10th CONFERENCE
on
**DYNAMICAL SYSTEMS
THEORY AND APPLICATIONS**
December 7-10, 2009. Łódź, Poland

CHAOTIC VIBRATIONS OF TWO-LAYERED BEAMS AND PLATES WITH GEOMETRIC, PHYSICAL AND DESIGN NONLINEARITIES. PART III.

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Abstract: A theory of nonlinear interaction of two-layered beams (part I) and plates (part II) taking into account design, geometric and physical nonlinearities is developed. Designed nonlinearity concerns activation and removal of one-sided constraints. Physical nonlinearity is associated with nonlinear relations between strains and stresses, whereas geometric nonlinearity is connected with nonlinear relations between deformations and displacements in the form proposed by von Karman for each of the beams. The theory is mainly developed in the first approximation on the Euler-Bernoulli hypothesis. When solving contact problems, the Winkler type relation between clamping and contact pressure is applied allowing the contact pressure to be removed from the quantities being sought. In order to solve strongly nonlinear partial differential equations, the finite difference method regarding space and time coordinates is applied. On each time step the iteration procedure, which improves the contact area between beams is applied and also the method of changeable stiffness parameters is used. A computational example regarding dynamic interaction of two beams depending on a gap between beams is given. Each beam is subjected to transversal sign-changeable load, and the upper beam is hinged, whereas the bottom beam is clamped. Analysis of chaotic vibrations of the beams package is carried out using the method of nonlinear dynamics and qualitative theory of differential equations. It has been shown that for some fixed system parameters and with an increase of the external load amplitude synchronization between two beams occurs with the upper beam vibration frequency. A qualitative analysis of the interaction of two non-coupled beams is also extended to the study of non-coupled plates. Charts of beam vibration types vs. control parameters $\{q_0, \omega_p\}$, i.e. the frequency and amplitude of excitation are constructed. The similar like competitions have been reported in the case of two-layered plates.

1. Introduction

In part III of the paper we are aimed on analysis of regular and chaotic dynamics of two-layered plates with geometric, physical and design nonlinearities.

In references [1]-[3] the approach devoted to solutions of the contact problems of nonlinear shells theory is proposed. It consists in removal of the contact pressure q_k from the unknown functions with a help of the Winkler type coupling. The mentioned approach is equivalent to that of formula (4.p.I), and it allows us to neglect the tedious task of constructing the Green function, and hence solutions can be found in equilibrium equations (1.p.I).

On the other hand, a study of chaotic vibrations of the contact problems of nonlinear mechanics of thin-walled structures is rather rarely presented [4]-[5]. The investigation of chaotic vibrations of nonlinear plates and shells is discussed in works [6]-[12]. However, modeling and analysis of chaotic vibrations of two-layered beams with the mentioned three types of nonlinearity belongs rather to pioneering works.

2. Chaotic vibrations of two uncoupled plates

Let us derive a system of differential equations of two layered uncoupled plates, when each of the layers satisfies the kinematic Kirchhoff hypothesis. The relative position of plates in space with the given coordinates O_{xyz}, is defined in the following way: a middle surface of the first plate lies in the $z=0$, and second – in the $z=-\frac{1}{2}(\delta_1-\delta_2)-h_k$ plane, where δ_i is the thickness of "i"-th plate, and h_k denotes the distance between two plates in a non-deformable state.

The system of governing equations is [4]

$$\begin{cases} h\dfrac{\gamma}{g}\dfrac{\partial^2 w_1}{\partial t^2}+\varepsilon_1\dfrac{\partial w_1}{\partial t}+A\left(w_1(x,y,t)\right)=q_1(x,y,t)-K\dfrac{E}{h}(w_1-w_2-h_k)\Psi(x,y,t), \\ h\dfrac{\gamma}{g}\dfrac{\partial^2 w_2}{\partial t^2}+\varepsilon_2\dfrac{\partial w_2}{\partial t}+A\left(w_2(x,y,t)\right)=q_2(x,y,t)-K\dfrac{E}{h}(w_1-w_2-h_k)\Psi(x,y,t), \end{cases} \tag{1}$$

with the following attached initial conditions

$$w_i(t,x,y)\big|_{t=0}=f_i(x,y), \qquad \left.\frac{\partial w_i}{\partial t}\right|_{t=0}=F_i(x,y), \qquad (i=1,2), \tag{2}$$

and system (1)-(2) is supplemented by one of the boundary conditions

$$w_i\big|_{\partial\Omega_i}=\left.\frac{\partial w_i}{\partial n_i}\right|_{\partial\Omega_i}=0, \tag{3}$$

$$w_1\big|_{\partial\Omega_1}=\left.\frac{\partial w_1}{\partial n_1}\right|_{\partial\Omega_1}=0, \qquad w_2\big|_{\partial\Omega_2}=\left.\frac{\partial^2 w_2}{\partial n_2^2}\right|_{\partial\Omega_2}=0, \tag{4}$$

$$w_i\big|_{\partial\Omega_i}=\left.\frac{\partial^2 w_i}{\partial n_i^2}\right|_{\partial\Omega_i}=0, \tag{5}$$

$$w_1\big|_{\partial\Omega_1} = \frac{\partial^2 w_1}{\partial n_1^2}\bigg|_{\partial\Omega_1} = 0\,, \qquad w_2\big|_{\partial\Omega_2} = \frac{\partial w_2}{\partial n_2}\bigg|_{\partial\Omega_2} = 0\,, \tag{6}$$

where $q_1\left(t,x,y\right) = q_0 \sin\left(\omega_p t\right)$ is the function of external load acing on the first plate, $q_2 = 0$.

