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MODELING AND ANALYTICAL/NUMERICAL ANALYSIS OF WEAR PROCESSES IN A MECHANICAL FRICTION CLUTCH. PART 1: DIFFERENTIAL WEAR MODEL

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Abstract: The presented work is first from the two devoted to modeling and analytical/numerical analysis of tribological processes appearing on the contact surface of shields of a mechanical friction clutch. Although the considered problems have been already studied earlier, however, simplified mathematical models have been used and applied. Unlike previous works, our work takes into account elasticity and wear of material of shields rubbing themselves, the presented mathematical model enables analysis of tribological processes in non-stationary conditions, and a general non-linear differential model of wear is applied. Analytical/numerical analysis is carried out with the qualitative and quantitative theories of differential and integral equations, including Laplace transformation. Many interesting results are obtained, illustrated and discussed.

1. Introduction

Contact phenomena on the joint of rubbing surfaces as friction and wear have the significant influence on the endurance and the speed of wearing of elements of a mechanical system and its dynamics. These issues in different kinds of friction connections were an object of many researchers and can be found in the works [1], [2], [4], [5], [6], [7], and others. In many monographs [3], [4], [5], [7] from the scope of friction and wear also essential testing methods and problems of the theory of wear in such systems were described.

Although the considered in this work problems have been already studied earlier, however, simplified mathematical models have been used and applied, and in addition (as a rule) individually. Namely: (i) in general clutches were treated as a friction connection of rigid bodies and hence an effect of wear and elasticity (flexibility) of material of contacting shields was omitted; (ii) most often wear was considered in stationary conditions; (iii) in contact issues during computer simulations an empirical linear wear model was most often used; (iv) for the simplification real contact pressure distribution was being omitted taking the identical pressure in each point of contact surface.

In this work we take into account elasticity and wear of material of shields rubbing themselves. The presented mathematical model enables analysis of tribological processes in non-stationary conditions. General non-linear model of wear is applied, where wear is modeled via non-linear (power) type function of contact pressure and relative sliding velocity with rates dependent on the model of wear, the step of lubricating and spreading on contacting surfaces. Equations modeling contact pressure (including its surface distribution) on the contact surface of shields are solved.

2. The Models of Wear and Mechanical Friction Clutch

In this work we assume that the model of wear w on the joint of rubbing surfaces has the form

$$\frac{dw}{dt} = K|V_r|^{\beta} P^{\alpha} , \qquad (1)$$

where K is a coefficient of wear of material, V_r is relative sliding velocity of surfaces touching each other, P is contact pressure and α , β are rates dependent on the model of wear, the step of lubricating and spreading on contacting surfaces.

Figure 1 shows the considered model of the mechanical friction clutch.

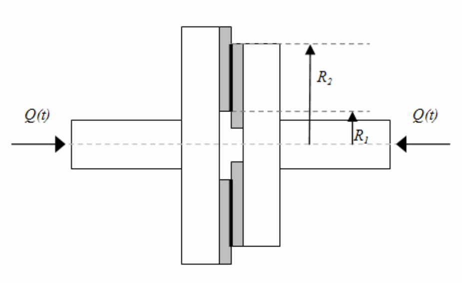

Fig. 1. Model of the considered mechanical friction clutch.

The friction contact between shields occurs in $R \in [R_1, R_2]$. Shields are being pressed with axial force $Q(t)$, and in each point of the joint of the surface of shields contact pressure is equal to $P(R,t)$. Coefficients of wear of material for the left and right shield are $K_1^{(w)}$ and $K_2^{(w)}$, however coefficients of the resilience of material of these shields are k_1 and k_2 , respectively.

