100$ and $F_{fr}(0) = 0.2$ for initial moment. Let us study first contact pressure distribution (Figure 2 on the left) as the function of the non-dimensional radius r of shields for different values of β parameter.

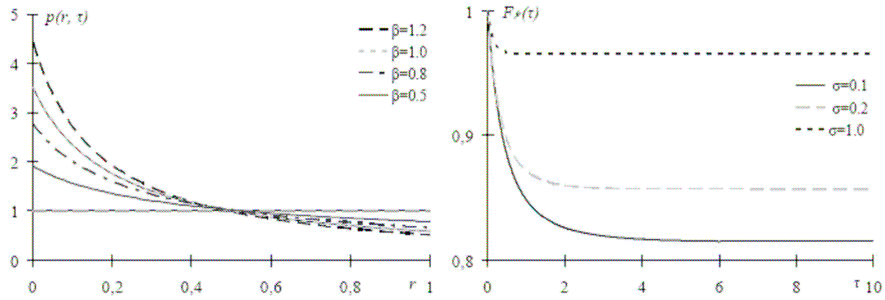

Figure 2. Contact pressures in equilibrium and changes of the friction torque moved by the clutch

Before beginning of the process of wearing of shields, the contact pressure distribution $p(r,0)$ is identical on the entire contact surface. However, contact pressure distributions $p(r,\infty)$ are different for various values of the β parameter. Figure 2 (on the right) shows changes of the friction torque moved by the clutch for various values of the geometrical ρ parameter. As can be seen, amendments of the contact pressure distribution during the wear causing reduction of the friction torque moved by the clutch. In Figure 3 we take into consideration process of wearing during vibrations of shields of clutch with a great coefficient of wear in order to observe changes in dynamics of the system. This dynamics is monitored via trajectories of motion in the system's phase for different (rather small) values of the angle velocities of excitation, possible to appear, for example in the set-up time of the system for the slowly acting harmonic excitation.

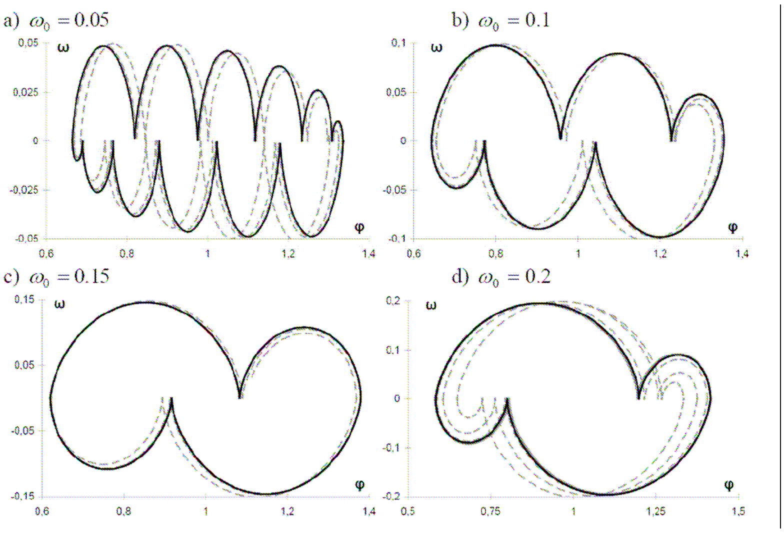

Figure 3. Phase trajectories of the system for various angular velocities including wear

As can be seen above, vibrations of the system depend both on angular velocity of harmonic excitation and friction torque moved by the clutch.

Figure 4 shows angular characteristics for various values of $F_{fr} = F_{fr}(0) = const$, (without wear process - $l_1 = 0$).

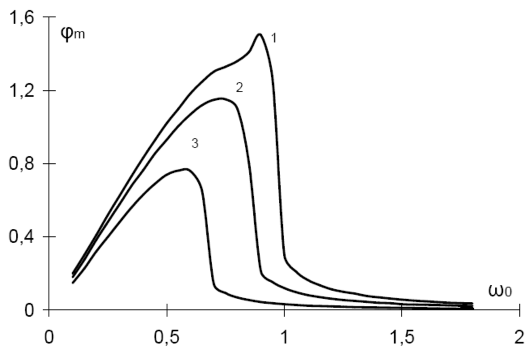

Figure 4. Angular characteristics of the system for various values of F_{fr};

curve 1 - $F_{fr} = 0.4$, curve 2 - $F_{fr} = 0.42$, curve 3 - $F_{fr} = 0.45$

Observe that the described resonance curves begin from the zero-dimensional vibration amplitude φ_m a little below value equal to zero for $\omega_0 = 0.1$, and are aspiring asymptotically in the scope apart from resonance up to the nought. They have tendency of the gentler course in the scope apart from resonance. For smaller values of the F_{fr} resonance amplitudes have greater values. Besides, it should be noticed the phenomenon of moving of resonance on the left for more and more great values of the F_{fr}.

5. Conclusions

The considered in this work issues allow to model and analyse wear processes on the contact surface of a mechanical friction clutch and the system dynamics. Unlike many previous works, here friction clutch is treated as a friction connection of elastic (not rigid) bodies and general non-linear differential wear model is applied. Besides, mathematical model describing wear processes and equation of motion of the system are used and applied together during computer simulations. The presented in this work numerical analysis shows influence of wear processes in friction clutch on the system dynamics.

Acknowledgments

This work has been supported by the Ministry of Science and Higher Education under the grant No. 1918/B/T02/2010/38 and MASTER Program of the Foundation for Polish Science.

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Drgania Układu Mechanicznego ze Sprzęgłem Ciernym

Praca przedstawia wyniki badań numerycznych poświęconych dynamice układu mechanicznego ze sprzęgłem ciernym. Podczas symulacji przedstawiono i wykorzystano nieliniowy model zużycia. Zbadano wpływ rozważanego modelu zużycia na rozkład nacisków na tarczach sprzęgłowych. Dynamikę układu monitorowano przy użyciu trajektorii ruchu w przestrzeni fazowej oraz zachowania układu wokół rezonansowej częstości kątowej. Przedstawione wyniki pokazują interesujące i zbliżone do rzeczywistych zachowania się rozważanego układu oraz wpływ procesu zużycia na jego dynamikę.