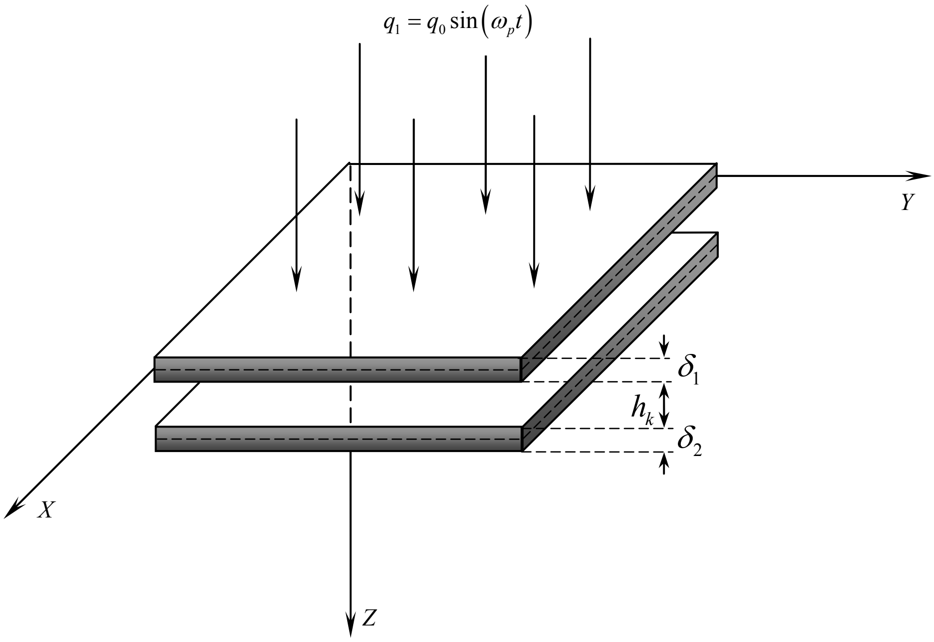


Fig. 1. Two plates with a gap

Let the plates occupy in R^2 the space $\Omega_i = \{(x,y)\big|\ 0 \le x \le a, \quad 0 \le y \le b\}$, $(i = 1,2)$, and $\partial\Omega_i$ be the associated space boundary in R^2, K is the known constant, and $\Psi\left(x,y,t\right)$ is the contact space Ω^* indicator:

$$\Psi\left(x,y,t\right) = \frac{1 = sign\left(w_1\left(x,y,t\right) - w_2\left(x,y,t\right) - h_1\right)}{2}.$$

As it has already been mentioned in reference [5], the differential operator $A\left(w_i\left(x,y,t\right)\right)$ $\left(i = 1,2\right)$ is nonlinear in general, but in this work each plate is treated as plastic and geometrically linear, and therefore

$$A\left(w_i\left(x,y,t\right)\right) = \nabla^4 w_i = \frac{\partial^4 w_i}{\partial x^4} + 2\frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial y^4}.$$

System (1) – (6) is transformed into a non-dimensional form through the following parameters

$$\bar{x} = \frac{x}{a}, \quad \bar{w} = \frac{w_i}{h}, \quad \bar{q}_1 = 2(1+\nu)\lambda_1^4 \frac{q_1}{E}, \quad \lambda_1 = \frac{a}{h}, \quad \bar{y} = \frac{y}{b}, \quad \bar{h}_i = \frac{\delta_i}{h},$$

$$\bar{K} = 12\left(1-\nu^2\right)\lambda_1^4 K, \qquad \bar{t} = \frac{h}{ab}\left(\frac{E}{1-\nu^2}\cdot\frac{g}{\gamma}\right)^{1/2} t.$$

System (1) is given in the following non-dimensional form

$$\begin{cases} \dfrac{\partial^2 w_1}{\partial t^2} + \varepsilon_1 \dfrac{\partial w_1}{\partial t} + \nabla^4 w_1 = q_1(x,y,t) - K\dfrac{E}{h}\left(w_1 - w_2 - h_k\right)\Psi(x,y,t), \\ \dfrac{\partial^2 w_2}{\partial t^2} + \varepsilon_2 \dfrac{\partial w_2}{\partial t} + \nabla^4 w_2 = q_2(x,y,t) - K\dfrac{E}{h}\left(w_1 - w_2 - h_k\right)\Psi(x,y,t), \end{cases} \tag{7}$$