Let us enrol equations on wear for the left shield $U_1^{(w)}(R,t)$ and the right shield $U_2^{(w)}(R,t)$ and axial displacements $U^{(1)}(R,t)$, $U^{(2)}(R,t)$ of each point of the surface of shields. We obtain

$$\frac{\partial U_1^{(w)}(R,t)}{\partial t} = K_1^{(w)} |V_r(R,t)|^{\beta} P^{\alpha}(R,t) , \quad \frac{\partial U_2^{(w)}(R,t)}{\partial t} = K_2^{(w)} |V_r(R,t)|^{\beta} P^{\alpha}(R,t) , \qquad (2)$$

$$U^{(1)}(R,t) = k_1 P(R,t)\ ,\ U^{(2)}(R,t) = k_2 P(R,t)\ . \tag{3}$$

Conditions of the contact of shields in the clutch have the form

$$U^{(1)}(R,t) + U^{(2)}(R,t) + U_1^{(w)}(R,t) + U_2^{(w)}(R,t) = E(t)\ , \tag{4}$$

where $E(t)$ is a function describing distance between shields. After differentiating of equation (4) with respect to the time and for $k = k_1 + k_2$, $K^{(w)} = K_1^{(w)} + K_2^{(w)}$ and $V_r(R,t) = \Omega_r(t)R$, we get

$$k\frac{\partial P(R,t)}{\partial t} + K^{(w)}\left|\Omega_r(t)\right|^{\beta} R^{\beta} P^{\alpha}(R,t) = \frac{dE(t)}{dt}\ . \tag{5}$$

Moreover, the following should be satisfied

$$Q(t) = 2\pi \int_{R_1}^{R_2} RP(R,t)dR\ . \tag{6}$$

Multiplying the equation (5) by RdR , integrating in interval $R \in [R_1, R_2]$ and taking into account differentiation regarding time equation (6) is cast to the form

$$\frac{dE(t)}{dt} = \frac{2K^{(w)}}{R_2^2 - R_1^2}\left|\Omega_r(t)\right|^{\beta} \int_{R_1}^{R_2} R^{1+\beta} P^{\alpha}(R,t)dR + \frac{k}{\pi(R_2^2 - R_1^2)}\frac{dQ(t)}{dt}\ . \tag{7}$$

Comparing relations (5) and (7) one gets

$$k\frac{\partial P(R,t)}{\partial t} + K^{(w)}R^{\beta}\left|\Omega_r(t)\right|^{\beta} P^{\alpha}(R,t) = \frac{2K^{(w)}}{R_2^2 - R_1^2}\left|\Omega_r(t)\right|^{\beta} \int_{R_1}^{R_2} R^{1+\beta} P^{\alpha}(R,t)dR + \frac{k}{\pi(R_2^2 - R_1^2)}\frac{dQ(t)}{dt}\ . \tag{8}$$

The friction torque moved by the clutch with a coefficient of friction $\mu(R, \Omega_r(t))$ between shields is as follows

$$M_{fr}(t) = \int_0^{2\pi} d\alpha \int_{R_1}^{R_2} \mu(R, \Omega_r(t))R^2 P(R,t)dR = 2\pi \int_{R_1}^{R_2} \mu(R, \Omega_r(t))R^2 P(R,t)dR\ . \tag{9}$$

3. Analytical Analysis

For $t = 0$ wear $U^{(w)}(R,0) = U_1^{(w)}(R,0) + U_2^{(w)}(R,0) = 0$, and according to equation (4) the relation

$$U^{(1)}(R,0) + U^{(2)}(R,0) = E(0) \tag{10}$$

doesn't depend on R and in consequence $P(R,0)$ it doesn't also depend on R . Taking into account this observation one obtains

$$Q(0) = 2\pi \int_{R_1}^{R_2} RP(R,0)dR \Rightarrow Q(0) = 2\pi P(R,0)\int_{R_1}^{R_2} RdR \Rightarrow Q(0) = 2\pi P(R,0)\frac{R_2^2 - R_1^2}{2}\ . \tag{11}$$

Then, on the basis of the relations (10) and (11) we have

$$P(R,0)=\frac{Q(0)}{\pi(R_2^2-R_1^2)}\,,\;\; E(0)=\frac{kQ(0)}{\pi(R_2^2-R_1^2)}\,. \tag{12}$$