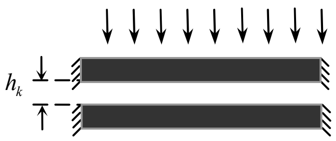
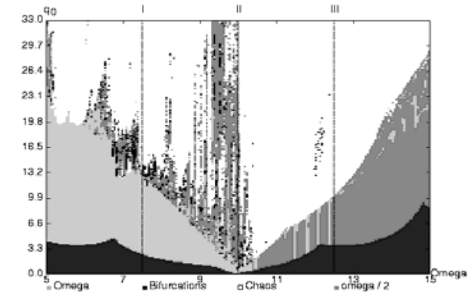
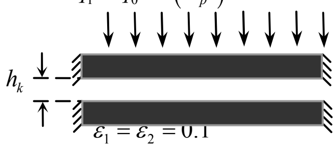
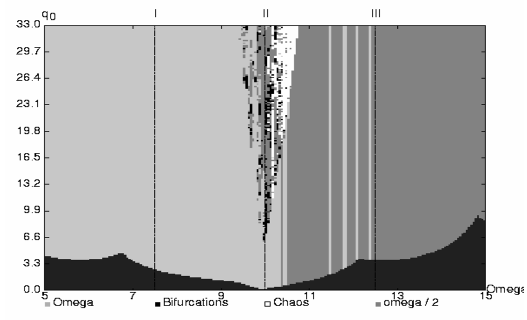
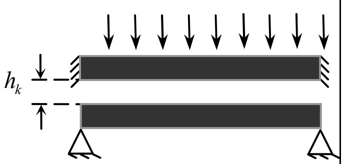
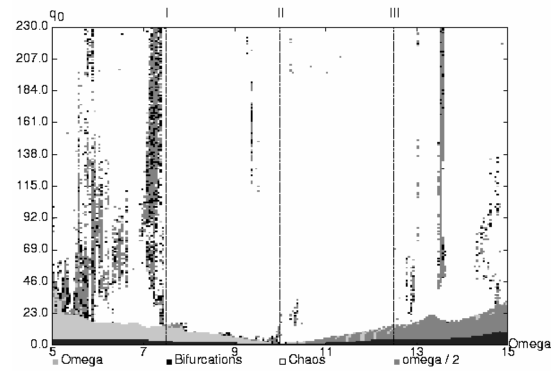
where bars are already omitted.

The obtained PDEs are reduced to the second order ODEs via the finite difference method with approximation $O\left(h^2\right)$. Further, the system is transformed to the first order ODEs and then solved via the fourth order Runge-Kutta method. Space and time steps are chosen via the Runge principle and $\Delta t = 0.001$, whereas grid step is 23x23 of space Ω.

Next, we study vibrations of two non-welded plates of constant thickness ($\bar{h}_1 = \bar{h}_2$), made from an isotropic material with Poisson's coefficient $\nu = 0.3$ subjected to uniformly distributed sign-changeable load. System behavior is investigated for two types of the boundary conditions (clamping-clamping (3) and clamping-hinge (4)) and for various values of the gap between plates ($h_k = 0.01$ and $h_k = 0.1$). In order to investigate two layered beams behavior, charts of vibration character are constructed (Table 1). Three vertical lines are introduced: $\omega_p = \omega_0$ frequency of linear vibration, left and right lines correspond to frequencies $\omega_p - \dfrac{\omega_p}{2}$ and $\omega_p + \dfrac{\omega_p}{2}$, respectively. The charts illustrate the whole possible nonlinear dynamics of two layered non-welded beams.

Analysis of the system dynamics for various gap parameters and boundary conditions showed increase of the gap between plates (Table 1a, b) and decreases of the chaotic vibrations zones, whereas zones of periodic and quasi-periodic vibrations are increased. In the case of boundary conditions (Table 1a, c) one may observe that the boundary condition change of the second beam (from clamping to hinge) causes an essential increase of chaotic zones, and the periodic and quasi-periodic zones decrease.

Table 1

a) $q_1 = q_0 \sin(\omega_p t)$  $\varepsilon_1 = \varepsilon_2 = 0.1$ $h_k = 0.01$	
b) $q_1 = q_0 \sin(\omega_p t)$  $\varepsilon_1 = \varepsilon_2 = 0.1$ $h_k = 0.1$	
c) $q_1 = q_0 \sin(\omega_p t)$  $\varepsilon_1 = \varepsilon_2 = 0.1$ $h_k = 0.01$	

Notation

Periodic vibrations	Period doubling bifurcation
Quasi-periodic vibrations	Chaos

3. Concluding remarks

The theory of nonlinear interaction of two-layered beams have been introduced. Then a series of computational examples regarding regular, bifurcational and chaotic dynamics of the investigated objects have been reported.

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