On the basis of the equation (7) for $t=0$ and after the appropriate transformations we obtain

$$\frac{dE(0)}{dt}=\frac{2K^{(w)}}{R_2^2-R_1^2}\left|\Omega_r(0)\right|^{\beta}\frac{Q^{\alpha}(0)}{\pi^{\alpha}(R_2^2-R_1^2)^{\alpha}}\frac{R_2^{2+\beta}-R_1^{2+\beta}}{2+\beta}+\frac{k}{\pi(R_2^2-R_1^2)}\frac{dQ(0)}{dt}\,. \tag{13}$$

The friction torque moved by the clutch in the initial moment $t=0$ is as follows

$$M_{fr}(0)=2\pi P(R,0)\int_{R_1}^{R_2}\mu(R,\Omega_r(t))R^2dR\,. \tag{14}$$

For more further analytical analysis let us introduce the following simplifications: $Q(t)=Q=const.$, $\Omega_r(t)=\Omega_r=const.$, $\alpha=\beta=1$. Then applying the Laplace transformation to equation (8), and after the appropriate transformations we get

$$\overline{P}(R,s)=\left(ks+K^{(w)}R\left|\Omega_r\right|\right)^{-1}\left(\frac{2K^{(w)}}{R_2^2-R_1^2}\left|\Omega_r\right|\int_{R_1}^{R_2}R^2\overline{P}(R,s)dR+\frac{kQ}{\pi(R_2^2-R_1^2)}\right). \tag{15}$$

On the basis of the statement

$$P(R,\infty)=\lim s\overline{P}(R,s)\;\text{ for }\; s\to 0\,, \tag{16}$$

and after next transformations we get

$$P(R,\infty)=\frac{1}{R}\left(\frac{2}{R_2^2-R_1^2}\int_{R_1}^{R_2}R^2P(R,\infty)dR\right). \tag{17}$$

In what follows we are going to solve integral equation (17) cast into the form

$$P(R,\infty)=\frac{A}{R}\,. \tag{18}$$

We prove first, that expression (18) is a solution to equation (17):

$$\frac{A}{R}=\frac{1}{R}\left(\frac{2}{R_2^2-R_1^2}\int_{R_1}^{R_2}R^2\frac{A}{R}dR\right)=\frac{1}{R}\left(\frac{2}{R_2^2-R_1^2}A\int_{R_1}^{R_2}RdR\right)=\frac{1}{R}\left(\frac{2}{R_2^2-R_1^2}A\frac{R_2^2-R_1^2}{2}\right)=\frac{A}{R}\,. \tag{19}$$

We are aimed on constant A determination. Substituting expression (18) to equation (6), we obtain constant $A=Q/2\pi(R_2-R_1)$, and finally we get

$$P(R,\infty)=\frac{Q}{2\pi(R_2-R_1)}\frac{1}{R}\,. \tag{20}$$

Let us place now relation (20) into equation (7). After appropriate transformations we receive

$$\frac{dE(\infty)}{dt} = K^{(w)} \left| \Omega_r \right| \frac{Q}{2\pi (R_2 - R_1)} . \tag{21}$$

Described in the above analytical analysis allows to appoint contact pressure distribution $P(R,0)$, distance between pressed shields $E(0)$, and the speed of approaching of shields $dE(0)/dt$ in the initial moment $t = 0$. After accepting additional simplifications, we also estimated the contact pressure distribution $P(R,\infty)$, and the speed of approaching of shields $dE(\infty)/dt$. Initial wears in each points of the contact of shields are equal zero.

4. Non-Dimensional Form

Let us introduce the following similarity coefficients: t_*, Ω_*, Q_*, U_*, P_*, M_*, non-dimensional time $\tau = t/t_*$, non-dimensional geometrical parameter $r = (R - R_1)/(R_2 - R_1)$, and other non-dimensional parameters: $l_1 = K^{(w)} R_2^{\beta} t_* \Omega_*^{\beta} P_*^{\alpha-1} \big/ (k(1+\rho)^{\beta})$, $l_2 = Q_*(1+\rho)^2 \big/ (\pi P_* R_2^2 (1+2\rho))$,
$k_1^{(w)} = K_1^{(w)} R_2^{\beta} t_* \Omega_*^{\beta} P_*^{\alpha} \big/ (U_*(1+\rho)^{\beta})$, $k_2^{(w)} = K_2^{(w)} R_2^{\beta} t_* \Omega_*^{\beta} P_*^{\alpha} \big/ (U_*(1+\rho)^{\beta})$,
$k^{(w)} = k_1^{(w)} + k_2^{(w)}$, $k_{fr} = 2\pi R_2^3 P_* \big/ (M_*(1+\rho)^3)$, $\rho = R_1/(R_2 - R_1)$, and finally the following non-dimensional functions: $p(r,\tau) = P((R_2 - R_1)(r+\rho), t_*\tau)/P_*$, $q(\tau) = Q(t_*\tau)/Q_*$,
$F_{fr}(\tau) = M_{fr}(t_*\tau)/M_*$, $f(r,\omega_r(\tau)) = \mu((R_2 - R_1)(r+\rho), \Omega_*\omega_r(\tau))$, $\omega_r(\tau) = \Omega_r(t_*\tau)/\Omega_*$,
$u_1^{(w)}(r,\tau) = U_1^{(w)}((R_2 - R_1)(r+\rho), t_*\tau)/U_*$, $u_2^{(w)}(r,\tau) = U_2^{(w)}((R_2 - R_1)(r+\rho), t_*\tau)/U_*$.
Then, we can obtain the associated non-dimensional relations

$$\frac{\partial p(r,\tau)}{\partial \tau} + l_1 \left| \omega_r(\tau) \right|^{\beta} (r+\rho)^{\beta} p^{\alpha}(r,\tau) = \frac{2 l_1}{(1+2\rho)} \left| \omega_r(\tau) \right|^{\beta} \int_0^1 (r+\rho)^{1+\beta} p^{\alpha}(r,\tau) dr + l_2 \frac{dq(\tau)}{d\tau} , \tag{22}$$

$$F_{fr}(\tau) = k_{fr} \int_0^1 f(r, \omega_r(\tau))(r+\rho)^2 p(r,\tau) dr , \tag{23}$$

$$\frac{\partial u_1^{(w)}(r,\tau)}{\partial \tau} = k_1^{(w)} \left| \omega_r(\tau) \right|^{\beta} (r+\rho)^{\beta} p^{\alpha}(r,\tau) , \quad \frac{\partial u_2^{(w)}(r,\tau)}{\partial \tau} = k_2^{(w)} \left| \omega_r(\tau) \right|^{\beta} (r+\rho)^{\beta} p^{\alpha}(r,\tau) , \tag{24}$$

$$p(r,0) = \frac{Q_*(1+\rho)^2 q(0)}{\pi P_* R_2^2 (1+2\rho)} , \quad \eta(0) = \frac{Q_* k (1+\rho)^2 q(0)}{\pi U_* R_2^2 (1+2\rho)} . \tag{25}$$

5. Numerical Computations

In this section, we present numerical computations applied to solve equations (22) - (24). For solving these equations, we divided the length of non-dimensional radius r on m even segments by taking $\Delta_r = 1/m$, $r_i = \Delta_r i$, $p(r,\tau) = p_i(\tau)$, $u_1^{(w)}(r,\tau) = u_{1i}^{(w)}(\tau)$, and $u_2^{(w)}(r,\tau) = u_{2i}^{(w)}(\tau)$. Integrals appearing in equations (22) and (23) are replaced with the sum using the method of trapezia (rates of the method are: $a_0 = a_m = 1/2$, $a_i = 1$, $i = 1,...,m-1$). In this way we yield obtain system of $3(m+1)$ ordinary first order differential equations. Solution to equations yield also friction torque moved by the clutch:

$$\begin{cases} \dfrac{dp_i(\tau)}{d\tau} = -l_1 |\omega_r(\tau)|^\beta (r_i+\rho)^\beta p_i^\alpha(\tau) + \dfrac{2l_1}{(1+2\rho)} |\omega_r(\tau)|^\beta \Delta_r \sum_{j=0}^{m} a_j (r_j+\rho)^{1+\beta} p_i^\alpha(\tau) + l_2 \dfrac{dq(\tau)}{d\tau}, \\ \dfrac{du_{1i}^{(w)}(\tau)}{d\tau} = k_1^{(w)} |\omega_r(\tau)|^\beta (r_i+\rho)^\beta p_i^\alpha(\tau), \\ \dfrac{du_{2i}^{(w)}(\tau)}{d\tau} = k_2^{(w)} |\omega_r(\tau)|^\beta (r_i+\rho)^\beta p_i^\alpha(\tau), \\ F_{fr}(\tau) = k_{fr} \Delta_r \sum_{j=0}^{m} a_j f(r_j, \omega_r(\tau))(r_j+\rho)^2 p_i^\alpha(\tau). \end{cases} . \quad (26)$$

6. Numerical and Analytical Results

Numerical calculations are carried out using the fourth order Runge-Kutta method with constant time step. We assumed the following values of similarity coefficients: $\Omega_* = \Omega_r$, $t_* = 1/\Omega_*$, $Q_* = Q(0)$, $U_* = E(0)$, $P_* = P(R,0)$, $M_* = M_{fr}(0)$, and the following initial non-dimensional parameters: $\omega_r(\tau) = 0.2$, $\rho = 0.2$, $l_1 = 0.5$, $k^{(w)} = 0.1$, $k_{fr} = 2$, $f(r, \omega_r(\tau)) = 0.1$, $\alpha = 1$, $\beta = 1$, $m = 100$.

Let us study first contact pressure distributions (Figure 2) as the function of the non-dimensional radius r of shields for different values of the β parameter. In the initial moment $\tau = 0$, before beginning of the process of wearing of shields, the contact pressure distribution $p(r,0)$ is identical on the entire contact surface. However, contact pressure distributions $p(r,\infty)$ are different for different values of the β parameter. Parameter α doesn't influence on the contact pressure distribution, however it has influence on the speed of settling of this distribution. For cases of the contact pressure distributions $p(r,0)$ and $p(r,\infty)$, and for the $\beta = 1$ numerical calculations overlaps with theoretical results. Figure 3 shows relations of the friction torque moved by the clutch for different values of the geometrical parameter ρ. As can be seen, amendments of the contact pressure distribution during the wear causing reducing of the friction torque moved by the clutch.

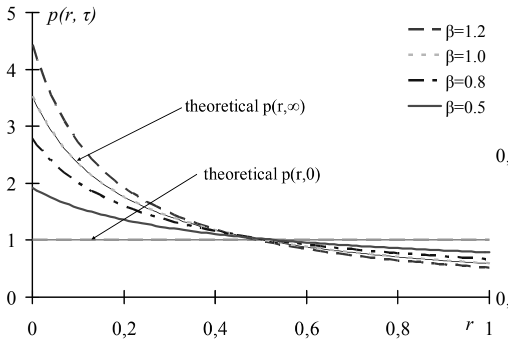

Fig. 2. Contact pressures in equilibrium.

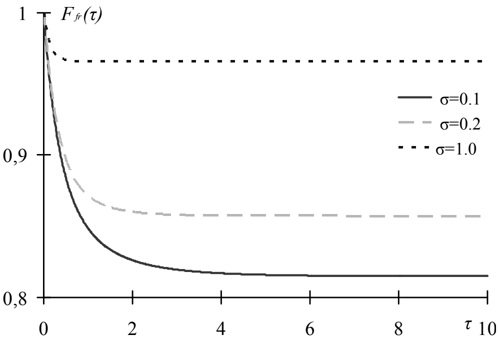

Fig. 3. Changes of the friction torque.

Figure 4 shows contact pressure distributions in the equilibrium, and Figure 5 shows wear distributions in the transient state (here $\tau = 10$) for different values of the geometrical parameter ρ. From the reported relations one may see, that these distributions depend on this parameter, which determines the geometry of the considered system. For smaller values of the ρ parameter differences of the values of both contact pressure and wear between contact surface borders ($r = 0$ and $r = 1$) are bigger.

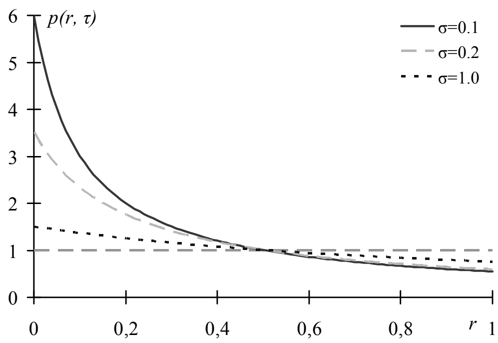

Fig. 4. Contact pressures in equilibrium.

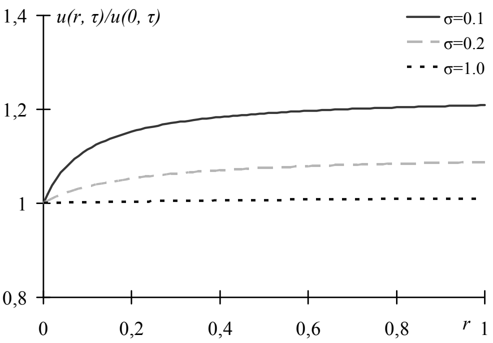

Fig. 5. Wears in transient state.

Figure 6 shows contact pressure distributions, whereas Figure 7 shows wear distributions in the transient state (here $\tau = 10$) for different values of the parameter β. From described relations, we can see, that these distributions depend on this parameter. For larger values of the β parameter differences of the values of contact pressure between contact surface borders ($r = 0$ and $r = 1$) are bigger, and differences of the values of wear between contact surface borders for greater values of the β parameter are smaller.

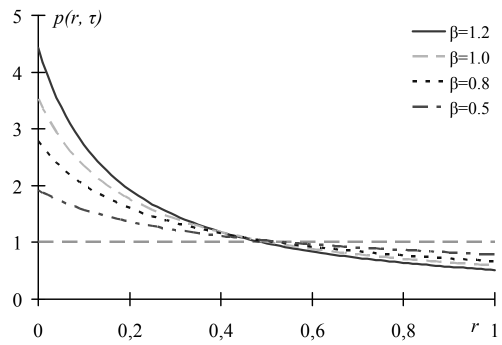

Fig. 6. Contact pressures in transient state.

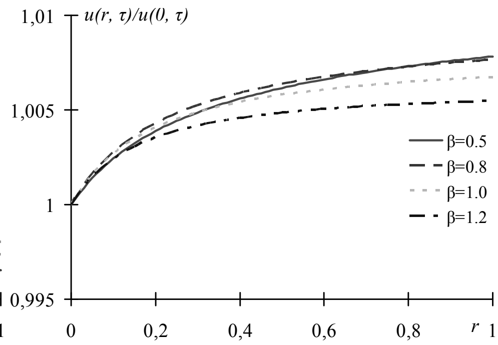

Fig. 7. Wears in transient state.

7. Conclusions

The considered in this work issues allow to modeling wear processes on the contact surface of a mechanical friction clutch. Unlike many previous works, here friction clutch was treated as a friction connection of elastic bodies, and hence an effect of wear and elasticity (flexibility) of material of contacting shields was considered. Here general non-linear differential model of wear is applied, where wear is modeled via non-linear function of contact pressure and relative sliding velocity. The presented modeling allows considering wear in non-stationary conditions and allows for obtaining real contact pressure distribution on the contact surface.

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